“Rational Appeasement,” Daniel Treisman,

International Organization.

Proofs of Propositions 4 and 5

Proposition 4: In the game with endogenous stakes, a deterrence equilibrium can exist only if the center’s fixed cost of fighting, k_c, is in the range: $\frac{\bar{t}}{a} - \frac{ap\,k_2}{a-p} \le k_c \le \min\,[\frac{p\,k_2}{a-p}, \frac{\bar{t}}{a}]$.

Proof: Denote the equilibrium value of t_1 “t_1*” and the equilibrium values of t_2 on the top, middle and bottom branches “t_2^t”, “t_2^m” and “t_2^b” respectively. In an on-equilibrium path DE, L_1 challenges, both strong and weak C’s fight and set strictly positive t_2’s, and L_2 acquiesces. Working backwards, L_2 only acquiesces at h_1 if $t_2^t \ge a\,k_c$ and

$-t_2^t \ge m_1(-t_2^t / a - k_2) \Leftrightarrow t_2^t \le \frac{am_1 k_2}{a - m_1}$. In equilibrium, I will show that both C’s play identically up to time 4, so the only consistent belief at h_1 is $m_1 = p$; the previous condition becomes $t_2^t \le \frac{ap\,k_2}{a-p}$. If both C’s set $t_1 = t_1$ *, and given that both play F at time 2, the weak C will set $t_2 = t_2^t$ if the strong C does (rather than reveal himself to be weak prompting a challenge). The strong C can never benefit by setting $t_2 < t_2^t$. Might he set $t_2 > t_2^t$ (in which case, he will optimize by setting $t_2 = \bar{t}$)? For a sequential equilibrium we just need to show that there is *some* belief such that he prefers not to deviate. We are interested throughout in defining the broadest set of parameter values such that an equilibrium can exist. It can be checked that the belief that supports a DE under the broadest range of $t_2^{\,t}$ is that any C who sets $t_2 \neq t_2^{\,t}$ C is weak. Given this belief, a DE exists if $t_1 + \frac{\overline{t}}{a} - 2k_c \leq t_1 + t_2^{\,t} - k_c \Leftrightarrow t_2^{\,t} \geq \frac{\overline{t}}{a} - k_c$.

In equilibrium, a weak C will always fight at time 2 if the strong C does (rather than reveal herself weak and get a maximum payoff of zero). For the strong C to fight at time 2, there must be some belief associated with A such that he prefers F. The belief that yields the broadest range of $t_2^{\,t}$ in DE is that any C who plays A is weak. This would support a DE so long as $\overline{t} - k_c \leq t_1 + t_2^{\,t} - k_c \Leftrightarrow t_1 + t_2^{\,t} \geq \overline{t}$. (If C deviates he will optimize by setting $t_2 = \overline{t}$.)

Given that both C's set $t_1 = t_1{*}$ in equilibrium, L_1's belief at time 1 must be the prior, p. For L_1 to play c, $p(-t_1{*} - k_1) + (1-p)(-k_1) \geq -t_1{*} \Leftrightarrow t_1{*} \geq \frac{k_1}{1-p}$. If the strong C sets $t_1 = t_1{*}$, the weak one will do the same since his payoff from this is $t_2^{\,t} - k_c > 0$ (recall that $a\,k_c \leq t_2^{\,t}$), while if he deviates and reveals himself to be weak, his highest payoff is 0. For the strong C to prefer to set $t_1 = t_1{*}$, there must be some belief that makes deviation unattractive. The belief that yields the broadest condition for equilibrium is that any C who deviates is weak. The strong C will prefer not to deviate only if the equilibrium payoff $t_1{*} + t_2^{\,t} - k_c \geq \overline{t} + \frac{\overline{t}}{a} - 2k_c \Leftrightarrow t_1{*} + t_2^{\,t} \geq \overline{t} + \frac{\overline{t}}{a} - k_c$. (If she deviates, she will optimize by setting $t_1 = t_2 = \overline{t}$, and both local actors will challenge.) So, for an on-equilibrium-path DE, we have the following necessary conditions: $t_2^{\,t} \geq a\,k_c$; $t_2^{\,t} \leq \frac{ap\,k_2}{a-p}$; $t_1 + t_2^{\,t} \geq \overline{t}$; $t_1{*} + t_2^{\,t} \geq \overline{t} + \frac{\overline{t}}{a} - k_c$; and $t_1{*} \geq \frac{k_1}{1-p}$, which together with $t_1{*} \leq \overline{t}$ and $t_2^{\,t} \leq \overline{t}$ imply:

$$\max\left[\frac{\overline{t}}{a} - \frac{ap\,k_2}{a-p},\ \frac{\overline{t}}{a} - \overline{t}\right] \leq k_c \leq \min\left[\frac{\overline{t}}{a},\ \frac{p\,k_2}{a-p}\right] \text{ and } k_1 \leq \overline{t}(1-p).$$

In an off-equilibrium-path DE, L_1 acquiesces because she correctly believes that if she challenged, both strong and weak C's would fight. Note that for both C's to play F at time 2 off the equilibrium path, both C's equilibrium strategies must set the same t_1* and t_2^t. Otherwise, the weak C reveals himself to be weak, L_2 would challenge, and the weak C would rather have appeased at time 2. Given this, at h_1 the only consistent belief for L_2 is the prior, $m_1 = p$, and the only consistent belief for L_1 at time 1 is also p. L_1's expected payoff from playing c is $p(-t_1 - k_1) + (1-p)(-k_1)$ and her payoff from playing a is $-t_1$. Thus, for her to play a, $t_1* \leq \frac{k_1}{1-p}$.

Given that both C's would fight off the equilibrium path at time 2, the weak C will set t_2 at the same level as the strong C to avoid revealing his weakness. The weak C will only fight at time 2 as required if L_2 will acquiesce. (If L_2 would challenge, the weak C would be better off playing A, even if this prompts a challenge.) L_2 will only acquiesce if

$m_1(-\frac{t_2^t}{a} - k_2) \leq -t_2^t \Leftrightarrow t_2^t \leq \frac{am_1k_2}{a-m_1} = \frac{apk_2}{a-p}$, and $t_2^t \geq a\,k_c$. Would the strong C set a

$t_2 = t_2^t \leq \frac{apk_2}{a-p}$? If not, an off-equilibrium-path DE cannot exist because the weak C would never fight. For a strong C to prefer a $t_2 = t_2^t$, there must be some belief associated with setting $t_2 > t_2^t$ that makes this unattractive. The belief that yields the broadest range of t_2^t is that any C who sets $t_2 > t_2^t$ is weak. Then C_S will prefer to set $t_2 = t_2^t \leq \frac{apk_2}{a-p}$ so long as

$t_1 * + t_2^t - k_c \geq t_1 * + \frac{\bar{t}}{a} - 2k_c \Leftrightarrow t_2^t \geq \frac{\bar{t}}{a} - k_c$. For the strong C to play F at time 2 off the equilibrium path, there must be a belief associated with A that makes him prefer F. The belief that supports the broadest range of t_2^s is that any C who plays A is weak. Then the strong C will prefer F only if $t_1{*}+t_2^{\,t}-k_c \geq \bar{t}-k_c \Leftrightarrow t_1{*}+t_2^{\,t} \geq \bar{t}$. If the strong *C* plays F at time 2, the weak *C* will prefer to do the same rather than reveal his weakness.

For the strong *C* to set $t_1 = t_1{*}$ there must be some belief that makes deviating unattractive. The belief that yields the loosest possible constraint is that any *C* who deviates is weak. Given this, C_S will only set $t_1 = t_1{*}$ if $t_1{*}+\max[\frac{p\,k_2}{1-p},\ \bar{t}-k_c] \geq \bar{t}+\frac{\bar{t}}{a}-2k_c$. The left-hand side is C_S's maximum payoff given that L_1 plays a ($\frac{p\,k_2}{1-p}$ is the maximum if C_W pools with C_S, $\bar{t}-k_c$ is the maximum if he does not); the right hand-side is her maximum payoff given that she sets $t_1 > t_1{*}$, prompting the belief she is weak.

So, for an off-equilibrium path DE, the following conditions are necessary:

$t_1{*} \leq \frac{k_1}{1-p};\ t_2^{\,t} \leq \frac{ap\,k_2}{a-p};\ t_2^{\,t} \geq a\,k_c;\ t_2^{\,t} \geq \frac{\bar{t}}{a}-k_c;$

$t_1{*}+t_2^{\,t} \geq \bar{t};$ and $t_1{*}+\max[\frac{p\,k_2}{1-p},\ \bar{t}-k_c] \geq \bar{t}+\frac{\bar{t}}{a}-2k_c$. Together with

$t_1{*} \leq \bar{t}$ and $t_2^{\,t} \leq \bar{t}$, these imply: $\bar{t}-\frac{ap\,k_2}{a-p} \leq \frac{k_1}{1-p}$, $\frac{\bar{t}}{a}-\frac{ap\,k_2}{a-p} \leq k_c \leq \min[\frac{p\,k_2}{a-p},\frac{\bar{t}}{a}]$ and

either $k_c > \min[\bar{t}-\frac{p\,k_2}{1-p},\frac{\bar{t}}{2a}-\frac{p\,k_2}{2(1-p)}]$ or $k_c < \bar{t}-\frac{p\,k_2}{1-p}$ and either

$\frac{1}{2}[\bar{t}+\frac{\bar{t}}{a}-\frac{p\,k_2}{1-p}-\frac{k_1}{1-p}] \leq k_c$ and $k_c > \bar{t}-\frac{p\,k_2}{1-p}$ or $\frac{\bar{t}}{a}-\frac{k_1}{1-p} \leq k_c < \bar{t}-\frac{p\,k_2}{1-p}$. So, for

both on- and off-equilibrium path deterrence equilibria, $\frac{\bar{t}}{a}-\frac{ap\,k_2}{a-p} \leq k_c \leq \min[\frac{p\,k_2}{a-p},\frac{\bar{t}}{a}]$ is necessary (though not sufficient). Q.E.D.

Proposition 5: At least one appeasement equilibrium exists if $\bar{t} - \frac{p\,k_2}{1-p} \le k_c \le \frac{p\,k_2}{1-p}$ and either

$\max[\frac{\bar{t}}{2a}, \ \frac{\bar{t}}{2a}(1+a) - \frac{p\,k_2}{2(1-p)}] \le k_c < \frac{\bar{t}}{a}$, or $k_c \ge \frac{\bar{t}}{a}$.

Proof: No off-equilibrium-path appeasement equilibria exist for the reason given in the text. In an on-equilibrium-path appeasement equilibrium, L_2 must choose a at h_2, both types of *C* must choose A at time 2, $t_2^m > 0$, and L_1 must choose c. Using the same notation as before, for L_2 to choose a at h_2, $-t_2^m \ge m_2(-t_2^m - k_2) \Leftrightarrow t_2^m \le \frac{m_2 k_2}{1-m_2}$, and $t_2^m \ge k_c$. In equilibrium, I will show that both types of *C* play identically up to time 4, so the belief at h_2 must be $m_2 = p$. Thus, $t_2^m \le \frac{m_2 k_2}{1-m_2}$ becomes $t_2^m \le \frac{p\,k_2}{1-p}$. For a strong *C* to set $t_2 = t_2^m$, there must be some belief associated with $t_2 \ne t_2^m$ that makes deviation unattractive. It can be checked that the belief that will yield the broadest range of possible equilibirum t_2^m's is that any *C* who sets $t_2 \ne t_2^m$ is weak. Then, C_S will prefer not to deviate only if $t_2^m \ge \bar{t} - k_c$ (assuming if he deviates he optimizes and sets $t_2 = \bar{t}$). If C_S sets $t_2 = t_2^m$, C_W will prefer to do the same rather than prompt the belief he is weak and get a payoff of zero. For the strong *C* to prefer to play A at time 2, there must be a belief associated with playing F that makes him prefer A. The belief that will yield the broadest range of possible equilibrium t_2^m's is that any *C* who plays F is weak. Given this, the strong *C* prefers A only if $t_2^m \ge \max[t_1 + \frac{\bar{t}}{a} - 2k_c, t_1 - k_c]$. But note that $t_2^m \ge t_1 - k_c$ is implied by $t_2^m \ge \bar{t} - k_c$ and $t_1 = \bar{t}$, required already, so this condition becomes:

$t_2^m \geq t_1 + \frac{\overline{t}}{a} - 2k_c$ if $\overline{t} > a\,k_c$. ($\overline{t} > a\,k_c$ is required for $t_1 + \frac{\overline{t}}{a} - 2k_c > t_1 - k_c$.) C_W will also prefer to play A here if C_S does, rather than prompt a challenge.

Given that both C's play A at time 2, L_1 will only play c if $t_1^* > 0$. For the strong C to prefer not to set $t_1 \neq t_1^*$, there must be a belief associated with this that makes deviation unattractive. The belief that yields the broadest conditions on t_1^* is that any C who deviates from $t_1 = t_1^*$ is weak. Suppose C_S deviates to some $t_1 > 0$ (in which case, I assume he optimizes by setting $t_1 = \overline{t}$). The greatest payoff C_S can get is $\max[\overline{t} + \frac{\overline{t}}{a} - 2k_c, \overline{t} - k_c]$. So for him not to deviate in this way, $\max[\overline{t} + \frac{\overline{t}}{a} - 2k_c, \overline{t} - k_c] \leq t_2^m$, which given $t_2^m \geq \overline{t} - k_c$ and $t_1 \leq \overline{t}$, reduces to $t_2^m \geq \overline{t} + \frac{\overline{t}}{a} - 2k_c$ if $\overline{t} > a\,k_c$.[1] Given that the strong C sets $t_1 = t_1^*$, the weak C will do the same rather than be believed weak. So, necessary conditions for an AE are:

$t_2^m \geq k_c$; $t_2^m \leq \frac{p\,k_2}{1-p}$; $t_2^m \geq \overline{t} - k_c$; $t_1^* > 0$; and $t_2^m \geq \overline{t} + \frac{\overline{t}}{a} - 2k_c$ if $\overline{t} > a\,k_c$. Together with $t_2^m \leq \overline{t}$ and $t_1^* \leq \overline{t}$, these imply: $\overline{t} - \frac{p\,k_2}{1-p} \leq k_c \leq \frac{p\,k_2}{1-p}$ and either $\frac{\overline{t}}{2a} \leq k_c < \frac{\overline{t}}{a}$ and $k_c \geq \frac{\overline{t}}{2a}(1+a) - \frac{p\,k_2}{2(1-p)}$ or $k_c \geq \frac{\overline{t}}{a}$. Q.E.D.

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[1] We have assumed $t_1 > 0$, but in fact this could be derived as follows. Suppose C_S deviates to $t_1 < 0$. L_1 would acquiesce, C would set $t_2 = \overline{t}$, L_2 would challenge, yielding a payoff to C of $\max\,[\overline{t} - k_c + t_1,\ t_1]$, which is never greater than the strong C's equilibrium payoff of $t_2^m > \overline{t} - k_c$. (Recall that in equilibrium $k_c \leq \overline{t}$.) So he would never set $t_1 < 0$.