Chapter 8

Prescriptive Marriage Systems and Permutation Matrices

Matrix Inverses and the Identity Matrix

1x = x1 = x for every number x in ordinary arithmetic. There is a similar matrix, called an *identity matrix,* that has the same property as 1. Multiplication by an identity matrix leaves the other matrix unchanged. Identity matrices have ones on the main diagonal and zeros everywhere else.

In ordinary arithmetic, all numbers except zero have inverses. For any non-zero number x, $xx^{-1} = x^{-1}x = 1$. Many but not all square matrices have inverses also. For matrices with inverses, $AA^{-1} = A^{-1}A = I$, where I is an identity matrix. t

Here is what a 4 by 4 identity matrix looks like.

$$I := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here is an arbitrary matrix A.

$$A := \begin{bmatrix} 2 & 2 & 5 & 1 \\ -1 & 7 & 2 & 6 \\ 7 & -4 & 0 & -3 \\ 0 & 2 & 2 & 0 \end{bmatrix}$$

I is like a one in arithmetic multiplication.

$$A \cdot I = \begin{bmatrix} 2 & 2 & 5 & 1 \\ -1 & 7 & 2 & 6 \\ 7 & -4 & 0 & -3 \\ 0 & 2 & 2 & 0 \end{bmatrix}$$

$$I \cdot A = \begin{bmatrix} 2 & 2 & 5 & 1 \\ -1 & 7 & 2 & 6 \\ 7 & -4 & 0 & -3 \\ 0 & 2 & 2 & 0 \end{bmatrix}$$

This is the inverse of A:

$$A^{-1} = \begin{bmatrix} -0.069 & 0.1 & 0.177 & 0.073 \\ -0.3 & 0.1 & 0.1 & 0.65 \\ 0.3 & -0.1 & -0.1 & -0.15 \\ 0.238 & 0.1 & -0.054 & -0.696 \end{bmatrix}$$

A matrix times its inverse equals the identity matrix.

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculating the inverses of matrices can be quite laborious. I will ask you to be able to compute the inverse of two by two matrices, matrices with two rows and columns.

=

$$\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}^{-1}$$

$$\frac{1}{(a11 \cdot a22 - a21 \cdot a12)} \cdot \begin{bmatrix} a22 & -a21 \\ -a12 & a11 \end{bmatrix}$$

The matrix does not have an inverse if the denominator of the fraction equals zero.

Permutation Matrices

Permutation matrices are square matrices consisting of 1's and 0's in which there is exactly one 1 in every row and column. The following, for example, are three by three permutation matrices.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

All permutation matrices have inverses. They are simply the transposes of the permutation matrices. If P is a permutation matrix, then $P^{-1} = P^t$.

Another feature of permutation matrices is that the product of two permutation matrices is always a permutation matrix. For example, consider the second and third permutation matrix above. Their product is the sixth permutation matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Definition: A *permutation* is an ordered arrangement of the members of a set.

For example, consider all the different ways of ordering the members of the set {a, b, c}. There are six different orders.

abc
acb
bac
bca
cab
cba

For any S, there are |S|! permutations. As was mentioned in the first chapter, |S| is the number of elements in set S. The "!" symbol is read "factorial" and refers to the product of all the numbers less than the preceeding number and greater than zero. For example, 3! = 3x2x1, 4! = 4x3x2x1, and so on.

Permutation matrices correspond to permutations. Each permutation matrix produces a permutation in the following way. Suppose that three members of a set {a,b,c} are ordered (a, b, c). Then

$$= (b \quad c \quad a)$$

$$(a \quad b \quad c) \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The effect of this permutation matrix is to change the order so that the first element becomes third, the second becomes first, and the third becomes second.

If this permutation is applied the second time, to the new permutation (a,b,c)P, then it still has the same effect, moving the first element to third place, etc.

$$(b\ \ c\ \ a)\cdot\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = (c\ \ a\ \ b)$$

$$(a\ \ b\ \ c)\cdot\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^2 = (c\ \ a\ \ b)$$

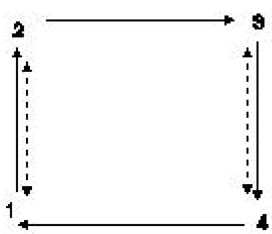
The general rule is as follows: If A and B are permutation matrices, then AB is a permutation whose effect is the result of applying A and then applying B to the altered order produced by A.

Permutation matrices are also incidence matrices and can be represented by graphs. Suppose we have the following two permutations on sets of size 4.

$$B := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

These two permutations can be represented by the following graphs. The graph for A is in solid lines and for B in dotted lines.



We can read from these graph what the effect of AB is. For example, A moves what is first place to second place and B moves what is in second place to first place. So the effect of AB must to leave what is in first place unchanged. A moves what is in second place to third place and B moves what is in third place to fourth place. So, the effect of AB must move what in second place to forth place. We can also show this by looking at the matrix AB.

$$A \cdot B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Kinship Relations Expressed Through Matrices

Kinship relations can be quite complex, and yet we seem to be able to master them without too much difficulty. Kinship terms inevitably group together different individuals with different biological relations to an individual. For example, the following individuals each have a different biological relationship to "ego" (the person from whose perspective we are examining the kinship system).They are all "uncles."

1. Ego's mother's brother
2. Ego's father's brother
3. Ego's mother's sister's husband
4. Ego's father's sister's husband

The following individuals may both be referred to as "grandfather."

1. Ego's mother's father
2. Ego's father's father

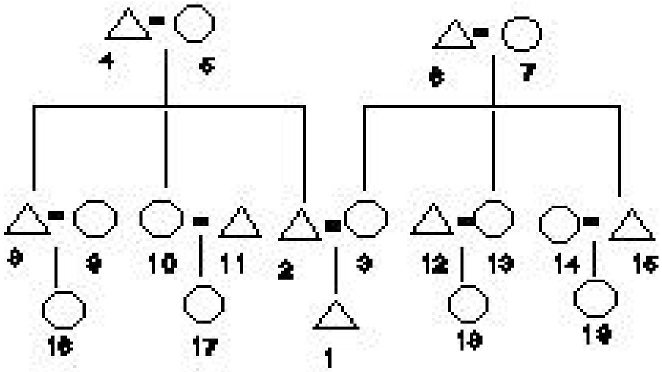
The following may both be referred to as "brother in law"

1. Ego's sister's husband
2. Ego's wife's brother
3. Ego's wife's sister's husband

Notice that in all these instances, kinship terms classify people in the same category who have quite a different relationship to ego. There is no obvious reason why three different types of relations should all be classified together as brother-in-law. Ego is biologically related to the children of his sister and her husband but unrelated to the children of his wife's brother and his wife.

Before we go on, I want to introduce a standard diagram for biological kinship relations, a kinship "tree."

Figure 1



In the types of diagrams represented by Figure 1, males are triangles and females are circles. The short horizontal lines connecting males and females are marriages. Vertical lines connect a married couple to their children. This diagram shows the parents and grandparents of a male child, his uncles and aunts, and his female first cousins.

These diagrams are graphs representing two different kinds of relationships. One is the parent-child relation and the other is the marriage relation. Let's define aCb if person a is the child of person b and aMb if person a is married to person b. Neither relation is reflexive and the M relation is symmetric.

We could also represent these two relations as matrices. Let C_{ij} = 1 if j is the child of i. The transpose of C, C^t, is a matrix of parenthood; C^t_{ij} = 1 if j is the parent of i. Let W_{ij} = 1 if i is married to j.

Other relationships can be represented as products of these matrices. For example, C^2 gives paths of length 2 with

respect to the "child of" relation. Therefore, C^2 gives the relation "grandchild of". C^2_{ij} = 1 if i is the grandchild of j. S= C^tC will show which pairs are siblings. S_{ij} = 2 if i and j share two parents. For half-siblings, S_{ij} = 1. The matrix A = C^tC^tC will show uncles and aunts.

The products of these matrices represent movement in kinship diagrams like Figure 1. The C matrix corresponds to paths from a parent to a child. C^2 corresponds to a path of length 2 from a grandparent to a grandchild. C^t corresponds to vertical paths up in the diagram. W corresponds to horizontal paths from men to women. W^t

corresponds to horizontal paths from women to men.

Kinship in other societies

Anthropologists have studied what they call clasificatory kinship systems, which are based on the dentification of biological kinship with social kinship and clans in interesting ways. A sociologist, Harrison White, was the first to give a complete mathematical description of these kinship systems. White described marriage systems that had the following eight properties.

1. The entire population of the society is divided into mutually exclusive groups, which we will call ***clans***. A person spends his or her whole life in the same clan. We will assume that there are n such clans.

2. There is a rule fixing the single clan among whose women the men of a given clan must find their wives. In other words, all the men in a given clan must find their wives in one clan.

3. Men from two different clans cannot marry women of the same clan.

4. All children of a couple are assigned to a single clan, uniquely determined by the clans of their mother and father.

5. Children whose fathers are in different clans must themselves be in different clans. In other words, all the children in a given clan have fathers in the same clan.

6. A man can never marry a woman of his own clan. A man can not only not marry his sister. He can't marry anyone in his sister's (and his own) clan.

7. Every person in the same society has some relative by marriage and descent in each other clan: i.e., the society is not split into groups not related to each other.

8. Whether two people who are related are in the same clan depends only on the kind of relationship, not on the clan either one belongs to. In other words, if some fathers and sons are in the same clan, then all fathers and sons are.

Or, if some men and their mother's brother's daughters are in the same clan, then this is true for all men and their mother's brother's daughters.

Matrix Representations of Kinship Rules

According to properties 3 and 4, there is a one-to-one correspondence between the husband's and wife's clan in these marriage systems. The men in each clan can marry women from only one of the other clans, and visa versa. This correspondence can be represented by a matrix.

Example 1

Let's take an example, the Kariera tribe in Western Australia. The rows of the following W matrix will represent the husbands and the columns the wives. The four clans are the Banaka, Burang, Palyeri, and Karimera.

$$W := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

This matrix tells us that Banaka men marry Burung Women, Burung men marry Banaka women, Palyeri men marry Karimera women, and Karimera men marry Palyeri women. The transpose of this matrix, which is also its inverse, gives us the clans of wives' husbands.

Properties 4 and 5 say that there is also a one-to-one correspondence between the clans of fathers and children, and this also can be represented by a matrix C.

$$C := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

These matrices are consistent with the eight properties. In any W matrix, there cannot be any 1s on the main diagonal because men and women of the same clan cannot marry (property 6). There is only one "1" in each row of W because of property 2. There must be only one "1" in each column of W because of property 3. Similarly, properties 4 and 5 guarantee that matrix C must have one "1" in each row and column.

Example 2

The following pair of W and C matrices also satisfy all the eight properties

$$W := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Each of these matrices also have one 1 in each row and column and the W matrix has no 1s on the main diagonal. The transpose (and inverse) of W gives the husband's clan. $C^t = C^{-1}$ gives the father's clan.

The following W and C matrices violate one or more of the eight properties.

$$W := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$W := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$W := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The first W matrix violates assumption 3 and assumption 6; men in clans 1 and 2 marry women from the same clan, and men in clan 2 marry women in their own clan. The second C matrix violates assumption 8; fathers in clan

3 belong to the same clan as their children, but this is not true for fathers in clans 1 and 2.

The third pair of W and C matrices violates assumption 7. In this system, no members of clans 1 and 2 will have any relatives in clans 3 and 4, and visa versa. For example, a man in clan 1 will have a wife in clan 2. Her father will be in clan 2 and her mother will be in clan 1. The man's children will be in clan 1. His sons will marry women in clan 2 and his daughters will marry men in clan 2. The children of his sons will be in clan 1 and the children of his daughters will be in clan 2. None of his relatives will be in clans 3 or 4.

All of the assumptions were expressed in terms of men and fathers, but this is arbitrary. We could have just as easily described things from a female point of view. The postulates would then read as follows:

1. same

2. There is a rule fixing the single clan among whose men the women of a given clan must find their husbands. In other words, all the women in a given clan must find their husbands in one other clan.

3. Women from two different clans cannot marry men of the same clan.

4. same.

5. Children whose mothers are in different clans must themselves be in different clans. In other words, all the children in a given clan have mothers in the same clan.

6. A woman can never marry a man of her own clan. A woman cannot only not marry her brother. She can't marry anyone in her brother's (and her own) clan.

7. same.

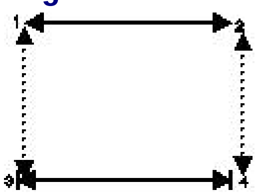
8. same

From the female point of view, we could use a H (husband) matrix giving the clan of the husband for women in each clan. This H matrix is just the transpose of the W matrix. We could also describe a different C matrix in which the clan of the children are given as a function of the clan of the wife. For consistency, we will use the male-centric W and C matrices.

Diagrams of Kinship Relations

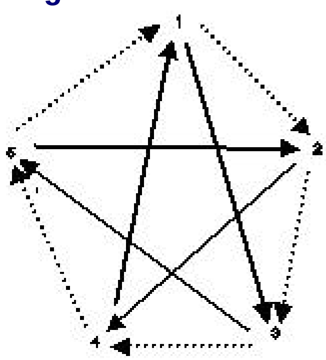
These matrices correspond to graphs of the marriage and kinship relations between clans. Let a solid line represent the marriage relation (where men find their wives) and a broken line the clan of each father's children. Then the diagram for the Kariera tribe is as follows.

Figure 2



For the second example, the diagram would look as follows:

Figure 3



Permutation Matrices

A square matrix with one 1 in every row and column and zeros elsewhere is called a *permutation matrix*.   A permutation is a one-to-one mapping of the elements of a set into each other. The diagrams in the last section are permutations because each clan is associated with just one other clan by the marriage and descent clan rules. This corresponds to the fact that there is exactly one solid (for clan of wife) and one broken (for clan of child) line from each of the clan-vertices. No diagram with more than one line (of a certain type) coming from a point can represent a permutation.

Second, the product of two permutation matrices is also a permutation matrix. In earlier chapters we learned that the products of matrices corresponded to paths in a graph. The product of two marriage-descent matrices gives the clan memberships of more distant connections.

For example, C gives the father's children's clan. C^2 gives the clan of the sons of his sons. C^3 gives the clan of the sons of his sons of his sons. In example 2, the child of a father in clan 1 is in clan 2, and the child of a father in clan 2 is in clan 3. Therefore, a man's son's children are in clan 3 if he is in clan 1. We can verify this by looking at C^2.

$$C := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$C_{13} = 1$, as we expect.

The transpose of C, which is also the inverse of C, gives the clan of the father as a function of the clan of the child. Therefore, $C^{-1}C^{-1} = C^{-2}$ gives the clan of a man's father's father.
The (3,1) element is one, as expected; men in clan 3 have father's fathers in clan 1.

$$C^{-2} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Every kinship relation can be represented by products of matrices. For example, the clan of a man's son is given by C and the clan of a man's wife is given by W. Therefore, C times W, CW, gives the clan of a man's daughter-in-law. A man in clan 1 has a son in clan 2, and a son in clan 2 has a wife in clan 4. We can see this in the matrix CW.

$$W := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$C \cdot W = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In the matrix CW, each row represents the clan of a man and each column represents the clan of his daughter in law.

C^{-1} gives a man's father's clan. W gives a wife's clan. $C^{-1}W$ gives the father's wife's = mother's clan. It also gives the clan of all of her siblings, including a child's maternal uncle and aunt (by blood, not marriage). His mother's brother (of clan $C^{-1}W$) will marry a woman of clan $C^{-1}WW$, which will give the clan of this aunt by marriage. His mother's sister (also of clan $C^{-1}W$) will marry a man of clan $C^{-1}WW^{-1} = C^{-1}$, which will be the clan of this uncle by marriage.

C^{-1}, which gives the clan of a child's father, also gives the clan of the father's siblings, the child's paternal uncle and aunt by blood. The father's brother (of clan C^{-1}) will marry a woman of clan $C^{-1}W$ and the father's sister (also of clan C^{-1}) will marry a man of clan $C^{-1}W^{-1}$. Therefore, a child's paternal uncle and aunt by marriage will have clans $C^{-1}W$ and $C^{-1}W^{-1}$.

In the Figure 3,

This gives the clan of a child's mother.

$$C^{-1} \cdot W = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This gives the clan of the father's sister's children.

$$C^{-1} \cdot W^{-1} \cdot C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

This gives the clan of mother's brother's children.

$$C^{-1} \cdot W \cdot C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

This gives the clan of the child's mother's mother

$$C^{-1} \cdot W \cdot C^{-1} \cdot W = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

This gives the clan of the father's mother.

$$C^{-2} \cdot W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Notice that we could simplify some expressions because the product of a matrix and its inverse is the identity matrix, and multiplication by the identity matrix I is just like multiplication by one.   The clan of the mother's sister's husband is $C^{-1}WW^{-1}$. WW^{-1} = I and $C^{-1}I = C^{-1}$, so the clan of the mother's sister's husband is C^{-1}. The clan of his children is the same as that of ego because $C^{-1}C$ = I, the identity matrix.

Anthropologists classify these societies according to whether or not individuals can marry their first cousins or other individuals in their first cousin's clan. They distinguish between *parallel* cousins and *cross* cousins.

Mother's sister's children and father's brother's children are parallel first cousins. Mother's brother's children and father's sister's children, on the other hand, are cross cousins.

It's clear from the diagram that a man cannot marry a parallel cousin or any member of her clan. She is always in his clan, and by property 6 a man can never marry someone from his own clan. He can marry his patrilateral or matrilateral cross cousin if their clan is given by the W matrix

I. Matrilateral cross-cousin marriage condition

$C^{-1}WC = W$ or, equivalently, $WC = CW$

II Patrilateral cross-cousin marriage condition

$C^{-1}W^{-1}C = W$ or, equivalently, $W^{-1}C = CW$

Let's look at some examples.

$$W := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W \cdot C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C \cdot W = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$W^{-1} \cdot C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

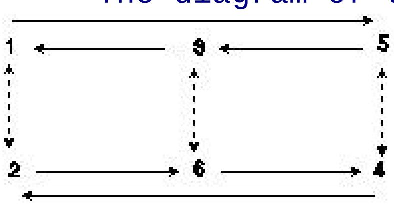
For this marriage system, the conditions for matrilateral cross-cousin marriage are met, and those for patrilateral cross-cousin marriage are not. A man may marry his mother's brother's daughter (or someone from her clan), but not his father's sister's daughter.

Now consider the following illustration.

$$W := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The diagram of this system is:



$$C^{-1} \cdot W^{-1} \cdot C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$C^{-1} \cdot W \cdot C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This system satisfies the conditions for patrilateral but not matrilateral cross-cousin marriage. A man may marry his father's brother's sister's (or someone in her clan) but not his mother's brother's daughter (or anyone else in her clan).

Bilateral cross-cousin marriage

A cousin can be both a matrilateral and patrilateral cross cousin if $C^{-1}W^{-1}C = C^{-1}WC$. This condition can be simplified through the following steps..

$C^{-1}W^{-1}C = C^{-1}WC$

$CC^{-1}W^{-1}C = CC^{-1}WC$, multiplying both sides of the equation on the left by C

$W^{-1}C = WC$, because $CC^{-1} = I$

$W^{-1}CC^{-1} = WCC^{-1}$, multiplying both sides of the equation on the right by C^{-1}

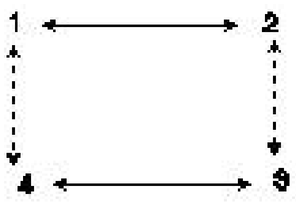
$W^{-1} = W$, because $CC^{-1} = I$

$W^{-1} = WW$

$I = W^2$ because $W^{-1}W = I$

A man's matrilateral and patrilateral cross cousins belong to the same clan if $W^2 = I$. A man can marry his (bilateral) cross cousins if, in addition, $C^{-1}W^{-1}C = C^{-1}WC = W$, or, equivalently, $WC = CW$.

The following is an illustration of a marriage system in which a man can marry his bilateral cross-cousins.



$$W := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$W^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$W \cdot C - C \cdot W = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that matrilateral and patrilateral cousins can belong to the same clan (when $W^2 = I$) without being marriageable, as in the following system.

$$W := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
W \cdot C - C \cdot W = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}
$$

FURTHER READING

Harrison White. An Anatomy of Kinship. Englewood Cliffs, N.J.: Prentice-Hall, 1963.

Ian Bradley and Ronald L. Meek. Matrices and Society. Princeton, N.J.: Princeton University Press, 1986. Chapter 4.

Robert K. Leik and Barbara F. Meeker. Mathematical Sociology. Englewood Cliffs, N.J.: Prentice-Hall, 1975. Chapter 5.

1. Which of the following pairs of C and W matrices violate one or more of the eight characteristics of prescribed marriage systems? If they violate assumptions, state which ones.

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad W := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

2. What kind of marriage systems are the following? Which kinds of cross cousin marriage do they permit?

$$C := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad W := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$