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The Invasion of the Physicists: Review of Duncan J. Watts, *Six Degrees*, and Albert-László Barábasi, *Linked*

Duncan Watts and Albert-László Barábasi are both physicists who have recently crashed the world of social networks, arousing some resentment in the process. Both have made a splash in the wider scientific community, as attested by their publications in high status science journals (*Science*, *Nature*). Both have analyzed some of the same very large networks (for example, the internet). Both use models from physics - Bose-Einstein condensation, percolation, and so on. Both have recently written scientific best-sellers: *Six Degrees* ranks 2,547 on the Amazon list, while *Linked* ranks 4,003. These similarities, however, obscure profound and important differences between the two models they initiated. Watts and Barábasi had different purposes in creating their models, and the models are applicable in different situations.

First, let me lay out the actors in this drama. The mathematician Paul Erdős and his colleague Alfréd Rényi played an important role in developing the model of a random graph[1], a graph in which either a fixed number of edges are randomly distributed among all the pairs of a set of nodes, or, alternatively, a graph in which every pair have the same independent probability of being connected (Bollabás 2001). This mathematically tractable but completely unrealistic model has been used as a baseline in epidemiology and other fields. Random networks completely lack structure: there is no tendency to form clusters (cliques); actors to differ in their prpensities of contacts; there are no tendencies for centralization or transitivity. In fact, no conceivable bias exists in random graphs – no leadership, no homophily of choice – no nothing. Compared to actual social networks, random graphs have a low level of clustering (no cliquing), small differences in degree among the vertices, and short distances between vertices In exchange for this unrealistic simplicity we can calculate other important features of networks such as the distribution of component sizes and average distances between nodes.

Duncan Watts's brilliance was in framing an excellent question: if Stanley Milgram's well-known "small world" experiment is correct, real social networks have two incompatible features – a high degree of clustering inconsistent with random graphs but also their relatively short path lengths. This conjunction of properties could be called "the small world dilemma" and it is as important as the "prisoner's dilemma" of game theory where individual rationality conflicts with observed levels of cooperation. In his earlier book *Small Worlds* (1999) Watts offers a precise definition of the dilemma as well as a simple demonstration that it can be solved without a centralized designer. A small world is defined as a network with high local density that simultaneously has path lengths of the same magnitude as random graphs.

Of course, it is easy to design such a network. A set of disjoint cliques each one is connected by one tie to one central position would have both high density and short

[1] I use the terms *graphs* and *networks* interchangeably.

distances between nodes. But since world-wide networks are not designed but emerge, Watts's next important contribution is to show that there is a very simple uncoordinated process requiring no overall designer that produces a small world. Watts starts with a model with only one of the properties of a small world, a network in which average path length is high but local density is also high. In one of his simulations (the simplest) this beginning model is a lattice in which individuals are connected to all neighbors within a certain distance. The microscopic process that creates a small world is random rewiring of this network. Each edge for each node is rewired with an independent probability β. If the edge of a node is rewired then another node is randomly selected as the other end of the edge. Watts shows that for small values of β the density of the network remains high but average distances between nodes approaches that of random graphs.

Watts's goal is primarily conceptual. He defines a small world and shows that it can be generated without central planning. The model (a lattice) and the process (random rewiring) are completely and utterly unrealistic idealizations. No social network is a lattice and no network is randomly rewired in the way that Watts specifies. Rather, the lattice is an idealized instance of a network with local clustering, and random rewiring is an idealization of some process that perturbs this network and produces connections between nodes that were previously quite distant. A "test" of Watts's model involves measuring average distances and local densities. When densities are high and distances are short the network is declared to be a "small world." There is no claim that the examined network is a randomly rewired lattice.

Barábasi proceeds in quite a different direction, although he too uses random networks as a beginning touchstone. For Watts, random networks lack clustering, but for Barábasi the defect of random networks is that they have too flat a distribution of nodal degree. In a random network the distribution of degree follows a Poisson (or binomial) distribution. Consequently, most nodes have nearly the average degree and there is little variability. Many real networks, on the other hand, appear to have *hubs* – positions with many more connections than would occur in a random network. Instead of having the Poisson distribution, degree in these networks appears to follow a power law. These distributions have "fat tails," with a greater variance than for a Poisson distribution (or even a theoretically infinite variance). Consequently, they are more centralized than random networks and mean path lengths are even shorter than for random networks of the same average degree. Barábasi calls these networks *scale-free* because the average degree does not characterize most vertices.

Barábasi's contribution is to suggest a plausible dynamic mechanism for why networks should have this scale-free shape: preferential attachment. A distribution with degree distributed approximately according to a power law will emerge from an initial small network if new nodes are continually being added that form attachments to existing nodes with probabilities proportional to the existing nodes' degree. In other words, the entering nodes tend to connect to the nodes that already have the most connections. This is a plausible scenario for many large networks. New web sites are most likely to link to popular and well-used sites. New members of a group may gravitate to its most popular members.

Note that the "beef" that Watts and Barábasi have with random networks is quite different. For Watts it was that they were not clustered enough. For Barábasi it was that they do not have hubs. Thus, the two approaches are useful in quite different situations. Small world models are useful when there is a natural tendency to cluster. Networks with a high degree of clustering may or may not be small worlds, depending on the patterns of weak distant ties in the network. Scale-free models are useful when preferential attachment is plausible in a dynamic growing network. Their different approaches to Milgram's experiment are instructive. In analyzing the small degrees of separation characteristic of large social networks Watts stresses the random connectivity imposed on a locally clustered social world. Barábasi stresses the distance-shortening effect of hubs.

The style of application is also quite different. Barábasi's scale-free network is a relatively precise model that is intended to be applied to real networks in all their detail. One can estimate the coefficient in the power law and test whether or not the network formed according to preferential attachment (although this latter is not often done). Even if the model is not precisely correct, Barábasi and his colleagues have worked out many of the implications of the model so that it can be useful as an alternative to random networks. Watts's small world models, designed to be conceptual illustrations of a theoretical point, are not expected to be accurate in their specifics.

In this review I have not included the related work by Barábasi, Watts, and their colleagues that is described in these books. In reaction to Barábasi Watts has developed the mathematics of random networks with fixed degree distributions. This enables him to approach the effects of large inequalities in degree from a direction different from Barábasi. He has also developed the mathematics of "two mode networks," networks in which there must be some clustering because relationships between actors are defined by their co-memberships in groups. He has used percolation theory to develop insights about the connectivity of networks. He has developed a thought-provoking model suggesting that social movements occur most readily in networks that are neither too sparsely or too densely connected. Barábasi and his colleagues have worked on issues of vulnerability of networks to attack and have found inspiration in Bose-Einstein condensation.

Watts and Barábasi have made their contributions at a receptive time and in an increasingly interconnected world in which diseases spread quickly and the internet facilitates long distance communication. Hopefully we can look forward to more insightful and challenging models from them in the future.

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