Answers to

Midterm exam   PS 30   November 2013

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, or digital devices of any kind are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part is weighted equally (12 points each).

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1	
2	
3	
4	
total	

Part 1. Ana and Boris are competitors in a reality TV show, "Survivor Galapagos."

a. Depending upon what happens in the show, they might have to face off in the Coconut Challenge. The Coconut Challenge looks like this: ①

	Boris plays L	Boris plays R
Ana plays U	✗ 3, 6 +	0, 0
Ana plays D	1, 2 +	✗ 2, 1

③

Make a prediction in the Coconut Challenge. What will be Ana's payoff? What will be Boris's payoff? (If you predict a pure strategy Nash equilibrium, write down Ana's payoff and Boris's payoff in that pure strategy Nash equilibrium. If you predict a mixed strategy Nash equilibrium, write down Ana's expected payoff and Boris's expected payoff in that mixed strategy Nash equilibrium.) (2 points)

NE: (U, L)   Note that for Boris, L s. dominates R ①
thus U " " D ②

payoffs are (3, 6)

b. Depending upon what happens in the show, they might have to face off in the Darwin Challenge. The Darwin Challenge looks like this:

[q]   [1-q]

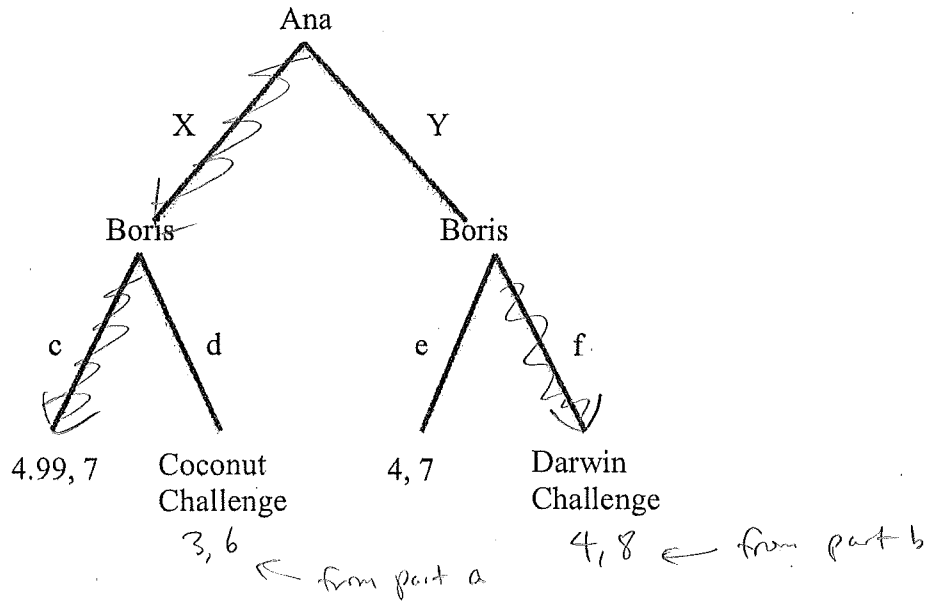|	Boris plays L	Boris plays R
Ana plays U	0, 16 +	✗ 8, 0
Ana plays D	✗ 8, 0	0, 16 +

[p]
[1-p]

No pure strategy NE

Make a prediction in the Darwin Challenge. What will be Ana's payoff? What will be Boris's payoff? (If you predict a pure strategy Nash equilibrium, write down Ana's payoff and Boris's payoff in that pure strategy Nash equilibrium. If you predict a mixed strategy Nash equilibrium, write down Ana's expected payoff and Boris's expected payoff in that mixed strategy Nash equilibrium.) (2 points)

$EU_{ana}(U) = 0 \cdot q + 8(1-q) = 8 - 8q$
$EU_{ana}(D) = 8 \cdot q + 0(1-q) = 8q$

$8 - 8q = 8q$
$8 = 16q \Rightarrow q = \frac{8}{16} = \frac{1}{2}$

$EU_{Boris}(L) = 16 \cdot p + 0(1-p) = 16p$
$EU_{Boris}(R) = 0 \cdot p + 16(1-p) = 16 - 16p$

$16p = 16 - 16p$
$32p = 16 \Rightarrow p = \frac{16}{32} = \frac{1}{2}$

So NE is (Ana plays U with prob $\frac{1}{2}$, Boris plays L with prob $\frac{1}{2}$).
D   R

Ana's payoff is $\frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 8 + \frac{1}{4} \cdot 8 + \frac{1}{4} \cdot 0 = 4$. Boris's payoff is $\frac{1}{4} \cdot 16 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 16$
= 8

c. Now say that Ana and Boris are in a larger game called the Survivor Showdown.

Ana
X   Y
Boris   Boris
c   d   e   f
4.99, 7   Coconut   4, 7   Darwin
          Challenge   Challenge
          3, 6   4, 8 ← from part b
          ← from part a

Note that if Ana plays X and then Boris plays d, they end up with the payoffs they get in the Coconut Challenge (which you found in part a. above). If Ana plays Y and then Boris plays f, they end up with the payoffs they get in the Darwin Challenge (which you found in part b. above). If Ana plays X and Boris plays c, then Ana gets payoff 4.99 and Boris gets payoff 7. Find the subgame perfect Nash equilibria of this game. (2 points)

SPNE shown in tree above

(X, cf)

d. Represent the Survivor Showdown as a strategic form game and find all (pure strategy) Nash equilibria of this game. (3 points)

	ce	cf	de	d f
X	*4.99, 7*	*4.99, 7*	3, 6	3, 6
Y	4, 7	4, 8*	*4, 7	*4, 8*

NE : (X, ce) (X, cf) (Y, df)

e. Now say that Boris can spend some of his Survivor Seashells to boost Ana's payoffs in the Darwin Challenge. If Boris spends k seashells, the Darwin Challenge now becomes:

$(q.)$ $(1-q)$

	Boris plays L	Boris plays R
(p) Ana plays U	0, 16	8 + k, 0
$(1-p)$ Ana plays D	8 + k, 0	0, 16

$EU_{Ana}(u) = 0 \cdot q + (8+k)(1-q) = (8+k)(1-q)$

$EU_{Ana}(D) = (8+k)q + 0(1-q) = (8+k)q$

$(8+k)(1-q) = (8+k)q \Rightarrow 1-q = q \Rightarrow q = \frac{1}{2}$

$EU_{Boris}(L) = 16p + 0(1-p) = 16p$

$EU_{Boris}(R) = 0 \cdot p + 16(1-p) = 16(1-p)$

$16p = 16(1-p) \Rightarrow p = 1-p$

$\Rightarrow p = \frac{1}{2}$

By spending Survivor Seashells, Boris can change Ana's payoffs in the Darwin Challenge. Thus Boris might be able to change Ana's actions in the Survivor Showdown. Boris wants to get the highest possible payoff in the Survivor Showdown while spending as few Survivor Seashells as possible.

How many Survivor Seashells will Boris spend? Assume that Ana and Boris will play a subgame perfect Nash equilibrium (SPNE) in the Survivor Showdown; if there is more than one SPNE because of ties, assume that they play the SPNE which is most favorable to Boris. Find the subgame perfect Nash equilibrium (SPNE) of the new version of the Survivor Showdown (with seashells) below. Again, if there is more than one SPNE, write down the one that Boris likes the best. (3 points)
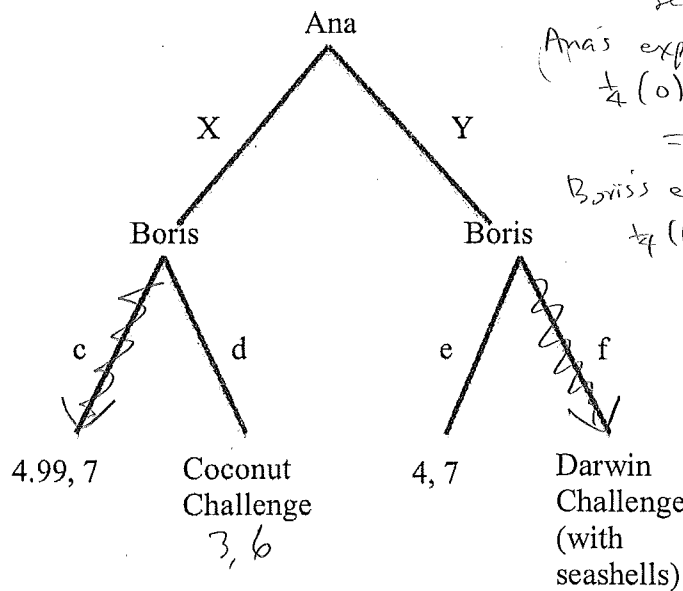
So the mixed NE with k seashells is the same as before.

Ana's expected payoff is

$\frac{1}{4}(0) + \frac{1}{4}(8+k) + \frac{1}{4}(8+k) + \frac{1}{4}(0)$

$= \frac{1}{2}(8+k) = 4 + \frac{k}{2}$

Boris's expected payoff is

$\frac{1}{4}(16) + \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(16) = 8$

So payoffs are

[Game tree:]

Ana

X / Y

Boris   Boris

c / d   e / f

4.99, 7   Coconut Challenge   4, 7   Darwin Challenge (with seashells)

3, 6 $4 + \frac{k}{2}$, 8

If $4 + \frac{k}{2} > 4.99$,

then Ana will pick Y in the SPNE and Boris will get 8, which is his best possible.

so $4 + \frac{k}{2} > 4.99$

or $\frac{k}{2} > 0.99 \Rightarrow k > 1.98$

So Boris should spend 2 seashells.

Part 2. Three children decide who gets the last candy by playing a three-person version of rock-paper-scissors. Each child throws a shape with her hand: each child chooses either rock (R), paper (P), or scissors (S). The children choose simultaneously. The rock breaks scissors, the paper covers rock, and the scissors cut paper.

Children who throw the same shape are tied. If all three throw different shapes (for example, if child 1 throws paper, child 2 throws rock, and child 3 throws scissors), the game is also tied.

The best thing, of course, is to beat the other two children and eat the last candy alone. The next best outcome is to win with a tie so that two winners can negotiate and share the candy. The worst thing is to be the sole loser. The second worst thing is to be one of the two losers, so that at least the two can cry together. If all three children tie by throwing the same shape or three different shapes, they are happier than losing but angrier than winning because they now have to play the game again.

a. Model this 3-person game as a strategic form game only using the numbers 5 (best), 3 (second-best), 0 (third-best), - 3 (second-worst), and - 5 (worst). If you like, use the "graph paper" below to keep things neat. (3 points)

	2R	2P	2S
1R	0,0,0	-3,5,-3	3,-5,3
1P	5,-3,-3	3,3,-5	0,0,0
1S	-5,3,3	0,0,0	-3,-3,5

3R

	2R	2P	2S
1R	-3,3,5	-5,3,3	0,0,0
1P	3,-5,3	0,0,0	-3,5,-3
1S	0,0,0	5,-3,-3	3,3,-5

3P

	2R	2P	2S
1R	3,3,-5	0,0,0	5,-3,-3
1P	0,0,0	-3,-3,5	-5,3,3
1S	-3,5,3	3,-5,3	0,0,0

3S

Sole winner	5
tie winner	3
tie	0
tie loser	-3
sole loser	-5

b. Does this game have pure strategy Nash equilibria? If it does, find them. If not, explain why not. (3 points)

no pure strategy NE

gets 3 or more. So there is no pure strategy NE.

If your opponents are playing the same thing, you can always deviate and get 5 by playing the shape which beats them both.

If your opponents are playing different shapes, then one of your opponents is beating your other opponent. Hence you can imitate the opponent who is beating the other and get 3. Hence you can always get 3. But there is no outcome in which everyone

c. Now let's assume Player 1 (one of the children) throws one of the shapes first. Player 2 then gets to throw. Lastly, Player 3 throws. Model this game as an extensive form game. (3 points)

1 ✱ tie for person 1 here
no matter what 1 does, she gets -5

R   P   S

2   2   2

R   P   S   R   P   S   R   P   S

3   3   3   3   3   3   3   3   3   3   S

R   R   R   R   R   R   0,0,0
P   S   R   S   R   P   S   R   P   S   P

R   3,3,-5
P   S   3,3,-5   P   -5,3,3
                 0,0,0   -3,-3,5

-3,-3,5   -5,3,3   R   0,0,0   5,-3,-3   0,0,0   -3,3,5   R -3,5,-3   5,-3-3
                   3,-5,3   -3,-3,5 -3,5,-3 0,0,0
                                                 5,-3-3   3,-5,3

0,0,0   -3,5,-3   3,-5,3   5,-3,-3   3,3,-5   0,0,0   -5,3,3   0,0,0   -3,-3,5
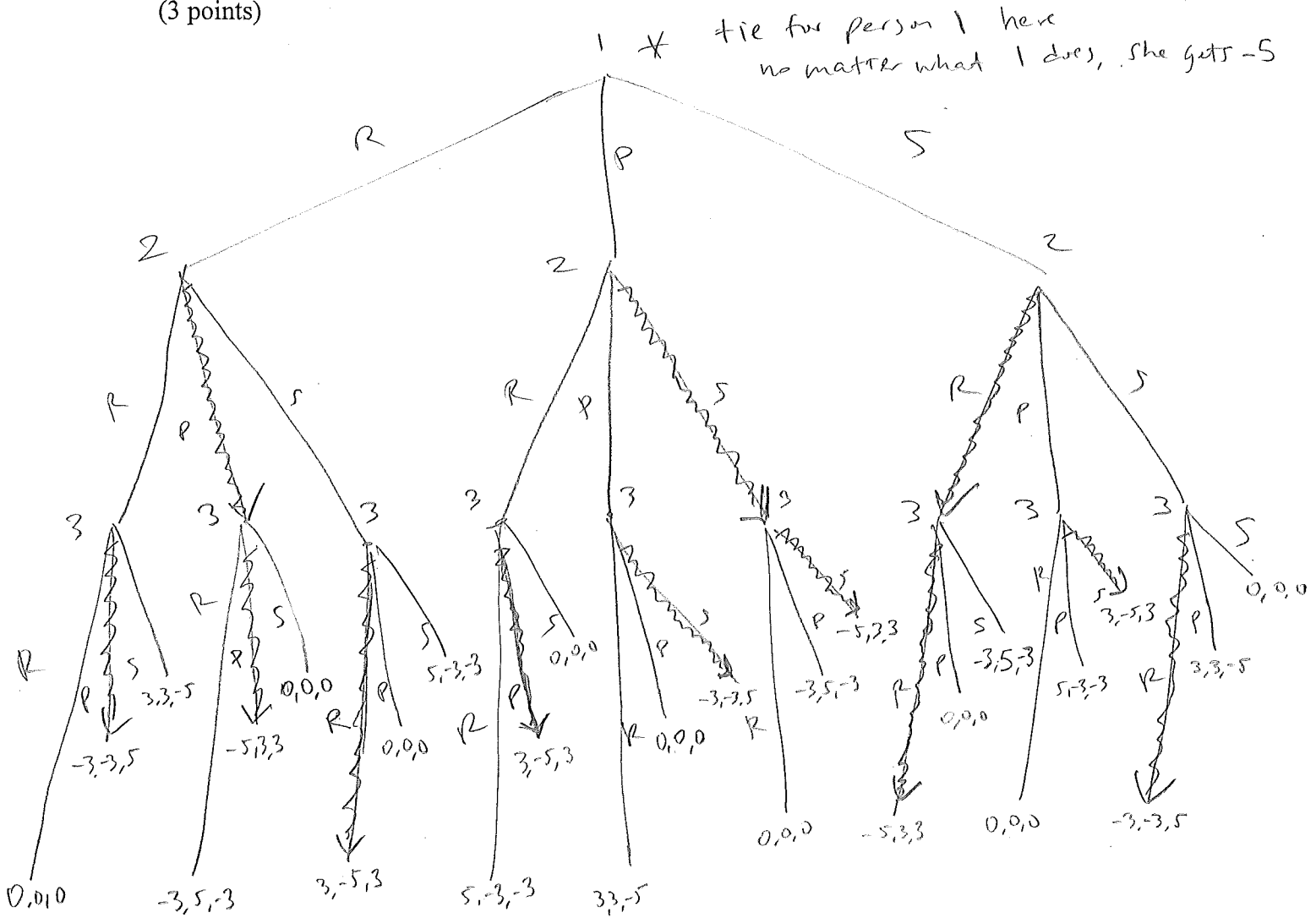
d. Find all subgame perfect Nash equilibria (SPNE). If there is more than one SPNE, you do not have to write down a separate tree for each SPNE—just put an asterisk in the tree where there is a tie and explain what is going on. (3 points)

SPNE shown above
there are three SPNE because 1 has a 3-way tie
at her node

Part 3. On the plane ride back from the Stagecoach Festival, two travelers lose the cowboy hats they bought at the festival. The hats are identical (they are both signed, limited-edition Toby Keith specials, with premium denim stitching). The loss of the hats is the airline's fault. The airline must compensate the travelers for their loss, but the airline does not know how much the hats are truly worth. Obviously, the travelers want to claim as high a value as possible.

An airline manager tries to settle the dispute. He asks each traveler to write down the true value of the hat. They do so simultaneously. If the travelers write down the same amount, then the airline reimburses that amount to both travelers. If they write down different amounts, then both travelers get nothing (since one of them must be fabricating). For simplicity, say that each traveler can choose from the following values: $20, $40, $60, $80, $100, or $120.

a. Model this as a strategic form game and find all (pure strategy) Nash equilibria. (3 points)

	20	40	60	80	100	120
20	20,20	0,0	0,0	0,0	0,0	0,0
40	0,0	40,40	0,0	0,0	0,0	0,0
60	0,0	0,0	60,60	0,0	0,0	0,0
80	0,0	0,0	0,0	80,80	0,0	0,0
100	0,0	0,0	0,0	0,0	100,100	0,0
120	0,0	0,0	0,0	0,0	0,0	120,120

NE:
(20,20)
(40,40)
(60,60)
(80,80)
(100,100)
(120,120)

b. Now say that the airline manager offers a "bonus scheme" to the two travelers. If the two travelers write down the same amount, then, just like before, the airline will reimburse that amount to both travelers. But now, if they write down different amounts, only the person who writes down the lower amount gets reimbursed; the person who writes down the higher amount gets nothing. The person who writes down the lower amount gets reimbursed an amount equal to her claim plus a 100 percent bonus (in other words, double her claim). For example, if traveler 1 writes down $40 and traveler 2 writes down $100, then traveler 1 gets $80 ($40 plus a bonus of $40) and traveler 2 gets nothing.

Model this as a strategic form game and find all (pure strategy) Nash equilibria. (3 points)

	20	40	60	80	100	120
20	20,20	40,0	40,0	40,0	40,0	40,0
40	0,40	40,40	80,0	80,0	80,0	80,0
60	0,40	0,80	60,60	120,0	120,0	120,0
80	0,40	0,80	0,120	80,80	160,0	160,0
100	0,40	0,80	0,120	0,160	100,100	200,0
120	0,40	0,80	0,120	0,160	0,200	120,120

NE:
(20,20)
(40,40)

c. Now say that the airline manager offers the same "bonus scheme" but this time with a 50 percent bonus instead of a 100 percent bonus. Again, if the two travelers write down the same amount, then, just like before, the airline will reimburse that amount to both travelers. If they write down different amounts, only the person who writes down the lower amount gets reimbursed; the person who writes down the higher amount gets nothing. The person who writes down the lower amount gets reimbursed an amount equal to her claim plus a 50 percent bonus. For example, if traveler 1 writes down $40 and traveler 2 writes down $100, then traveler 1 gets $60 ($40 plus a bonus of $20) and traveler 2 gets nothing.

Model this as a strategic form game and find all (pure strategy) Nash equilibria. (3 points)

	20	40	60	80	100	120
20	20,20	30,0	30,0	30,0	30,0	30,0
40	0,30	40,40	60,0	60,0	60,0	60,0
60	0,30	0,60	60,60	90,0	90,0	90,0
80	0,30	0,60	0,90	80,80	120,0	120,0
100	0,30	0,60	0,90	0,120	100,100	150,0
120	0,30	0,60	0,90	0,120	0,150	120,120

NE: (20,20)
(40,40)
(60,60)

d. Now say the airline manager is sympathetic to the travelers and hates the bonus scheme. The airline manager must still use the bonus scheme because she is ordered to do so by her superiors. The manager can, however, decide the percentage amount of the bonus (although her superiors want the bonus to be as high as possible). Say that the manager wants the travelers to receive the greatest possible compensation. What is the highest possible percentage bonus that the manager can choose such that the travelers get maximum compensation? (3 points)

(120,120) can be a NE as long as no one wants to deviate. If the bonus is 20% or less, then a person's (payoff from deviating to 100) is 120 or less. Hence (120,120) will be a NE.

If the bonus is higher than 20%, then (120,120) will not be a NE.

So 20% is the highest possible bonus which makes (120,120) a NE.

Part 4. Say we have the three person extensive form game below.

```
                          1
                     a /     \ b
                      /       \
                    2           2
                 r / \ s     c / \ d
                  /   \       /   \
              1,4,1  7,3,7  3,2,0   3
                                 e / \ f
                                  /   \
                              5,7,4   2,1,5
```

a. Write this as a strategic form game. (2 points)

	rc	rd	sc	sd
a	1,4,1	1,4,1	7,3,7	7,3,7
b	3,2,0	5,7,4	3,2,0	5,7,4

e   ①

②

	rc	rd	sc	sd
③ a	1,4,1	1,4,1	7,3,7	7,3,7
b	3,2,0	2,1,5	3,2,0	2,1,5

f

b. Make a prediction in this strategic form game by iteratively eliminating (strongly or weakly) dominated strategies. Please indicate the order of elimination. (2 points)

① For 3, f w. dominates e

② For 2, rc w. dominates rd, sc, and sd

③ For 1, b w. dominates a.

c. Say we have the following three person strategic form game.

②

	2 plays a	2 plays b	2 plays c
1 plays rd	4, 4, 4	1, 3, 6	6, 9, 6
1 plays re	4, 4, 4	1, 3, 6	3, 1, 4
1 plays sd	4, 4, 4	5, 5, 0	6, 9, 6
1 plays se	4, 4, 4	5, 5, 0	3, 1, 4

3 plays x

③

|	2 plays a	2 plays b	2 plays c
1 plays rd	4, 4, 4	1, 3, 6	7, 7, 7
1 plays re	4, 4, 4	1, 3, 6	7, 7, 7
1 plays sd	4, 4, 4	5, 5, 0	7, 7, 7
1 plays se	4, 4, 4	5, 5, 0	7, 7, 7

3 plays y

Make a prediction in this game by iteratively eliminating (strongly or weakly) dominated strategies. Please indicate your order of elimination. (2 points)

① For 1, sd. w. dominates rd, re, and se

② For 2, c s. dominates a and b

③ For 3, y s. dominates x.

So we predict 1 plays sd, 2 plays c, and 3 plays y.

d. Usually we take an extensive form game and turn it into a strategic form game (as in part a. above). Now let's go in the opposite direction. The strategic form game in part c. above was created from an extensive form game. Write down the extensive form game. (If there is more than one possible answer, just write down one of them.) Please note that the player who moves first in the extensive form game is not necessarily player 1—it could be that player 2 or player 3 moves first. Find all subgame perfect Nash equilibria of this extensive form game. (2 points)

SPNE indicated by shaded arrows

(sd, c, y)

4, 4, 4
1, 3, 6
5, 5, 0
6, 9, 6
3, 1, 4
7, 7, 7

Note that if 2 plays a, it doesn't matter what anyone else does (you get 4,4,4). So 2 goes first. If 2 chooses b, then 3 doesn't matter. So 1 chooses after 2 chooses b, and 1 chooses between r and s. If 2 chooses c, then if 3 chooses y, then 1 doesn't matter. So 3 chooses after 2 chooses c. If 3 chooses x, then what matters is if 1 chooses d or e. Hence we have this tree.

e. Say we have the following three person strategic form game.

①

	2 plays a	2 plays b	2 plays c
1 plays rd	0, 2, 0	0, 2, 0	0, 2, 0
1 plays re	0, 2, 0	0, 2, 0	0, 2, 0
1 plays sd	4, 1, 7	4, 1, 7	4, 1, 7
1 plays se	4, 1, 7	4, 1, 7	4, 1, 7

3 plays x   ②

①

	2 plays a	2 plays b	2 plays c
1 plays rd	5, 5, 5	5, 4, 2	3, 9, 3
1 plays re	5, 5, 5	8, 0, 5	3, 9, 3
1 plays sd	4, 1, 7	4, 1, 7	4, 1, 7
1 plays se	4, 1, 7	4, 1, 7	4, 1, 7

③   ⑤

3 plays y

Make a prediction in this game by iteratively eliminating (strongly or weakly) dominated strategies. Please indicate your order of elimination. (2 points)

① For 2, c w. dominates a and b.

② For 3, y w. dominates x.

③ For 1, sd, s, dominates rd and re.

Prediction: 1 plays rd or re, 2 plays c, 3 plays y.

f. The strategic form game in part e. above was created from an extensive form game. Write down the extensive form game. (If there is more than one possible answer, just write down one of them.) Please note that the player who moves first in the extensive form game is not necessarily player 1—it could be that player 2 or player 3 moves first. Find all subgame perfect Nash equilibria of this extensive form game. (2 points)
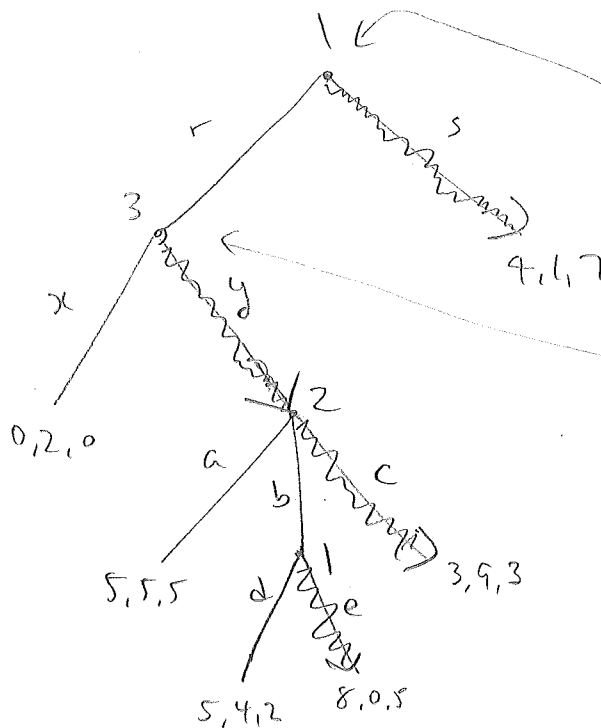
If 1 plays s, then it doesn't matter what anyone else does. So 1 goes first. If 1 chooses r, then if 3 chooses x, nothing else matters. So 3 chooses between x and y next, etc.

SPNE indicated by arrows

(se, c, y)