Answers to

Midterm exam PS 30 November 2011

Name:

TA:

Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has four parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1	
2	
3	
4	
total	

1. Consider the following game.

	2a	2b	2c	2d
1a	3, 2	0, 4	1, 5	1, 4
1b	2, 1	2, 2	3, 1	1, 0
1c	4, −1	1, 0	2, −1	2, 1
1d	2, 3	0, 2	4, 0	1, 4

a. Eliminate as many strategies as possible using the method of iterative elimination of (strongly or weakly) dominated strategies. Please specify the order of elimination. (4 points)

(1) 1c s. dominates 1a

(2) 2b s. dominates 2c

(3) 1c s. dominates 1d

(4) 2b s. dominates 2a

b. In the remaining game, find all (pure strategy and mixed strategy) Nash equilibria. (4 points)

		(q) $\frac{1}{2}$	(1-q) $\frac{1}{2}$
		2b	2d
$\frac{1}{3}$ (p)	1b	* 2, 2+	1, 0
$\frac{2}{3}$ (1-p)	1c	1, 0	* 2, 1+

pure strategy NE: (1b, 2b) (1c, 2d)

$Eu_1(1b) = 2q + 1(1-q) = 1+q$ ‖ $1+q = 2-q$

$Eu_1(1c) = 1q + 2(1-q) = 2-q$ ‖ $2q = 1$ $\boxed{q = \frac{1}{2}}$

$Eu_2(2b) = 2p + 0(1-p) = 2p$ ‖ $2p = 1-p$ $\boxed{p = \frac{1}{3}}$

$Eu_2(2d) = 0p + 1(1-p) = 1-p$ ‖ $3p = 1$

mixed strategy NE: (1 plays 1b with prob $\frac{1}{3}$, 1c $\frac{2}{3}$, 2 plays 2b with prob $\frac{1}{2}$, 2d $\frac{1}{2}$).

c. Now consider the same game but with a few small changes. Now in a few places, a person's payoff is a variable x or a variable y. In this game, the Nash equilibria might depend on the values of x and y.

	2a	2b	2c	2d
1a	3, 2	0, 4	1, 5	1, 4
1b	x, 1	2, 2	3, 1	1, 0
1c	4, −1	1, 0	2, −1	2, 1
1d	2, y	0, 2	4, 0	1, 4

Below are possible values of x and y. Circle all which give you the same Nash equilibria as what you found in part b. earlier.

x=0 y=0	x=1 y=0	x=2 y=0	x=3 y=0	x=4 y=0	x=5 y=0	x=6 y=0
x=0 y=1	x=1 y=1	x=2 y=1	x=3 y=1	x=4 y=1	x=5 y=1	x=6 y=1
x=0 y=2	x=1 y=2	x=2 y=2	x=3 y=2	x=4 y=2	x=5 y=2	x=6 y=2
x=0 y=3	x=1 y=3	x=2 y=3	x=3 y=3	x=4 y=3	x=5 y=3	x=6 y=3
x=0 y=4	x=1 y=4	x=2 y=4	x=3 y=4	x=4 y=4	x=5 y=4	x=6 y=4

For example, if this game has the same Nash equilibria as what you found in part b. when $x = 2$ and $y = 4$, then circle $\substack{x=2\\y=4}$. Please explain your work. (4 points)

no matter what x and y are,

(1b, 2b) and (1c, 2d) will be the only pure strategy NE

no matter what x and y are,

the iter. elim of s. dominated strategies works out the same way as before.

2. Suppose a town collects donations from citizens to fund a public program. The town has three citizens. Each decides simultaneously either to donate 0, 1, or 5. If the town gathers a total of 4 or more, then they build a new park. If the town gathers a total of 9 or more, then they build the park and create a jobs program (the jobs are for maintaining the park). If the town gathers a total of less than 4, then no project is built.

If the park is built, then each player receives 4 minus his own donation. If the town creates the park and the jobs program, then each receives 5 minus his own donation. If no project is built, then each player receives 0 minus his own donation. For example, if person 1 donates 1 and the park and jobs program is built, then person 1 gets a payoff of 4. If person 1 donates 5 and no project is built, then person 1 gets a payoff of -5.

a. Represent this situation as a strategic form game. (2 points)

	0	1	5
0	0,0,0	0,-1,0	4,-1,4
1	-1,0,0	-1,-1,0	3,-1,4
5	-1,4,4	-1,3,4	0,0,5

0

	0	1	5
0	0,0,-1	0,-1,-1	4,-1,3
1	-1,0,-1	-1,-1,-1	3,-1,3
5	-1,4,3	-1,3,3	0,0,4

1

	0	1	5
0	4,4,-1	4,3,-1	5,0,0
1	3,4,-1	3,3,-1	4,0,0
5	0,5,0	0,4,0	0,0,0

5

b. Iteratively eliminate (strongly or weakly) dominated strategies. (2 points)

0 s. dominates 1
0 s. dtes 5 } for everyone.

c. Find all pure strategy Nash equilibria. Which projects does the town enact? (2 points)

(0,0,0)

	0	1	5
0	N	N	P
1	N	N	P
5	P	P	J

0

	0	1	5
0	N	N	P
1	N	N	P
5	P	P	J

1

	0	1	5
0	P	P	J
1	P	P	J
5	J	J	J

5

Now say that player one believes government should alleviate unemployment. She enjoys a payoff of 5 minus her donation if they erect the park and 8 minus her donation if the employment program succeeds. Player two is a staunch libertarian. She believes there should be no public park. But, she becomes happier if the unemployed are no so idle. Hence receives 0 minus her donation if the park is built and 3 minus her donation if the park and jobs program are funded. Player three is an exercise enthusiast and therefore gets 6 minus her donation if there is a park. But she doesn't agree with the jobs program and receives 2 minus her donation if the park and jobs program are funded.

d. Represent this situation as a strategic form game. (2 points)

	0	1	5
0	0,0,0	0,-1,0	5,-5,6
1	-1,0,0	-1,-1,0	4,-5,6
5	0,0,6	0,-1,6	3,-2,2

0

	0	1	5
0	0,0,-1	0,-1,-1	5,-5,5
1	-1,0,-1	-1,-1,-1	4,-5,5
5	0,0,5	0,-1,5	3,-2,1

1

	0	1	5
0	5,0,1	5,-1,1	8,-2,-3
1	4,0,1	4,-1,1	7,-2,-3
5	3,3,-3	3,2,-3	3,-2,-3

5

e. Iteratively eliminate (strongly or weakly) dominated strategies. (2 points)

For plyr 1, 0 s. dominates 1
For plyr 2, 0 s. dominates 1 and 0 s. dominates 5.
For plyr 3, 0 s. dominates 1.

(because spending $1 never makes a project happen)

So we have left:

	0
0	0,0,0
5	0,0,6

0

	0
0	5,0,1
5	3,3,-3

5

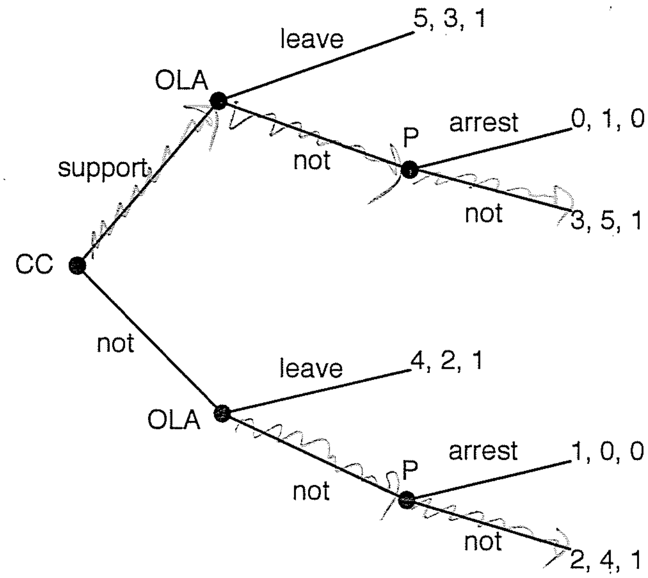
Thus for plyr 1, 0 w. dominates 5.
Thus for plyr 3, 5 s. dominates 0.

f. Find all pure strategy Nash equilibria. Which projects does the town enact? (2 points)

PSNE: (5, 0, 0) (0, 0, 5)

the town builds only the park.

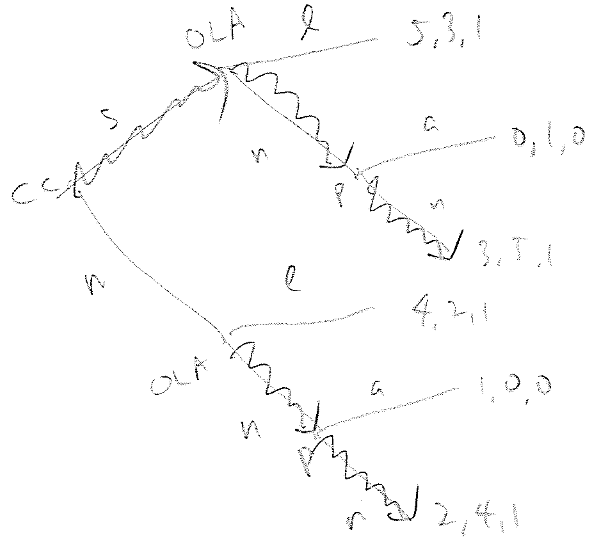
3. The Occupy movement has camped out at City Hall for almost a month now. The Los Angeles City Council (CC) chooses to support it or not. Then Occupy Los Angeles (OLA) chooses whether to leave or not. If OLA decides to leave, then the game ends. However, if OLA decides not to leave, then the police (P) decides whether to arrest the protesters. Thus the game looks like this. Here the payoffs are given as (CC, OLA, P).



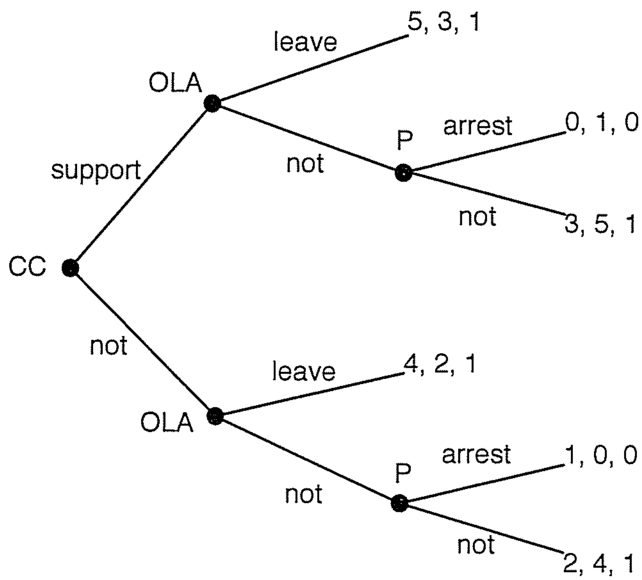
Note that the best thing for the CC is to support the movement but then have OLA leave. The worst thing for the CC is to support the movement but then have OLA arrested, because that looks hypocritical. The CC would like the movement to leave peacefully and dislikes arrests. The OLA wants the support of the CC and would rather stay, but would rather leave than be arrested. The P simply doesn't want to arrest anyone because of the potential chaos.

a. Find the subgame perfect Nash equilibria of this game. (4 points)

SPNE: (Support, not not, not not)



SPNE



b. The game is printed again above for convenience. Represent it as a strategic form game. (4 points)

a / a

	l / l	l / n	n / l	n / n
s	5,3,1	5,3,1	0,1,0	0,1,0
n	4,2,1	1,0,0	4,2,1	1,0,0

a / n

	l / l	l / n	n / l	n / n
s	5,3,1	5,3,1	0,1,0	0,1,0
n	4,2,1	2,4,1	4,2,1	2,4,1

n / a

	l / l	l / n	n / l	n / n
s	5,3,1	5,3,1	3,5,1	3,5,1
n	4,2,1	1,0,0	4,2,1	1,0,0

n / n

	l / l	l / n	n / l	n / n
s	5,3,1	5,3,1	3,5,1	3,5,1
n	4,2,1	2,4,1	4,2,1	2,4,1

c. Find all Nash equilibria of this strategic form game. (4 points)

NE: (s, l/l, a/a) (s, l/n, a/a) (s, n/l, a/a) (s, l/l, a/n) (s, l/n, a/n) (n, n/n, a/n)

(n, n/l, n/a) (n, n/n, n/a) (s, n/n, n/n)

SAME

4. The coal industry and the oil industry each decide whether to lobby Congress or not. Lobbying costs 100 but if you lobby, you get a benefit. If you are the only industry which lobbies, you get a benefit of 400. If both industries lobby, then both get benefits of 200. So the game looks like this.

	Oil lobbies	Oil does not
Coal lobbies	100, 100	300,0
Coal does not	0, 300	0, 0

a. Now say that a citizens' group tries to shame the coal industry and oil industry into not lobbying. The citizens' group can spend money shaming the coal industry and can spend money shaming the oil industry. For every dollar the citizens' group spends on shaming an industry, the payoff to the industry decreases by one. For example, if the citizens' group spends 60 dollars on shaming the coal industry and 12 dollars on shaming the oil industry, the resulting game is

	Oil lobbies	Oil does not
Coal lobbies	40, 88	240,0
Coal does not	0, 288	0, 0

The citizens' group wants to get rid of industry lobbying but has a limited budget. It wants to change the game so that the only pure strategy Nash equilibrium is the one in which no one lobbies, but it wants to spend as little as possible in total to do so. How much money should the citizens' group spend on shaming the coal industry and how much should it spend on shaming the oil industry? (Assume that the citizens' group can spend only whole dollar amounts; in other words, it cannot spend 23.15 dollars.) Write down the resulting game and show that its only pure strategy Nash equilibrium is the one in which no one lobbies. (2 points)

	lobbies	not
lobbies	$100 - S_c$, $100 - S_{oil}$	$300 - S_c$, 0
not	0, $300 - S_{oil}$	0, 0

For (not, not) to be a NE, $0 \geq 300 - S_c$ $\quad S_c \geq 300$

$0 \geq 300 - S_{oil}$ $\quad S_{oil} \geq 300$

If $S_c = 300$ $S_{oil} = 300$, we have

	lobbies	not
lobbies	−200, −200	0, 0
not	0, 0	0, 0

if $S_c = 300$, $S_{oil} = 300$, then NE are (not, not), (lobbies, not), (not, lobbies) to make it so that (not, not) is the <u>only</u> NE,

we need to set $s_C = 301$, $s_{oil} = 301$.

We get this game.

	lobbies	not
lobbies	-201, -201	-1, 0*
not	*0, -1	*0, 0*

now the only pure strategy NE is (not, not).

b. Here is the original game again.

	Oil lobbies	Oil does not
Coal lobbies	100, 100	300,0
Coal does not	0, 300	0, 0

Now say that the citizens' group has a smaller budget and has a more limited goal. It simply wants to get rid of the Nash equilibrium in which both industries lobby, again by spending as little as possible. How much money should the citizens' group spend on shaming the coal industry and how much should it spend on shaming the oil industry? Write down the resulting game and find its pure strategy Nash equilibria. (3 points)

	lobbies	not
lobbies	$100 - S_c$, $100 - S_{oil}$	$300 - S_c$, 0
not	0, $300 - S_{oil}$	0, 0

to make (lobbies, lobbies) not a NE,
it's enough to make $100 - S_c < 0$

so set $S_c = 101$ and $S_{oil} = 0$

	lobbies	not
lobbies	−1, 100+	* 199, 0
not	* 0, 300+	0, 0

now (lobbies, lobbies) is not a NE.
the pure strategy NE is (not, lobbies).

c. Here is the original game again.

	Oil lobbies	Oil does not
Coal lobbies	100, 100	300,0
Coal does not	0, 300	0, 0

Now say that the citizens' group wants to create chaos in the system. It wants to alter the game so that there exists a mixed Nash equilibrium in which the coal industry lobbies with probability 1/2 and doesn't lobby with probability 1/2, and the oil industry similarly lobbies with probability 1/2 and doesn't lobby with probability 1/2. How much money should the citizens' group spend on shaming the coal industry and how much should it spend on shaming the oil industry? Write down the resulting game and find all (mixed strategy and pure strategy) Nash equilibria. (3 points)

		$\frac{1}{2}$ (q) lobbies	$\frac{1}{2}$ (1-q) not
$\frac{1}{2}$ (p)	lobbies	$100 - S_c$, $100 - S_{oil}$	$300 - S_c$, 0
$\frac{1}{2}$ (1-p)	not	0, $300 - S_{oil}$	0, 0

$EU_1(\text{lobbies}) = (100 - S_c)\frac{1}{2} + (300 - S_c)\frac{1}{2} = 50 - \frac{1}{2}S_c + 150 - \frac{1}{2}S_c = 200 - S_c$

$\| \quad EU_1(\text{not}) = 0 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} = 0$

$0 = 200 - S_c$

$S_c = 200$

$EU_2(\text{lobbies}) = (100 - S_{oil})\frac{1}{2} + (300 - S_{oil})\frac{1}{2} = 200 - S_{oil}$

$EU_2(\text{not}) = 0 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} = 0$

$0 = 200 - S_{oil}$

$S_{oil} = 200$

	lobbies	not
lobbies	-100, -100	*100, 0*
not	*0, 100*	0, 0

pure strategy NE: (not, lobbies) (lobbies, not)

d. Is it possible for the citizens' group to change the game (by shaming) so that the only Nash equilibrium of the game is a mixed strategy Nash equilibrium? Why or why not? (2 points)

	l	n
l	$100 - S_c$, $100 - S_{oil}$	$300 - S_c$, 0
n	0, $300 - S_{oil}$	0, 0

If $S_c > 300$ or $S_{oil} > 300$ then we have a dominated strategy and thus there will be a pure strategy NE.

If $S_c < 100$ or $S_{oil} < 100$, then we have a dominated strategy and thus there will be a pure strategy NE.

mixed strategy NE: (1 plays lobbies with prob $\frac{1}{2}$, not $\frac{1}{2}$, 2 plays lobbies with prob $\frac{1}{2}$, not prob $\frac{1}{2}$)

So we must have $100 \leq S_c \leq 300$ and $100 \leq S_{oil} \leq 300$.

then we have:

	$100 - S_c$, $100 - S_{oil}$	*$300 - S_c$, 0*
	0, $300 - S_{oil}$	0, 0

in which case we have pure strategy NE.

e. Here is the original game again.

	Oil lobbies	Oil does not
Coal lobbies	* 100, 100 +	* 300,0
Coal does not	0, 300 +	0, 0

Now say the citizens' organization is taken over by an industry spokesperson. This spokesperson wants to bankrupt the citizens' organization and spend as much of its money as possible as long as the Nash equilibrium in the original game still remains a Nash equilibrium. How should this spokesperson spend the money? Write down the resulting game and find its pure strategy Nash equilibria. (2 points)

	lobby	not
lobby	$100 - S_c$, $100 - S_{oil}$	$300 - S_c$, 0
not	0, $300 - S_{oil}$	0, 0

$100 - S_c \geq 0$ $\quad$ $S_c = 100$

$100 - S_{oil} \geq 0$ $\quad$ $S_{oil} = 100$

	lobby	not
lobby	*0, 0+	*200, 0+
not	*0, 200+	0, 0

NE: (lobby, lobby) (lobby, not) (not, lobby).