

Answers to
Midterm exam

PS 30

November 2005

Name:

TA:

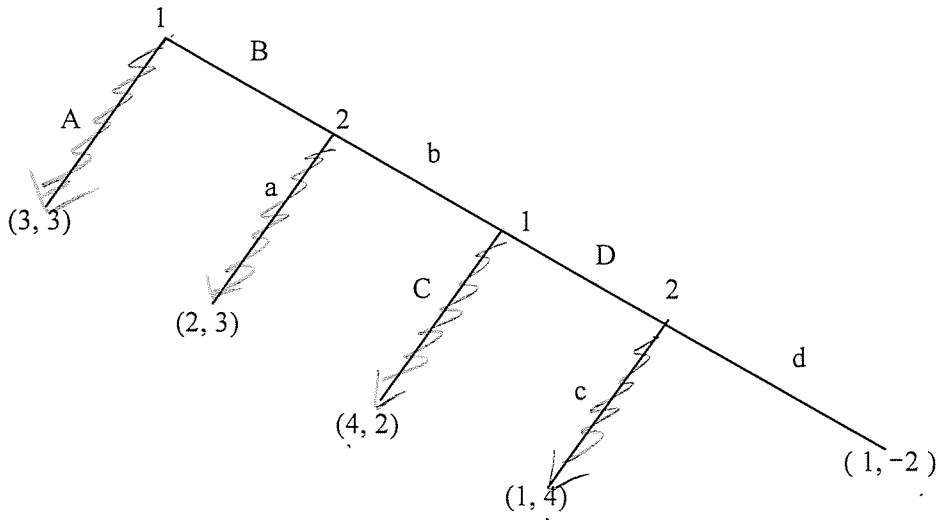
Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are four sections of this exam. Each section is weighted equally (12 points for each section). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the section you have questions about (if you have a question on Section II, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

Part I. Consider the following game. Note that payoffs are written as (Player 1, Player 2).

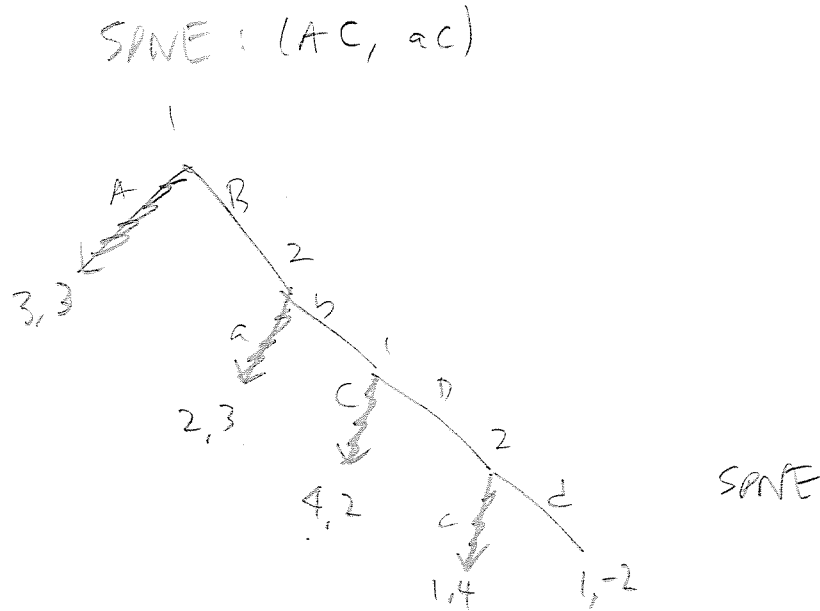


a. Represent this game as a strategic form game and find all pure strategy Nash equilibria. (4 points)

	ac	ad	bc	bd
AC	$(3, 3)$	$(3, 3)$	$(3, 3)$	$(3, 3)$
AD	$(3, 3)$	$(3, 3)$	$(3, 3)$	$(3, 3)$
BC	$(2, 3)$	$(2, 3)$	$(4, 2)$	$(4, 2)$
BD	$(2, 3)$	$(2, 3)$	$(1, 4)$	$(1, -2)$

NE : (AC, ac) (AC, ad) (AD, bc) (AD, bd)

b. Find the subgame perfect Nash equilibrium (write down the strategies for each player). (4 points)



c. Find all Nash equilibria (pure strategy and mixed strategy) of the following game. (4 points)

		$\frac{2}{3}(q)$	$\frac{1}{3}(1-q)$	
		2a	2b	
$\frac{1}{3}(p)$	1a	3, 0	*4, 2*	
$\frac{2}{3}(1-p)$	1b	*4, 1*	2, 0	

Pure NE : (1a, 2b)
 strategy (1b, 2a)

$$EU_1(1a) = 3q + 4(1-q) = 3q + 4 - 4q = 4 - q$$

$$EU_1(1b) = 4q + 2(1-q) = 4q + 2 - 2q = 2 + 2q$$

$$4 - q = 2 + 2q \Rightarrow 2 = 3q \Rightarrow \boxed{q = \frac{2}{3}}$$

$$EU_2(2a) = 0p + 1(1-p) = 1 - p$$

$$EU_2(2b) = 2p + 0(1-p) = 2p$$

$$1 - p = 2p \Rightarrow 1 = 3p \Rightarrow \boxed{p = \frac{1}{3}}$$

Mixed Strategy NE : (1 plays 1a with prob $\frac{1}{3}$, 1b with prob $\frac{2}{3}$, 2 plays 2a with prob $\frac{2}{3}$, 2b with prob $\frac{1}{3}$)

Part II. Consider the following three person game. Each player chooses a number, either 1 or 2. The chosen numbers are added up and if the sum is 3 or 6, Players 2 and 3 both have to give \$3 to Player 1. If the sum is 4 then Player 1 and Player 3 both have to give \$1 to Player 2. If the sum is 5 then Player 1 and Player 2 both have to give \$1 to Player 3.

a. Write this as a strategic form game. (4 points)

	1	2
1	*6, -3, -3	*-1, 2, -1
2	-1, 2, -1	*-1, -1, 2
	1	2

	1	2
1	*-1, 2, -1	-1, -1, 2
2	*-1, -1, 2	*6, -3, -3
	1	2

b. Are any strategies strongly or weakly dominated? If so, which ones? (4 points)

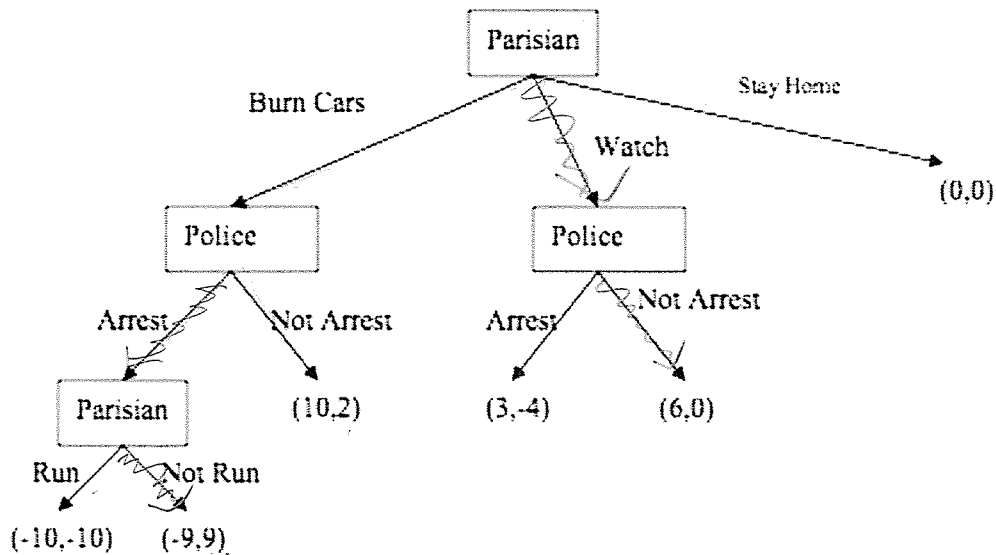
no strategies are dominated (weakly or strongly)

c. Find all pure strategy Nash equilibria. (4 points)

NE : (1, 1, 2) (2, 1, 2)

Part III. What would the rational Parisian do?

The following game describes loosely the interactions between Parisians and the Police in France this week. The Parisians are in a foul mood this week but still have to ponder strategic considerations before deciding on which strategy to adopt – and so do the Police. Help us figure out what the outcome will be (in this simple version of reality). Here payoffs are written as (Parisian, Police).



3.2

a. How many strategies does the Parisian have? 6 (0.5 points). Write down 3 of them. (1.5 points)

Burn cars, run
Burn cars, not

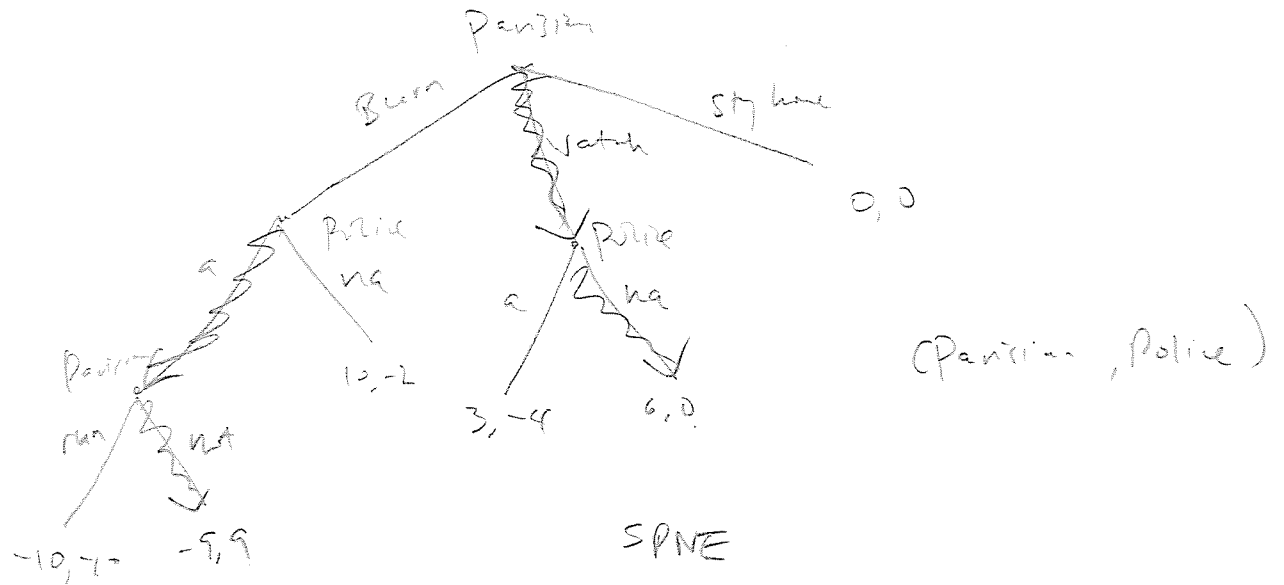
Watch, run

2.2

b. How many strategies does the Police have? 4 (0.5 points). Write down 3 of them. (1.5 points)

arrest, arrest
arrest, not
not, arrest

c. Find the Subgame Perfect Nash Equilibrium of this game (4 points).

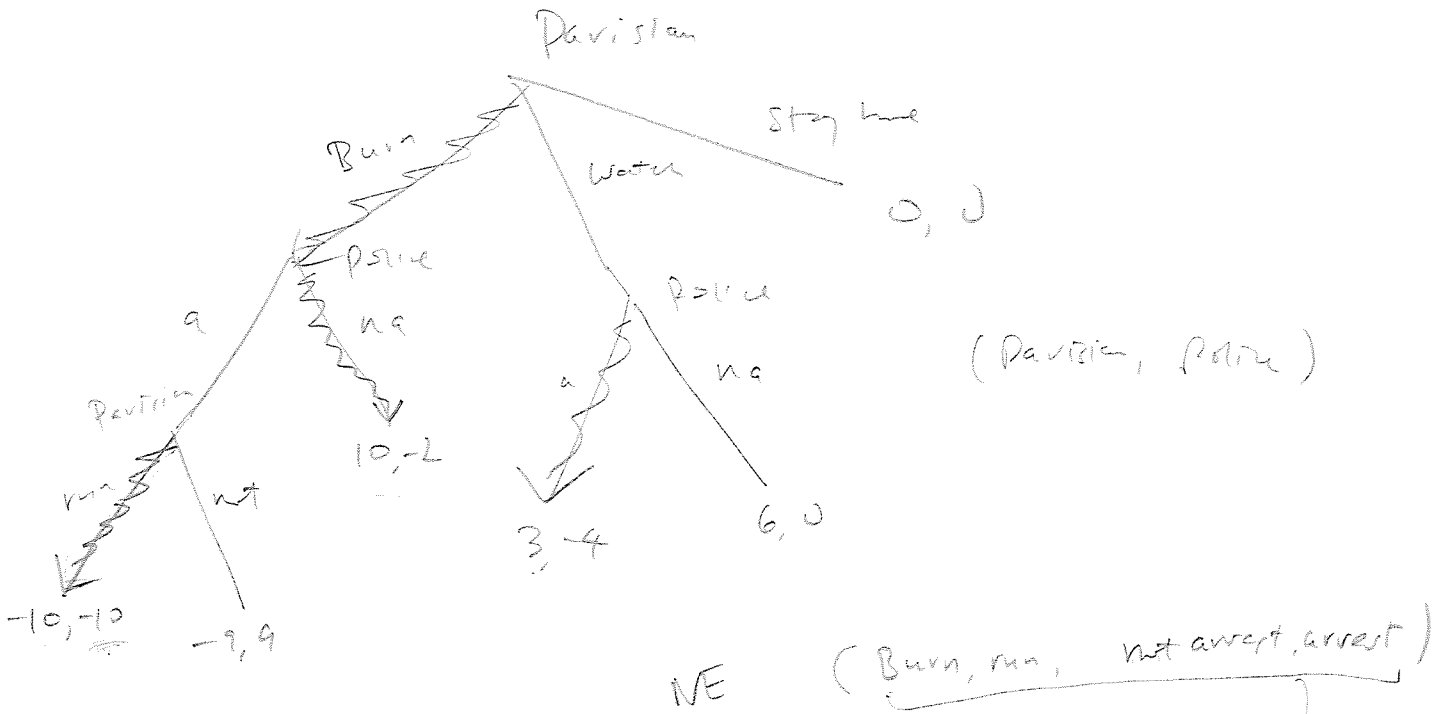


SPNE : (watch, not, arrest, notarrest)

d. If you were a reporter for the Daily Bruin reporting from the field, describe the events you would observe. (2 points)

The Parisian watched
 the Police do not arrest.

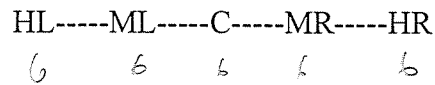
e. Write down a Nash Equilibrium that is not subgame perfect. [Hint: This is a hard question if you do it systematically but guessing is OK.] (2 points)



	if Burn		if watch	
	a	na	a	na
Burn run	-10, -10	-10, -10	*10, -2+	*10, -2+
Burn not	-9, 9+	-9, 9+	*10, -2	*10, -2
Watch run	*3, -4	*6, 0+	3, -4	6, 0+
Watch not	*3, -4	*6, 0+	3, -4	6, 0+
Stay here run	0, 0+	0, 0+	0, 0+	0, 0+
Stay here not	0, 0+	0, 0+	0, 0+	0, 0+

SPNE

Part IV. There are 30 voters in a district. Six are Hard Left (HL), six are Moderate Left (ML), six are centrists (C), six are Moderate Right (MR), and six are Hard Right (HR). We can place these voters on a left-right spectrum as follows.



There are two major candidates, Ann and Barbara, and one minor candidate Chuck. Each major candidate chooses her position to maximize the number of votes she receives. Chuck, the minor candidate, always chooses position C and does not care about how many votes he receives. In other words, the minor candidate Chuck is not a player in the game, since he always chooses position C.

Each voter votes for the candidate whose position is closest to his own. If two or more candidates are equally close, then votes are split equally among the two or more candidates which are closest.

For example, say Ann chooses HL and Barbara chooses HL. Then the HL voters are split between Ann and Barbara and the ML voters are split three ways between Ann, Barbara, and Charlie. Thus Ann gets half of the HL voters and one third of the ML voters, for a total of 5 votes.

For another example, if Ann chooses HL and Barbara chooses MR, then Ann gets 9 votes total (all 6 HL voters and half of the ML voters) and Barbara gets 12 votes total (all of the MR and HR voters).

Remember that Chuck is not a player. The sole "purpose" of Chuck is to suck votes away from the two major candidates.

Model this as a strategic form game and find all pure strategy Nash equilibria. (12 points)

	HL	ML	C	MR	HR
HL	5, 5	6, 6	8, 11	9, 12 ⁺	9, 9
ML	6, 6	6, 6	*12, 9	*12, 12 ⁺	*2, 9
C	11, 8	9, 12 ⁺	10, 10	9, 12	11, 8
MR	*12, 9	*12, 12 ⁺	*12, 9	6, 6	6, 6
HR	9, 9	9, 12 ⁺	8, 11	6, 6	5, 5

6 6 C 6 6
 A B ?

pure strategy
 NE: (ML, MR)
 (MR, ML)

(continue your work on this page if you need it)