Midterm answers PS 30 May 2002

1. Consider the following game.

	$2a$	$2b$	$2c$
$1a$	$0,9$	$3,0$	$1,5$
$1b$	$1,2$	$5,4$	$4,3$
$1c$	$0,6$	$2,1$	$6,7$

a. Find all pure strategy Nash equilibria of this game.
Using our $*$ and $+$ notation to indicate best responses, we get

	$2a$	$2b$	$2c$
$1a$	$0,9+$	$3,0$	$1,5$
$1b$	$*1,2$	$*5,4+$	$4,3$
$1c$	$0,6$	$2,1$	$*6,7+$

So $(1b,2b)$ and $(1c,2c)$ are the pure strategy Nash equilibria.

b. Find all mixed strategy Nash equilibria of this game. (Note: an answer like "$p = 2/3, q = 2/5$" is not sufficient. Please write down in a sentence which strategies are played with what probability.)
First you have to realize that $1b$ strongly dominates $1a$ and hence person 1 will never play $1a$. We iteratively eliminate $2a$ (once $1a$ is eliminated, $2c$ strongly dominates $2a$). We thus have the following game left over.

	$2b$	$2c$
$1b$	$5,4$	$4,3$
$1c$	$2,1$	$6,7$

Say that person 1 plays $1b$ with probability p and $1c$ with probability $1-p$. Say that person 2 plays $2b$ with probability q and $2c$ with probability $1-q$.
If person 1 plays $1b$, her expected utility is $5q+4(1-q)$. If person 1 plays $1c$, her expected utility is $2q+6(1-q)$. To find the "switchover probability," we set these equal: $5q+4(1-q) = 2q+6(1-q)$, or in other words $4+q = 6-4q$. Hence $5q = 2$ and so $q = 2/5$.
If person 2 plays $2b$, his expected utility is $4p+1(1-p)$. If person 2 plays $2c$, his expected utility is $3p+7(1-p)$. To find the "switchover probability," we set these equal: $4p+1(1-p) = 3p+7(1-p)$, or in other words $1+3p = 7-4p$. Hence $7p = 6$ and so $p = 6/7$.
Thus in the mixed Nash equilibrium, person 1 plays $1b$ with probability $6/7$ and $1c$ with probability $1/7$; person 2 plays $2b$ with probability $2/5$ and $2c$ with probability $3/5$.

c. Use the method of iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can't eliminate any more).
We did this already: $1b$ strongly dominates $1a$ and once $1a$ is eliminated, $2c$ strongly dominates $2a$.

2. Person 1 and Person 2 are competing for the affections of the extremely attractive Sandy. Person 1 and Person 2 plan to show up at Sandy's house at the same time on Saturday night to ask Sandy out. Since this is southern California, a crucial decision for both Person 1 and Person 2 is what kind of car to drive to Sandy's house. Person 1 is wealthy and can either have the butler prepare the impressive Lamborghini or the cute VW. Person 2 ordinarily drives a pickup truck, but can borrow a Lexus for the night from a roommate. If Sandy sees the Lamborghini and the Lexus pull up, Sandy chooses the Lamborghini because it is of course more impressive. If Sandy sees the Lamborghini and the pickup truck, Sandy chooses the pickup truck because the Lamborghini is clearly trying too hard. If Sandy sees the VW and the Lexus, Sandy chooses the Lexus because it is more impressive than the VW. If Sandy sees the VW and the pickup truck, Sandy chooses the VW because it is obviously more comfortable than the pickup truck.

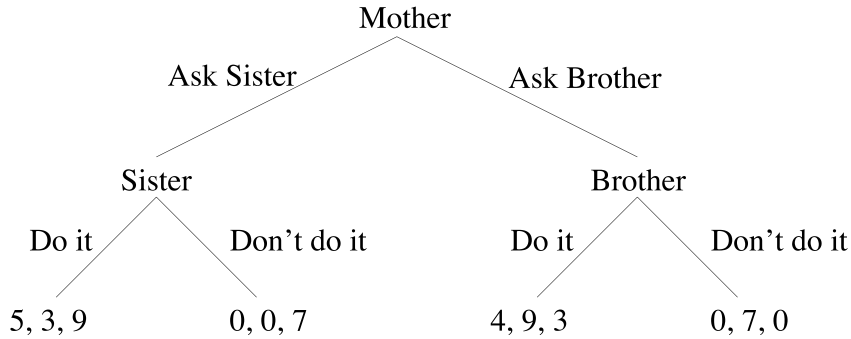
a. Model this as a strategic form game between Person 1 and Person 2 (assume Sandy is not a player).
Say that taking out Sandy yields a payoff of 1 and not taking Sandy out yields a payoff of 0. We then get the following game.

	Pickup truck	Lexus
Lamborghini	0,1	1,0
VW	1,0	0,1

b. Find all (pure strategy and mixed strategy) Nash equilibria of this game.
It is easy to see that there are no pure strategy Nash equilibria of this game. For mixed strategy Nash equilibria, say that person 1 plays Lamborghini with probability p and VW with probability $1-p$. Say that person 2 plays Pickup truck with probability q and Lexus with probability $1-q$.
If person 1 plays Lamborghini, her expected utility is $0q+1(1-q)=1-q$. If person 1 plays VW, her expected utility is $1q+0(1-q)=q$. To find the "switchover probability," we set these equal: $1-q=q$ and hence $q=1/2$.
If person 2 plays Pickup truck, his expected utility is $p+0(1-p)=p$. If person 2 plays Lexus, his expected utility is $0p+1(1-p)=1-p$. To find the "switchover probability," we set these equal: $p=1-p$ and so $p=1/2$.
Thus in the mixed Nash equilibrium, person 1 plays Lamborghini with probability 1/2 and VW with probability 1/2; person 2 plays Pickup truck with probability 1/2 and Lexus with probability 1/2.

3. Mother can ask either Sister or Brother to do the dishes while she goes out shopping. If Mother asks Sister, she can either do it or not do it. If Mother asks Brother, he can either do it or not do it. Since Sister does a better job, Mother prefers Sister doing the dishes over Brother doing the dishes. However, Mother prefers Brother doing the dishes over them not being done.
Both Sister's and Brother's preferences are like this: the best thing is for the other person to do the dishes; the second best thing is for Mother to ask the other person and have the other person not do it (since then the other person will get blamed). The third best thing is to do the dishes, and the worst thing is to be asked to do the dishes but then not do it (since you will get in trouble).

So the game looks like this (payoffs are written as (Mother, Sister, Brother)):



a. Represent this as a strategic form game.
The main thing here is to set up the players and strategies correctly. It is crucial that this is represented as a 3 person game.

	You do it	You don't
Mom asks you	5, 3, 9	0, 0, 7
Mom asks brother	4, 9, 3	4, 9, 3

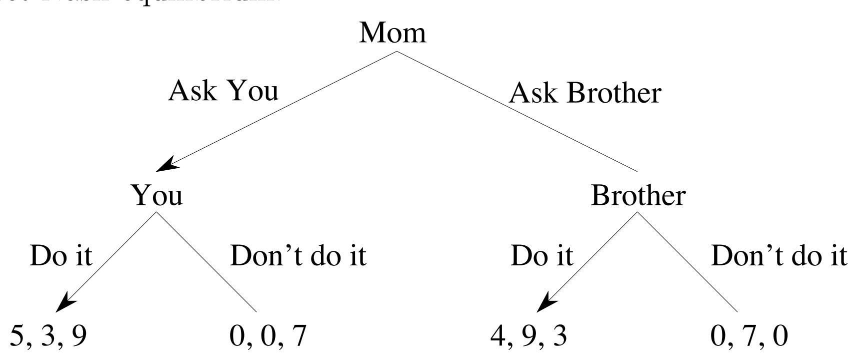
Brother does it

	You do it	You don't
Mom asks you	5, 3, 9	0, 0, 7
Mom asks brother	0, 7, 0	0, 7, 0

Brother doesn't

b. Find all (pure strategy) Nash equilibria.
By checking all the strategy profiles, we can see that (Mom asks you, You do it, Brother does it), (Mom asks you, You do it, Brother doesn't), and (Mom asks brother, You don't, Brother does it) are the three Nash equilibria.

c. Find all subgame perfect Nash equilibria.
By "backward induction," we find that (Mom asks you, You do it, Brother does it) is the subgame perfect Nash equilibrium.



4. Consider the "cross-out game." In this game, one writes down the numbers 1, 2, 3. Person 1 starts by crossing out any one number or any two adjacent numbers: for example, person 1 might cross out 1, might cross out 1 and 2, or might cross out 2 and 3. Then person 2 also crosses out either one number or two adjacent numbers. For example, starting from 1, 2, 3, say person 1 crosses out 1. Then person 2 can either cross out 2, cross out 3, or cross out both 2 and 3. Play continues like this. Once a number is crossed out, it cannot be crossed out again. Also, if for example person 1 crosses out 2 in her first move, person 2 cannot then cross out both 1 and 3, because 1 and 3 are not adjacent (even though 2 is crossed out). The winner is the person who crosses out the last number.

a. Model this as an extensive form game. Show a subgame perfect Nash equilibrium of this game by drawing appropriate arrows in the game tree.
Please see the attached sheet.

b. Now instead of just three numbers, say that you start with m numbers. In other words, you have the numbers $1, 2, 3, \ldots, m$. Can person 1 always win this game? (Hint: look at $m = 4$, $m = 5$, etc. first to get some ideas.)
Person 1 can always win this game. His strategy is follows. If m is even, then person 1 starts by crossing out the two middle numbers. If m is odd, then person 1 starts out by crossing out the middle number. Henceforth, whatever number(s) person 2 crosses out, person 1 responds by crossing out the "mirror image" of those number(s) on the "other side" (if person 2 crosses out the number x, person 1 crosses out $m - x + 1$). Play continues like this, with each time person 1 responding in this manner to person 2. It is easy to see that person 1 is guaranteed to cross out the last number.
For example, say that $m = 7$. Person 1 crosses out 4, the middle number. If person 2 crosses out 1, then person 1 responds by crossing out 7. If person 2 then crosses out 5 and 6, then person 1 responds by crossing out 2 and 3.
Notice that this game is not related to the Nim game in the homework (there is no issue of multiples of 3, for example). Answers of the form "Person 1 will always win by crossing out the middle numbers" receive no credit without a full explanation like the one above.