Last name	First name	Student ID number

Part 1

Midterm exam PS 30 November 2012

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part is weighted equally (12 points each). Each part is stapled separately (for ease in grading, since the parts are graded separately). Please make sure that your name and student ID number is on each part.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1	
2	
3	
4	
total	

Part 1. Ann, Betty and Cindy are sisters who win a free ticket to Paris. Unfortunately, this ticket is just for two persons. Each person names the sister whom they want to go with. For example, Ann can name either Betty or name Cindy. Betty can name either Ann or Cindy.

If two names match, those matched persons go to Paris together. For example, if Ann names Cindy and Cindy names Ann, then Ann and Cindy go to Paris together (Betty stays home). A person who does not match stays at home. If nobody matches, the sisters give the ticket to their parents and the three sisters stay at home.

Say that a sister doesn't care about which sister she goes with. Each sister simply prefers going to Paris over staying at home. So a sister gets payoff 1 if she goes to Paris, and a sister gets payoff 0 if she stays at home.

a. Model this as a strategic form game. For each sister, use the payoffs 0 or 1. (3 points)

b. Find all pure strategy Nash equilibria of this game. (3 points)

c. Now, assume that Ann does not want go to Paris with Betty. Ann thinks Betty will only go shopping in Paris instead of visiting museums and experiencing its diverse culture. So Ann would rather stay at home than go to Paris with Betty. But Ann still prefers going to Paris with Cindy rather than staying at home.

Also, Betty does not want to go to Paris with Cindy. Betty thinks Cindy is too picky and always complains about something. So Betty would rather stay at home than go to Paris with Cindy. Betty still prefers going to Paris with Ann rather than staying at home.

Cindy really cares about her parents. For Cindy, the best thing is if her parents get the ticket. Cindy also prefers staying at home over going to Paris because she enjoys spending time at home with her parents. If she goes to Paris, Cindy is indifferent between Ann and Betty as her partner.

Model this as a strategic form game. For each sister use the payoffs 1 (best), 0 (second-best), or -1 (worst). (2 points)

d. Make a prediction in this game using the iterated elimination of weakly dominated strategies. (2 points)

e. Find all pure strategy Nash equilibria of this game. (2 points).

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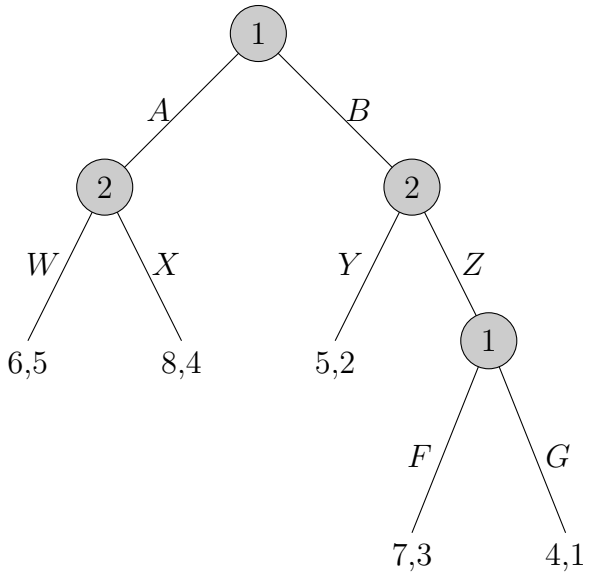
Part 2

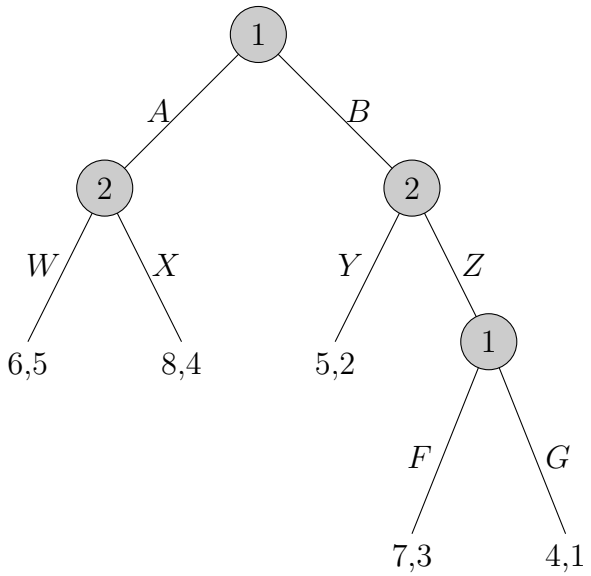
Part 2. Consider the following extensive form game.



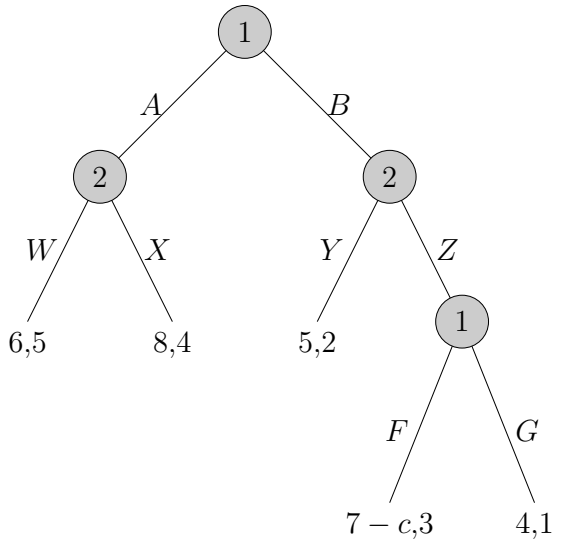
a. Represent this as a strategic form game. (3 points)

b. Find all pure strategy Nash equilibria of this game. (2 points)

c. Find all subgame perfect Nash equilibria of the game. We write the game again for convenience. (2 points)



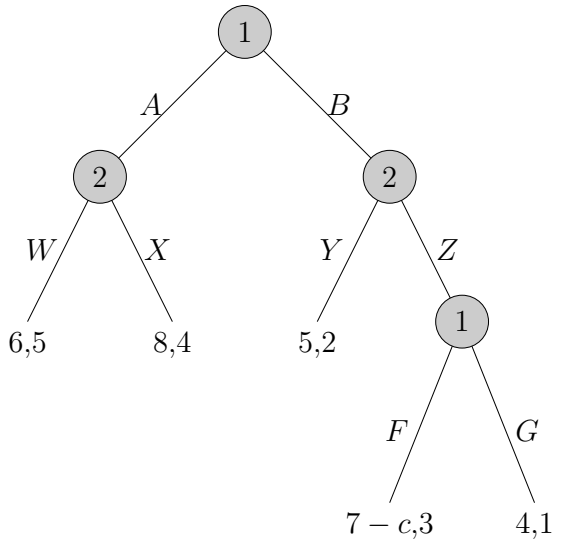
Now change the game slightly so that the payoff 7 is now the payoff $7 - c$. The new game is below.



d. Fill in the table below. (2 points)

If	then the subgame perfect Nash equilibrium is/are:
c = 0	
c = 2	
c = 4	

Here is the game again.



e. For most values of c, there is only one subgame perfect Nash equilibrium. For example, if $c = 0$, there is only one subgame perfect Nash equilibrium.

However, for some "special" value(s) of c, there exists more than one subgame perfect Nash equilibrium. Find all such "special" value(s) of c. For each "special" value of c, find all subgame perfect Nash equilibria. (3 points)

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Part 3

Part 3. Consider the following game.

	left	right
up	4, 2	3, 5
down	3, 4	5, 3

a. Find all (pure strategy or mixed strategy) Nash equilibria of this game. (2 points)

b. Now change the game by doubling all payoff numbers. Write down this new game and find all (pure strategy or mixed strategy) Nash equilibria. Do the Nash equilibria change when all payoff numbers are doubled? If so, show how they change. If not, explain why they do not. (2 points)

c. Now go back to the original game and say that all payoffs are multiplied by k, where k is a positive number ($k > 0$). Thus we have the following game. (For example, when $k = 2$, all payoff numbers are doubled.)

	left	right
up	$4k, 2k$	$3k, 5k$
down	$3k, 4k$	$5k, 3k$

Do the (pure strategy or mixed strategy) Nash equilibria you found in part a. change when k changes? If so, show how they change. If not, explain why they do not. (4 points)

d. Now say that we have the original game and we multiply k to person 1's payoffs when she plays "up" and we multiply k to person 2's payoffs when he plays "left." Again, k is a positive number ($k > 0$). Thus we have the following game.

	left	right
up	$4k, 2k$	$3k, 5$
down	$3, 4k$	$5, 3$

Note that if k is very large (for example, if $k = 10$) then "up" strongly dominates "down" and "left" strongly dominates "right."

What is the largest k such that there exists a mixed strategy Nash equilibrium of this game (in other words, a Nash equilibrium in which at least one person mixes)? (4 points)

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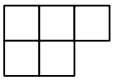
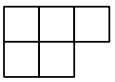
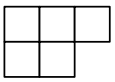
Part 4

Part 4. Two people play the game of Battleship on a board with five squares, as shown below.

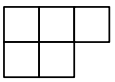


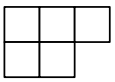
Player 1 fires a missile into one of the five squares. She has five possible strategies. The following are two examples.





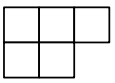
Player 2 places a battleship on the board. The battleship occupies two adjacent squares and can be oriented vertically or horizontally. He has five possible strategies. The following are two examples.

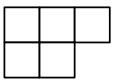




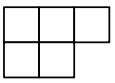
If player 1's missile lands where player 2's battleship is, then player 1 wins: player 1 gets payoff 1 and player 2 gets payoff 0. If player 1's missile misses player 2's battleship, then player 2 wins: player 1 gets payoff 0 and player 2 gets payoff 1.

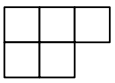
For example, here is a situation in which player 1 wins.





Here is a situation in which player 2 wins.





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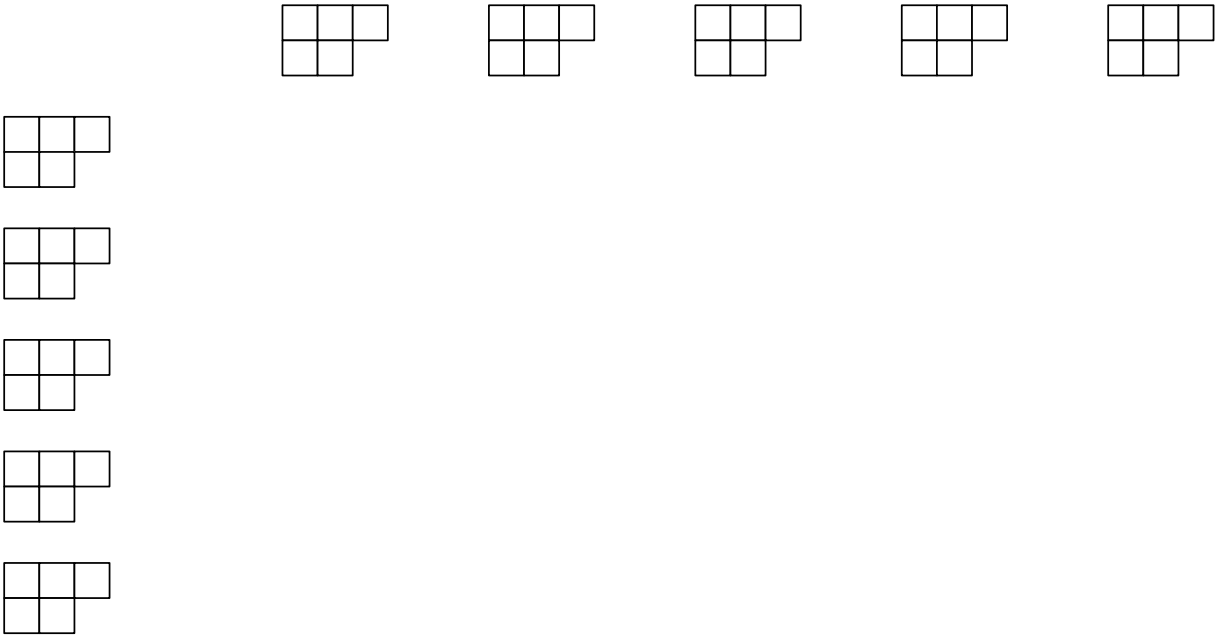
a. Represent this game as a strategic form game. (4 points)



b. Use the method of iterative elimination of strongly or weakly dominated strategies. You should be able to reduce it down to a 2×2 game (a game in which player 1 has two strategies and player 2 has two strategies) by trying out different orders of elimination. Please write down the order of elimination. (4 points)

c. Write down the remaining 2×2 game and find all (pure strategy and mixed strategy) Nash equilibria of this 2×2 game. (4 points)