Midterm exam   PS 30   November 2009

Name:

TA:

Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has four parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1	
2	
3	
4	
total	

Part 1. Consider the following game:

	2a	2b	2c
1a	4, 3	4, 5	3, 4
1b	3, 4	3, 10	2, 5
1c	9, 5	1, 2	7, 6

a. (4 points) Find all pure strategy Nash equilibria of this game.

b. (4 points) Use the method of iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can't eliminate any more). Indicate what kind of domination is appropriate at each step.

c. (4 points) After you have iteratively eliminated as much as you can, find all mixed strategy Nash equilibria of the remaining game. (Note: an answer like $p = 2/3$, $q = 2/5$ is not sufficient. Please write down in a sentence which strategies are played with what probability.)

Part 2. One day while lazily riding the bus, dreaming of faraway places, you notice a rather scary looking individual take something out of a woman's purse. The woman does not notice, but the person to the left of you and the person to the right of you also see the same thing. You would feel really bad if the pickpocket got away because all three of you stayed quiet. The only thing worse than that is if you confront the pickpocket alone and possibly get into a fight. You think to yourself: "two of us could confront him but it would still be risky for me." You would prefer that one of the others say something or even better that all three of you speak out. The best thing for you, however, is if the other two confront him and you can stay quiet—the pickpocket gets caught and you don't have to risk your new Armani glasses. Curiously enough, you also notice and admire the new sunglasses of the two people next to you and conclude, "they're probably thinking just like me."

a. (3 points) Model this as a strategic form game only using the numbers $\{10, 8, 5, 1, -5, -10\}$ as payoffs.
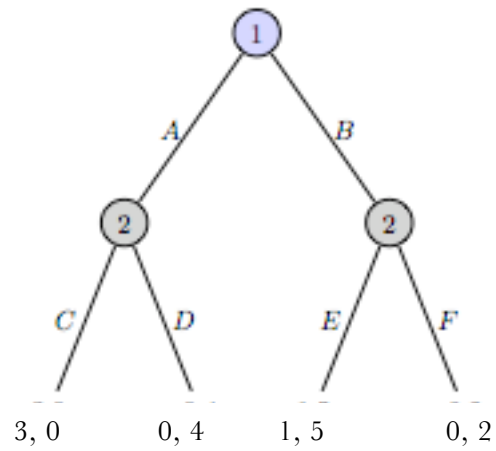
b. (2 points) Find all pure strategy Nash equilibria of this game.

c. (2 points) Does this game have a mixed strategy Nash equilibrium? If it does, find it. If not, explain why not.

d. (3 points) Now say that the three of you have vigilante urges and each of you wants to get sole credit for confronting the pickpocket. Now the best thing is to confront the pickpocket alone and the worst thing is to stay quiet and have the other two confront him, which makes you look like a coward. In other words, the situation is the same as in part a. above, but what was the best is now the worst and what was the worst is now the best. All other preferences remain the same. Model this as a strategic form game only using the numbers {10, 8, 5, 1, -5, -10} as payoffs.

e. (2 points) Find all pure strategy Nash equilibria of this new game.

Part 3. Consider the following game. Player 1 chooses first, and her alternatives are A and B. Then player 2 chooses between C and D if Player 1 chose A, and player 2 chooses between E and F if player 1 chose B. Payoffs are shown as (Player 1's payoff, Player 2's payoff).
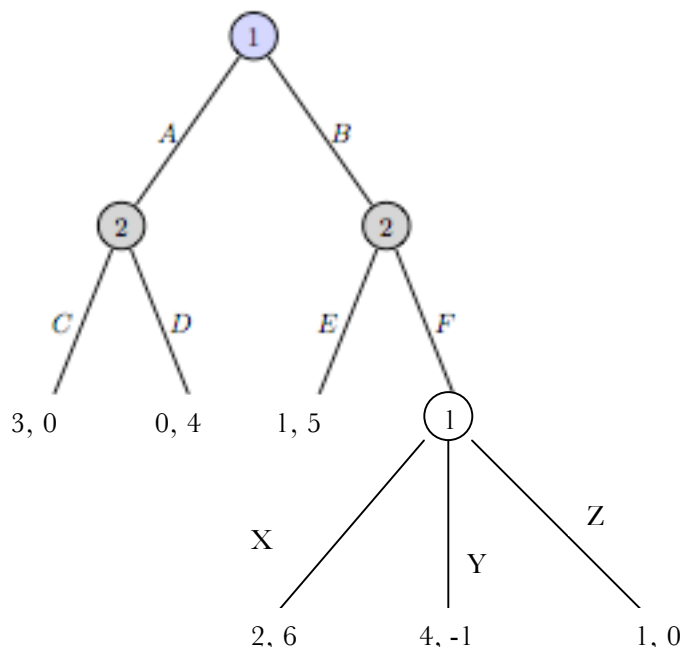

a. (2 points) Write this extensive form game as a strategic form game.

b. (2 points) Find all of the pure strategy Nash equilibria of this game.

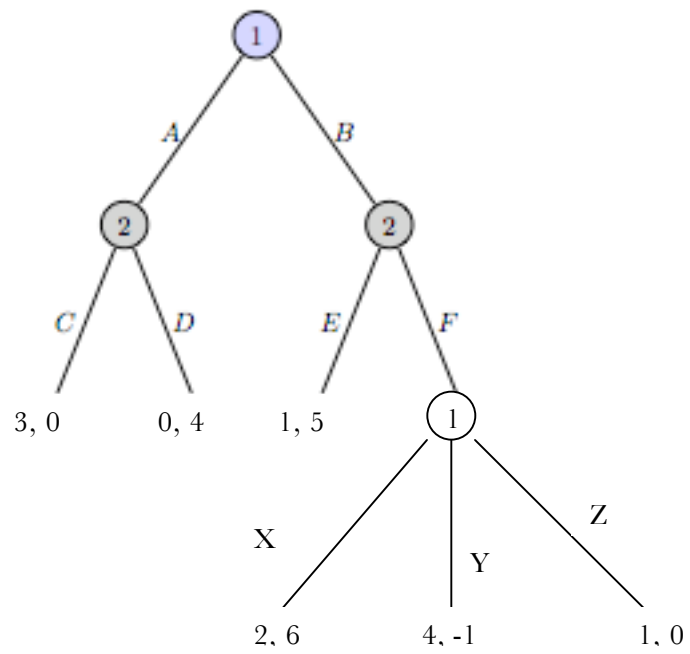c. (2 points) Find all subgame perfect Nash equilibria of this game.

d. (2 points) Now change the game a bit. Now when player 1 plays B and player 2 plays F, then player 1 gets to choose among X, Y, and Z.

Write this as a strategic form game.

e. (2 points) Find all of the pure strategy Nash equilibria of this game.

f. (2 points). Find all subgame perfect Nash equilibria of this game. The game is written again below for your convenience.

```
                         (1)
                        /    \
                     A /      \ B
                      /        \
                   (2)          (2)
                  /   \        /    \
                C/     \D    E/      \F
                /       \    /        \
             3, 0     0, 4  1, 5      (1)
                                     / | \
                                  X/  |  \ Z
                                  /   |Y  \
                               2, 6  4,-1  1, 0
```

Part 4. Each of two gamblers has two possible actions, **Bet** and **Challenge**. Each pair of actions results in the gamblers receiving amounts of money shown below. So for example if Gambler 1 chooses **Bet** and Gambler 2 chooses **Challenge**, then Gambler 1 receives nothing and Gambler 2 receives $4.

	Bet	**Challenge**
Bet	2, 2	0, 4
Challenge	4, 0	1, 1

Now suppose the gamblers actually care about each other's welfare and thus the preferences of each gambler are represented by the following payoff function:

$$m_i + \alpha m_j$$

where m_i is the amount of money received by gambler i for each outcome in the game above, m_j is the amount of money the other player receives, and α is a positive number. For example, gambler 1's payoff when they play (**Bet**, **Challenge**) is $0 + 4\alpha$.

a. (2 points) Write this as a strategic form game.

b. (2 points) Find all pure and mixed strategy Nash equilibria of this game when $\alpha=1/2$.

c. (4 points) Take the game you wrote down in part a. Consider the following statement (where K and L are two numbers):

"If $\alpha < K$, then the game does not have a mixed strategy Nash equilibrium. If $L < \alpha$, then the game does not have a mixed strategy Nash equilibrium. If $K < \alpha$ and $\alpha < L$, then the game does have a mixed strategy Nash equilibrium."

Do there exist K and L which make this statement true? If so, find K and L (your answer will look something like "K=2/27, L=7/5"; in other words, give explicit numerical values). If not, explain why not.

d. (2 points) Is it possible for this game to have more than two pure strategy Nash equilibria? If so, find the value(s) of α such that the game has more than two pure strategy Nash equilibria. If not, explain why not.

e. (2 points) Is it possible for this game to have no pure strategy Nash equilibria? If so, find the values of α such that the game has no pure strategy Nash equilibria. If not, explain why not.