

Midterm exam

PS 30

May 2005

Name:

TA:

Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are four sections of this exam. Each section is weighted equally (12 points for each section). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the section you have questions about (if you have a question on Section II, hold up two fingers). When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will not be accepted. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

Part I. Consider the following game.

| | 2a | 2b | 2c |
|----|-------|-------|-------|
| 1a | 4, -1 | 3, 0 | -3, 1 |
| 1b | -1, 1 | 2, 2 | 2, 3 |
| 1c | 2, 1 | -1, 1 | 0, 4 |
| 1d | 4, 6 | -3, 0 | 1, 4 |

(a) Find all pure strategy Nash equilibria (4 points).

(b) Use the method of iterative elimination of strongly and weakly dominated strategies and eliminate as many strategies as possible (4 points).

(c) After you have eliminated as many strategies as possible, find the mixed strategy Nash equilibria of the REMAINING game (4 points).

Part II.

Consider the following game. In this game, when Person 1 plays 1b and Person 2 plays 2b, both people get payoff x . For example, if $x=5$, then 2b strongly dominates 2a.

| | | |
|----|------|--------|
| | 2a | 2b |
| 1a | 2, 3 | 7, 6 |
| 1b | 5, 4 | x, x |

(a) What is the largest value of x such that there are no strongly dominated strategies? (4 points)

(b) What is the smallest value of x such that (1b, 2b) is a Nash equilibrium? (4 points)

(c) Assume that $x < 0$. Find all (pure strategy and mixed strategy) Nash equilibria of this game (4 points).

Part III.

The US (Player 1), Turkey (Player 2) and Spain (Player 3) are all in an alliance together and are trying to decide if they should invade Iraq or not. Each country has one vote and can either vote to invade or vote to not invade. They will only invade if the vote to invade is unanimous. The US always prefers to invade and will be unhappy if there is no invasion. Turkey's happiness, on the other hand, is entirely connected to what Spain does. It does not really care if there is an invasion or not; its only concern is that it votes exactly as Spain does. Turkey wants to vote to invade only if Spain also does; otherwise it does not want to go to war. Spain, much like Turkey, doesn't care if there's an invasion or not. It wants to vote to invade only if either Turkey *or* the US (or both) votes to invade.

(a) Using 10 as the high payoff and 0 as the low payoff, model this as a strategic form game. (2 points)

(b) Find the pure strategy Nash equilibria of this game. (2 points)

(c) Are there any weakly or strongly dominated strategies? If so, what are they? What prediction do you get using the method of iterative elimination of weakly and strongly dominated strategies? (2 points)

(d) Now change the game slightly. What happens if we say that Spain is now more like the US and it does care about going to war? Spain now opposes the war and is happy as long as there is no invasion. The other players' preferences remain the same. Model this as a strategic form game, again using 10 as the high payoff and 0 as the low payoff. (2 points)

(e) Find the pure strategy Nash equilibria of this game. (2 points)

(f) Are there any weakly or strongly dominated strategies? If so, what are they? What prediction do you get using the method of iterative elimination of weakly and strongly dominated strategies? (2 points)

Part IV.

You are taking your date to a concert but as you are driving you realize that you are getting late. You have to choose whether to speed on the PCH highway or obey the posted limit.

PCH is sometimes patrolled by cops however, and if they are on patrol and you speed, they will give you a citation and that will result in a big embarrassment for you; you will also get to the concert even later than if you had obeyed the limit. If they are not on patrol and you speed, you will get there on time and impress your date. If you don't speed, you don't care what the cops do.

Cops can either patrol PCH or go get doughnuts. If they patrol and catch you speeding, they get a reward (and are very happy). If you speed on PCH while they are having doughnuts they will look like they are not doing their job, and they will be quite unhappy. If you are not speeding, they would rather get doughnuts than patrol.

(a) Model this as a strategic form game. For the cops' payoffs, use only the numbers 1, 2, 3, 4. For your payoffs, use only the numbers 1, 2, 3. (4 points)

(b) Find all (pure strategy and mixed strategy) Nash equilibria of this game. (6 points)

(c) Say that the cops' payoff for catching speeders doubles. How would this affect the cops' behavior? How would it affect your behavior? (2 points)