

Midterm exam      PS 30      May 2002

*This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Those involved in academic dishonesty will be summarily given a grade of F for the course; I will also write a letter recommending expulsion. Partial credit will be given: math mistakes will not jeopardize your grade. The problems all have the same weight. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.*

*Make sure that your complete name is on each page. Number each page like this: "page 1 of 5," "page 2 of 5," and so forth (if you have 5 total pages for example). Write only on the front of each page. Staple your pages together when you turn them in and write your name down on the log. Good luck!*

1. Consider the following game.

	$2a$	$2b$	$2c$
$1a$	0, 9	3, 0	1, 5
$1b$	1, 2	5, 4	4, 3
$1c$	0, 6	2, 1	6, 7

a. Find all pure strategy Nash equilibria of this game.

b. Find all mixed strategy Nash equilibria of this game. (Note: an answer like " $p = 2/3, q = 2/5$ " is not sufficient. Please write down in a sentence which strategies are played with what probability.)

c. Use the method of iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can't eliminate any more).

2. Person 1 and Person 2 are competing for the affections of the extremely attractive Sandy. Person 1 and Person 2 plan to show up at Sandy's house at the same time on Saturday night to ask Sandy out. Since this is southern California, a crucial decision for both Person 1 and Person 2 is what kind of car to drive to Sandy's house. Person 1 is wealthy and can either have the butler prepare the impressive Lamborghini or the cute VW. Person 2 ordinarily drives a pickup truck, but can borrow a Lexus for the night from a roommate. If Sandy sees the Lamborghini and the Lexus pull up, Sandy chooses the Lamborghini because it is of course more impressive. If Sandy sees the Lamborghini and the pickup truck, Sandy chooses the pickup truck because the Lamborghini is clearly trying too hard. If Sandy sees the VW and the Lexus, Sandy chooses the Lexus because it is more impressive than the VW. If Sandy sees the VW and the pickup truck, Sandy chooses the VW because it is obviously more comfortable than the pickup truck.

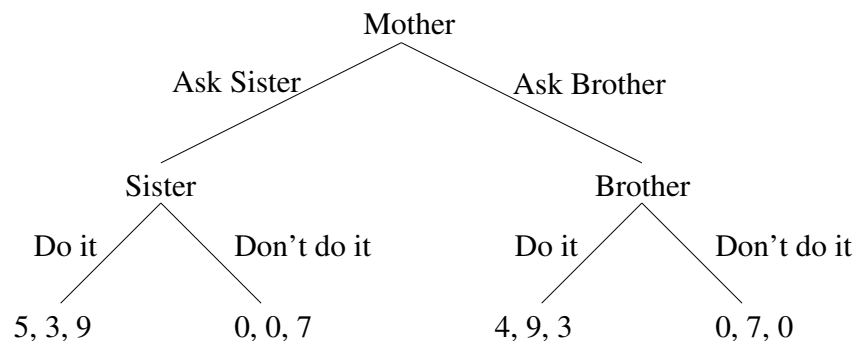
a. Model this as a strategic form game between Person 1 and Person 2 (assume Sandy is not a player).

b. Find all (pure strategy and mixed strategy) Nash equilibria of this game.

3. Mother can ask either Sister or Brother to do the dishes while she goes out shopping. If Mother asks Sister, she can either do it or not do it. If Mother asks Brother, he can either do it or not do it. Since Sister does a better job, Mother prefers Sister doing the dishes over Brother doing the dishes. However, Mother prefers Brother doing the dishes over them not being done.

Both Sister's and Brother's preferences are like this: the best thing is for the other person to do the dishes; the second best thing is for Mother to ask the other person and have the other person not do it (since then the other person will get blamed). The third best thing is to do the dishes, and the worst thing is to be asked to do the dishes but then not do it (since you will get in trouble).

So the game looks like this (payoffs are written as (Mother, Sister, Brother)):



a. Represent this as a strategic form game.

b. Find all (pure strategy) Nash equilibria.

c. Find all subgame perfect Nash equilibria.

4. Consider the “cross-out game.” In this game, one writes down the numbers 1, 2, 3. Person 1 starts by crossing out any one number or any two adjacent numbers: for example, person 1 might cross out 1, might cross out 1 and 2, or might cross out 2 and 3. Then person 2 also crosses out either one number or two adjacent numbers. For example, starting from 1, 2, 3, say person 1 crosses out 1. Then person 2 can either cross out 2, cross out 3, or cross out both 2 and 3. Play continues like this. Once a number is crossed out, it cannot be crossed out again. Also, if for example person 1 crosses out 2 in her first move, person 2 cannot then cross out both 1 and 3, because 1 and 3 are not adjacent (even though 2 is crossed out). The winner is the person who crosses out the last number.

a. Model this as an extensive form game. Show a subgame perfect Nash equilibrium of this game by drawing appropriate arrows in the game tree.

b. Now instead of just three numbers, say that you start with  $m$  numbers. In other words, you have the numbers 1, 2, 3,  $\dots$ ,  $m$ . Can person 1 always win this game? (Hint: look at  $m = 4$ ,  $m = 5$ , etc. first to get some ideas.)