Homework 8 PS 30 November 2013

1. Say that we have a threshold model in which there are 5 people. If the total number of other people who participate is greater or equal to a person's threshold, the person wants to participate also. If the total number of other people who are participating is less than a person's threshold, the person does not want to participate.

a. Say that one person has threshold 1, two people have threshold 2, and two people have threshold 4. Find all of the pure strategy Nash equilibria.

b. Now say that one of the threshold 2 people becomes a threshold 0 person. Find all of the pure strategy Nash equilibria. Does this change guarantee some level of participation?

2. Say that you have a group of 50 people who can either buy a color fax machine or not buy. No one wants to buy a color fax machine if no one else has one (because there would be no one to exchange color faxes with). In fact, each person will buy one only if at least 6 other people buy them. Thus each person has a threshold of 6.

a. Find the two pure strategy Nash equilibria.

b. Now say that you are a sales rep for the color fax machine company. You can offer discount coupons to potential customers. If you give 1 discount coupon to someone, that decreases their threshold by 1. For example, if you give 6 discount coupons to a single person, you can make that person have threshold 0. If you give 2 discount coupons to a single person, you can make that person have threshold 4. Obviously, you can guarantee that everyone will buy a color fax machine by giving all 50 people six coupons each, but that would be silly (the company would get no profits). Using the fewest possible number of coupons, how can you guarantee that everyone will buy a color fax machine?

3. [from Spring 2003 final] Say that we have 10 people. Each person is thinking about whether or not to join a revolt or not. Each person has a threshold: five people have threshold 4 and five people have threshold 6.

a. Find all pure strategy Nash equilibria of this game.

b. Say that you have some discount coupons which lower the cost of revolting and hence lower a person's threshold. For example, if I give 3 coupons to a person with threshold 4, she now has threshold 1. If I give 1 coupon to a person with threshold 6, he now has threshold 5. By giving out coupons, I can change the game so that the only Nash equilibrium is one in which everyone revolts. I want to do this by giving out the fewest number of coupons. How do I distribute the coupons (who gets coupons, and how many does each person get)?

c. Now say that you can give tickets which raise the cost of revolting and hence raise a person's threshold. For example, if I give 3 tickets to a person with threshold 4, she now has threshold 7. If I give 2 tickets to a person with threshold 6, he now has threshold 8. By giving out tickets, I can change the game so that the only Nash equilibrium is one in which no one revolts. I want to do this by giving out the fewest number of tickets. How do I distribute the tickets (who gets tickets, and how many does each person get)?

4. Say that there are four people: Alicia, Betsy, Carlos, and Davis. Each can choose whether to wear platform sandals or not. Alicia is very fashion-forward and will wear them even if no one else wears them; in fact, if more than one other person wears them, she won't wear them anymore because she hates being part of a crowd. Betsy also likes fashion but is not as cutting-edge: she will wear them if at least one other person wears them, but like Alicia, hates being "part of the crowd" and will not wear them if more than two other people wear them. Carlos thinks of himself as hip, but is kind of slow on the uptake and will wear them if at least two others wear them. Still, Carlos has at least some fashion pride and will not wear them if everyone else wears them. Finally, Davis gets his fashion tips from the JC Penney catalog and will wear them if everyone else wears them.

a. Say that at the beginning, no one wears platform sandals (they have just hit the market). Show that at first the sales of platform sandals steadily grow, but eventually sales "cycle" between high and low in a never-ending "fashion cycle."

b. Are there any pure strategy Nash equilibria of this game?

5. Say that there are three men A, B, and C and three women X, Y, and Z. Each of these six people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman X best, woman Y next, and woman Z least. Man B prefers woman Y best, woman Z next, and woman X least. Man C prefers woman Y best, woman X next, and woman Z least. Woman X prefers man B best, man A next, and man C least. Woman Y prefers man A best, man B next, and man C least. Woman Z prefers man A best, man C next, and man B least.

a. Say that man A is matched with woman X, man B is matched with woman Y, and man C is matched with woman Z. Is this matching stable?

b. Write down all possible matchings and determine which of them are stable and which are not stable.

c. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?

6. Say that there are four men, A, B, C, and D and four women W, X, Y, and Z. Each of these eight people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman W best, X next, Y, next, and Z least. Man B's preference ordering (from best to worst) is Y, X, W, Z. Man C's preference ordering is Y, Z, X, W. Man D's ordering is Y, Z, X, W. Woman W's preference ordering (from best to worst) is D, C, B, A. Woman X's preference ordering is C, B, A, D. Woman Y's ordering is D, A, C, B. Woman Z's ordering is B, C, D, A.

a. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?