Homework IEE1149 Summer 2015

1. Read "You Yawn, We All Yawn—And Empathy May Explain Why" by Alison McCook (available at the "Media Clips" link on the course website). Is the explanation here of why a person yawns a rational choice explanation? Why or why not?

2. Read "Which Price is Right?" by Charles Fishman (available at the "Media Clips" link). The article mentions a behavior which seems to be a violation of our simple model of individual choice as we defined it in class. Point out the behavior and explain why a rational choice model (at least the most obvious one) cannot capture this behavior. What do you think explains the behavior?

3. Read "Does Blanket 'Don't Go to Graduate School!' Advice Ignore Race and Reality?" by Tressie McMillan Cottom (available at the "Media Clips" link). Can you make McMillan Cottom's argument using our simple model of individual choice? Or is McMillan Cottom making a different kind of argument?

4. Say that you and a friend are meeting for lunch. Both you and your friend can either be late or on time. If both of you are on time, you each get a utility of 3. If one is on time and the other is late, the prompt one gets a utility of 1 (since she has to wait around doing nothing) and the tardy one gets a utility of 4 (since she doesn't have to wait). However, if both are late, you don't find each other and you each get a utility of 0.

a. Model this as a strategic form game.

5. Say you and a friend each privately choose a whole number between 0 and 5 (that is: 0, 1, 2, 3, 4, or 5). If you both choose the same number, I will give you both that number times \$100. If your number is exactly one less than your opponent's, however, you will get your opponent's number times \$100 plus a bonus \$100 and your opponent will get nothing. In any other case, both of you get nothing. So for example, if you both choose the number 5, I will give you both \$500. If you choose 4 and your opponent chooses 5, you will get \$600 and your opponent nothing. If you choose 3 and your opponent chooses 5, you both get nothing.

a. Model this as a strategic form game.

b. Read the article "Hollywood's Death Spiral" by Edward Jay Epstein on the web site. Can you use this game to think about the situation described in the article?

6. Ann and Bob are each trying to win a prize in a school raffle (lottery). Each can buy either 0, 1, 2, or 3 raffle tickets. Ann and Bob are the only two people in the raffle, and each ticket has an equal chance of winning. So for example, if Ann buys 2 tickets and Bob buys 3 tickets, then Ann has a 2/5 chance of winning and Bob has a 3/5 chance of winning (if no one buys any tickets, the raffle is cancelled). The prize is worth $60, and both Ann and Bob care about their "expected payoffs": for example, if Ann has a 2/5 chance of winning, her expected payoff is $24. Model the following situations with strategic form games.

a. Say that raffle tickets are free. What does the game look like?

b. Now say that raffle tickets cost $6 each. What does the game look like?

c. Now say that raffle tickets cost $10 each. What does the game look like?

7. Say that Spy 1 is trying to listen in on Spy 2. There are three rooms, A, B, and C, arranged in a line like this: A—B—C. In other words, A is on the left, B is in the middle, and C is on the right. Each spy must decide independently and simultaneously which room to enter. Their payoffs are determined as follows. If they both choose the same room, then they will see each other, a bloody gun battle will ensue, and both get payoff -10. If they are in adjacent rooms (for example, if Spy 1 is in room A and Spy 2 is in room B) then Spy 1 can set up her eavesdropping equipment and can intercept Spy 2's communications; hence Spy 1 gets a payoff of 5 and Spy 2 gets a payoff of -5. If they are not in adjacent rooms and they are not in the same room (for example, if Spy 1 is in room A and Spy 2 is in room C) then the distance between them is too great for the eavesdropping equipment to work; Spy 1 gets no secrets and Spy 2 gets to keep hers, and so both get a payoff of 0.

a. Model this as a strategic form game.

8. Say that persons 1, 2, and 3 each decide whether to go to restaurant A or restaurant B. Person 1 wants the dinner group to be as large as possible. For person 1, the worst thing is if she goes to a restaurant alone, the best thing is if all three go to the same place, and going with one person (it doesn't matter which) is OK, neither best or worst. Person 2 is the exact opposite; she wants the dinner group to be as small as possible. All person 3 cares about is going to the same place as person 1, since he likes person 1.

a. Model this as a strategic form game.

b. Now say that person 3 loses interest in person 1 and becomes grouchy like person 2. Model this as a strategic form game.

9. Say that there are two people, a security guard and a thief. The security guard can either be vigilant or relax. The thief can either steal or do nothing. If the guard is vigilant, then the thief would rather do nothing than steal. If the guard is relaxed, however, the thief would rather steal than do nothing. If the thief steals, the guard would rather be vigilant than be relaxed. If the thief does nothing, however, the guard would rather be relaxed than vigilant.

a. Model this as a strategic form game. Feel free to choose payoffs which make sense to you.

10. Read "Running Mates: The Clark-Lieberman Iowa Jailbreak" by William Saletan (at the "Media Clips" link). Model the situation as a strategic form game.

11. In the movie *Return to Paradise* (see the "Media Clips" link), Sheriff, Tony, and Lewis went to Malaysia and did various illegal things. After Sheriff and Tony left, Lewis was charged and scheduled to be executed. If either Sheriff or Tony returns to Malaysia and admits shared responsibility, Lewis's sentence will be reduced and he will live. If both Sheriff and Tony return, then each will have to serve three years in prison in Malaysia. If only one returns, then that person will have to serve six years in prison. The two players, Sheriff and Tony, can each either decide to go back to Malaysia or stay in New York. Model the situation as a game with two players (Sheriff and Tony). Feel free to choose payoffs which make sense to you (that's what makes the problem kind of interesting).

Earlier you set up the various games, and now you make predictions for them by the two methods of (1) iteratively eliminating dominated strategies and (2) finding Nash equilibria. For clarity's sake, the "new parts" are in bold.

12. Say that you and a friend are meeting for lunch. Both you and your friend can either be late or on time. If both of you are on time, you each get a utility of 3. If one is on time and the other is late, the prompt one gets a utility of 1 (since she has to wait around doing nothing) and the tardy one gets a utility of 4 (since she doesn't have to wait). However, if both are late, you don't find each other and you each get a utility of 0.

a. Model this as a strategic form game.

b. **Are there strongly or weakly dominated strategies in this game?**

c. **Find the (pure strategy) Nash equilibria of this game.**

13. Say you and a friend each privately choose a whole number between 0 and 5 (that is: 0, 1, 2, 3, 4, or 5). If you both choose the same number, I will give you both that number times $100. If your number is exactly one less than your opponent's, however, you will get your opponent's number times $100 plus a bonus $100 and your opponent will get nothing. In any other case, both of you get nothing. So for example, if you both choose the number 5, I will give you both $500. If you choose 4 and your opponent chooses 5, you will get $600 and your opponent nothing. If you choose 3 and your opponent chooses 5, you both get nothing.

a. Model this as a strategic form game.

b. Read the article "Hollywood's Death Spiral" by Edward Jay Epstein on the web site. Can you use this game to think about the situation described in the article?

c. **"Solve" this game by iteratively eliminating weakly dominated strategies.**

d. **Find the (pure strategy) Nash equilibria of this game.**

14. Ann and Bob are each trying to win a prize in a school raffle (lottery). Each can buy either 0, 1, 2, or 3 raffle tickets. Ann and Bob are the only two people in the raffle, and each ticket has an equal chance of winning. So for example, if Ann buys 2 tickets and Bob buys 3 tickets, then Ann has a 2/5 chance of winning and Bob has a 3/5 chance of winning (if no one buys any tickets, the raffle is cancelled). The prize is worth \$60, and both Ann and Bob care about their "expected payoffs": for example, if Ann has a 2/5 chance of winning, her expected payoff is \$24. Model the following situations with strategic form games.

a. Say that raffle tickets are free. What does the game look like? **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

b. Now say that raffle tickets cost \$6 each. What does the game look like? **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

c. Now say that raffle tickets cost \$10 each. What does the game look like? **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

15. Say that Spy 1 is trying to listen in on Spy 2. There are three rooms, A, B, and C, arranged in a line like this: A—B—C. In other words, A is on the left, B is in the middle, and C is on the right. Each spy must decide independently and simultaneously which room to enter. Their payoffs are determined as follows. If they both choose the same room, then they will see each other, a bloody gun battle will ensue, and both get payoff -10. If they are in adjacent rooms (for example, if Spy 1 is in room A and Spy 2 is in room B) then Spy 1 can set up her eavesdropping equipment and can intercept Spy 2's communications; hence Spy 1 gets a payoff of 5 and Spy 2 gets a payoff of -5. If they are not in adjacent rooms and they are not in the same room (for example, if Spy 1 is in room A and Spy 2 is in room C) then the distance between them is too great for the eavesdropping equipment to work; Spy 1 gets no secrets and Spy 2 gets to keep hers, and so both get a payoff of 0.

a. Model this as a strategic form game.

b. **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

16. Say that persons 1, 2, and 3 each decide whether to go to restaurant A or restaurant B. Person 1 wants the dinner group to be as large as possible. For person 1, the worst thing is if she goes to a restaurant alone, the best thing is if all three go to the same place, and going with one person (it doesn't matter which) is OK, neither best or worst. Person 2 is the exact opposite; she wants the dinner group to be as small as possible. All person 3 cares about is going to the same place as person 1, since he likes person 1.

a. Model this as a strategic form game.

b. **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

c. Now say that person 3 loses interest in person 1 and becomes grouchy like person 2. Model this as a strategic form game.

d. **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

17. Say that there are two people, a security guard and a thief. The security guard can either be vigilant or relax. The thief can either steal or do nothing. If the guard is vigilant, then the thief would rather do nothing than steal. If the guard is relaxed, however, the thief would rather steal than do nothing. If the thief steals, the guard would rather be vigilant than be relaxed. If the thief does nothing, however, the guard would rather be relaxed than vigilant.

a. Model this as a strategic form game. Feel free to choose payoffs which make sense to you.

b. **What are the (pure strategy) Nash equilibria of this game?**

18. Read "Running Mates: The Clark-Lieberman Iowa Jailbreak" by William Saletan (at the "Media Clips" link). Model the situation as a strategic form game.

a. **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

19. Look at Clip 1 of *Return to Paradise* on the web site (look under the "Media Clips" link). Model the situation as a game with two players (Sheriff and Tony). Feel free to choose payoffs which make sense to you (that's what makes the problem kind of interesting).

a. **In your version of the game, are there strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

20. Read “Report Calls Recycling Costlier Than Dumping” by Eric Lipton at the “Media Clips” link. At the end of the story, Lipton states that if people don’t recycle much, then it is too costly to have a recycling program, but if people recycle a lot, it becomes economically worthwhile. Model the situation as a strategic form game (say with just two people for simplicity) and show that there are two Nash equilibria: one in which both people recycle a lot and one in which both people recycle little.

21. Read “Drifter Jailed on Girls’ Lies Set Course of Desperation” by H. G. Reza, Christine Hanley, and James Ricci at the “Media Clips” link. Model the situation as a strategic form game. Is this a Prisoners’ Dilemma? Why is it standard procedure for police to interview witnesses separately?

22. Read the excerpt from Richard Wright’s *Black Boy* at the “Media Clips” link. Model this as a strategic form game and interpret the situation and the outcome in terms of the game and your predictions given the game.

23. Read “9 Questions about Syria You Were Too Embarrassed to Ask” by Max Fisher at the “Media Clips” link. On page 6, under the “You didn’t answer my question” heading, a situation concerning chemical weapons is described. Model this as a strategic form game.

24. Find all Nash (mixed strategy and pure strategy) equilibria to this version of the “Chicken” game:

		$[q]$	$[1-q]$
		2 swerves	2 doesn’t
$[p]$	1 swerves	1, 1	0, 5
$[1-p]$	1 doesn’t	5, 0	-10, -10

Chicken

25. Find all Nash equilibria to the “Early-late” game, which looks like this:

		$[q]$	$[1-q]$
		2 arrives early	2 arrives late
$[p]$	1 arrives early	1, 1	-5, -1
$[1-p]$	1 arrives late	-1, 0	3, 3

Early-late

26. Say you have an admirer whom you don't like very much. You can either go to the library or the coffee shop to study. You prefer the coffee shop but you want to avoid your admirer. Your admirer can also go to the library or coffee shop to study. Your admirer prefers the library but wants to be where you are more than anything else. So the game looks like:

	Admirer goes to library	Admirer goes to coffeeshop
You go to library	0, 3	4, 0
You go to coffeeshop	6, 0	0, 1

a. Find all (pure strategy *and* mixed strategy) Nash equilibria of this game.

b. Now say that you begin to actually enjoy your admirer's company. The game is now:

	Admirer goes to library	Admirer goes to coffeeshop
You go to library	4, 3	0, 0
You go to coffeeshop	0, 0	6, 1

Find all (pure strategy *and* mixed strategy) Nash equilibria of this game.

27. Person 1 and Person 2 are competing for the affections of Sandy. Person 1 and Person 2 plan to show up at Sandy's house at the same time on Saturday night to ask Sandy out. Since this is southern California, a crucial decision for both Person 1 and Person 2 is what kind of car to drive to Sandy's house. Person 1 is wealthy and can either have the butler prepare the impressive Lamborghini or the cute VW. Person 2 ordinarily drives a pickup truck, but can borrow a Lexus for the night from a roommate. If Sandy sees the Lamborghini and the Lexus pull up, Sandy chooses the Lamborghini because it is of course more impressive. If Sandy sees the Lamborghini and the pickup truck, Sandy chooses the pickup truck because the Lamborghini is clearly trying too hard. If Sandy sees the VW and the Lexus, Sandy chooses the Lexus because it is more impressive than the VW. If Sandy sees the VW and the pickup truck, Sandy chooses the VW because it is obviously more comfortable than the pickup truck.

a. Model this as a strategic form game between Person 1 and Person 2 (assume Sandy is not a player).

b. Find all (pure strategy and mixed strategy) Nash equilibria of this game.

28. Consider the following game.

	$2a$	$2b$	$2c$
$1a$	$0, 9$	$3, 0$	$1, 5$
$1b$	$1, 2$	$5, 4$	$4, 3$
$1c$	$0, 6$	$2, 1$	$6, 7$

a. Find all pure strategy Nash equilibria of this game.

b. Use the method of iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can't eliminate any more).

c. After you have iteratively eliminated as much as you can, find all mixed strategy Nash equilibria of the "remaining" game. (Note: an answer like "$p = 2/3, q = 2/5$" is not sufficient. Please write down in a sentence which strategies are played with what probability.)

29. In a simplified version of "Battleship," say that there are four spaces, numbered $1, 2, 3, 4$. Person 1 chooses to fire a missile at one of these four spaces. Person 2 has a ship which is two spaces long, and chooses where to put the ship on the board: she can either put it on spaces 1 and 2, on spaces 2 and 3, or on spaces 3 and 4. The people make their choices simultaneously.

a. Model this as a strategic form game and use the method iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can't eliminate any more).

b. Find all mixed-strategy and pure-strategy Nash equilibria of the remaining game.

c. Now say that there are 5 spaces, numbered $1, 2, 3, 4, 5$. Model this as a strategic form game and find all mixed-strategy and pure-strategy Nash equilibria. Make a prediction in this game like you did before (iteratively eliminate dominated strategies, and then find pure strategy or mixed strategy Nash equilibria of the remaining game).

30. Say that a seller tries to sell a car to a buyer. The car is worth \$1000 to the seller and \$2000 to the buyer, and both people know this. First, the seller proposes a price of either \$1800 or \$1200 to the buyer. Given this price, the buyer can either accept or reject the offer. Model this as an extensive form game.

a. Find all (pure strategy) Nash equilibria.

b. Which Nash equilibria are subgame perfect?

31. Say that you and your brother divide up 4 oranges in the following manner. First, you divide the oranges up into two piles: the left pile and the right pile. For example, you could put one orange in the left pile and three in the right pile. Then, your brother decides which of the two piles he wants; he keeps that pile and you get the other pile. Both of you like oranges.

a. Model this as an extensive form game and find the subgame perfect Nash equilibria. How will the oranges be divided?

32. Say that country A can either attack or not attack country B. Country B cannot defend itself; the only thing it can do is to set off a "doomsday device" which obliterates both countries. The worst thing for both countries is to be obliterated; for country A, the best thing is to attack and not be obliterated; for country B, the best thing is to not be attacked.

a. Think of country A moving first and model this as an extensive form game. Find all pure strategy Nash equilibria and find the subgame perfect Nash equilibria.

b. Now think of country B moving first and model this as an extensive form game. Find all pure strategy Nash equilibria and find the subgame perfect Nash equilibria.

33. The game called "Nim" goes like this: there are a pile of five stones on the ground. Player 1 can take either 1 or 2 stones. Then player 2 can take either 1 or 2 stones. They continue taking either 1 or 2 stones in turn until all the stones are gone. The player who takes the last stone (or stones) wins.

a. Model this as an extensive form game and find the subgame perfect Nash equilibria.

b. What happens when you start with n stones instead of five?

34. Say we have a version of the "centipede" game. Each person can either move "down" or "across."

$$\begin{array}{ccccccccc}
1 & -a- & 2 & -a- & 1 & -a- & 2 & -a- & 8,8 \\
| & & | & & | & & | & & \\
d & & d & & d & & d & & \\
| & & | & & | & & | & & \\
2,2 & & 1,8 & & 4,4 & & 2,16 & &
\end{array}$$

Just to be clear, the game goes from left to right. First, person 1 chooses whether to play d or a. If she plays d, then the game ends and both players get 2. If person 1 plays a, then person 2 chooses between d or a. If person 2 chooses d, then person 1 gets 1 and person 2 gets 8. If person 2 chooses a, then person 1 gets to choose between d and a, and so forth.

a. Find the subgame perfect Nash equilibrium.

b. Represent this game in strategic (matrix) form.

c. Solve this strategic form game using iterated elimination of weakly dominated strategies. Show the order of elimination.

35. [from Spring 2004 midterm] Boyfriend and Girlfriend are on a 7 day cruise, which starts on Monday and goes until Sunday. Both know that this cruise is their last hurrah. Both know that once Sunday comes around and the cruise ends, if they are still together, the game ends because Boyfriend will be forced to dump Girlfriend (because otherwise he would be disowned by his wealthy parents).
On Monday evening, Boyfriend decides whether to break up or continue in the relationship. If Boyfriend decides to continue, then on Tuesday evening, Girlfriend decides whether to break up or to continue. If Girlfriend decides to continue, then on Wednesday evening, Boyfriend decides whether to continue. The game continues like this, with Boyfriend and Girlfriend taking turns deciding whether to continue or to break up (Boyfriend gets to decide on Monday, Wednesday, and Friday, and Girlfriend decides on Tuesday, Thursday, and Saturday). If anyone decides to break up, the game is immediately over. If no one decides to break up, then they make it to Sunday, and Girlfriend is left heartbroken.
Each person enjoys the others company, and gets a payoff of 5 for every day the relationship continues. However, no person wants to be dumped; if a person breaks up the relationship, the other person (the dumped one) gets -10 added to her payoff. For example, if they make it all the way to Sunday, Boyfriend's payoff is 35 and Girlfriend's payoff is 25.

a. Represent this as an extensive form game.

b. Find the subgame perfect Nash equilibrium.

36. [from Spring 2002 midterm] Consider the "cross-out game." In this game, one writes down the numbers 1, 2, 3. Person 1 starts by crossing out any one number or any two adjacent numbers: for example, person 1 might cross out 1, might cross out 1 and 2, or might cross out 2 and 3. Then person 2 also crosses out either one number or two adjacent numbers. For example, starting from 1, 2, 3, say person 1 crosses out 1. Then person 2 can either cross out 2, cross out 3, or cross out both 2 and 3. Play continues like this. Once a number is crossed out, it cannot be crossed out again. Also, if for example person 1 crosses out 2 in her first move, person 2 cannot then cross out both 1 and 3, because 1 and 3 are not adjacent (even though 2 is crossed out). The winner is the person who crosses out the last number.

a. Model this as an extensive form game. Show a subgame perfect Nash equilibrium of this game by drawing appropriate arrows in the game tree.

b. Now instead of just three numbers, say that you start with m numbers. In other words, you have the numbers $1, 2, 3, \ldots, m$. Can person 1 always win this game? (Hint: look at $m = 4$, $m = 5$, etc. first to get some ideas.)

37. Read "It's Not Just About Bonds, It's About Who's on Deck" by Jack Curry on the "Media Clips" website. Model the situation using extensive form games and use the games to show that who bats behind Barry Bonds influences a pitcher's decision of how to pitch to Bonds.

38. Richard Nixon subscribed to a foreign-policy principle which he called the "madman theory" or the "theory of excessive force" (see "The Trials of Henry Kissinger" on the Media Clips website, or read "Nixon's Nuclear Ploy" by William Burr and Jeffrey Kimball, also on the Media Clips website). We will explain this theory using an extensive form game.
Say that the US is facing an adversary (for example North Vietnam). The US makes the first move: it can either threaten North Vietnam or do nothing. If the US does nothing, then the game ends and "nothing happens." If the US makes a threat, North Vietnam can either comply or not comply. If North Vietnam complies, then the game ends. If North Vietnam does not comply, then the US can either strike militarily or not.

a. Say the payoffs are like this: if the US does nothing, then both get payoff 0. If the US threatens and North Vietnam complies, then the US gets payoff 10 and North Vietnam gets payoff -5. If the US threatens, North Vietnam does not comply, and the US strikes, then the US gets payoff -10 and North Vietnam gets payoff -10. If the US threatens, North Vietnam does not comply, and the US does not strike, then the US gets payoff -5 and North Vietnam gets payoff 10.
The idea here is that the best thing for the US is for the US to make a threat and North Vietnam to comply; this way the US doesn't have to perform the military strike. The worst thing for the US is to have to perform the military strike, since this would risk expanding the war. The best thing for North Vietnam is to openly defy the US and "call the bluff" of the US; the worst thing is the military strike. The second-worst thing is to comply to the US threat, and the second-best thing is to not be threatened at all.

Model this as an extensive form game and find the subgame perfect Nash equilibrium.

b. Now say that Nixon, by purposefully showing the world that he is a "madman," convinces everyone that the US payoff from a military strike is 5, not -10. In other words, Nixon tells the world that as far as he is concerned, he would enjoy a military strike. Model this as an extensive form game (all other payoffs are the same) and find the subgame perfect Nash equilibrium.

c. Compare the two situations and explain why Nixon wants to show the world that he is a "madman."

39. Say we have two basketball players, Nash and Yao. Nash has the ball 20 feet from the basket and Yao is defending. Nash can either take a jump shot or drive the basket. Yao can either come out and contest the shot, or can stay in and protect the basket. The game looks like this.

	Yao comes out	Yao protects
Nash takes jump shot	0.3, 0.7	0.4, 0.6
Nash drives	0.5, 0.5	0.3, 0.7

For example, if Nash takes a jump shot and Yao comes out, then Nash has a 30 percent chance of scoring and Yao has a 70 percent chance of successfully defending.

a. Find the mixed strategy Nash equilibrium of this game. In the mixed Nash equilibrium, what is Nash's overall probability of scoring?

b. Now say that Nash practices over the summer and improves his jump shot. Now the game looks like this.

	Yao comes out	Yao protects
Nash takes jump shot	0.4, 0.6	0.5, 0.5
Nash drives	0.5, 0.5	0.3, 0.7

Find the mixed strategy Nash equilibrium of this new game. In the mixed Nash equilibrium, what is Nash's overall probability of scoring?

c. After Nash improves his jump shot, does he go to his jump shot more often? After Nash improves his jump shot, is it more successful? After Nash improves his jump shot, is his drive more successful? Why does an improved jump shot improve other parts of Nash's game?

40. Consider the "Battle of the Sexes" game below.

	$2a$	$2b$
$1a$	$2, 1$	$0, 0$
$1b$	$0, 0$	$1, 2$

a. Find all Nash equilibria (pure strategy and mixed strategy) of this game.

b. Are any strategies in this game weakly or strongly dominated?

41. Consider the following game.

	$2a$	$2b$	$2c$	$2d$	$2e$
$1a$	$63, -1$	$28, -1$	$-2, 0$	$-2, 45$	$-3, 19$
$1b$	$32, 1$	$2, 2$	$2, 5$	$33, 0$	$2, 3$
$1c$	$54, 1$	$95, -1$	$0, 2$	$4, -1$	$0, 4$
$1d$	$1, -33$	$-3, 43$	$-1, 39$	$1, -12$	$-1, 17$
$1e$	$-22, 0$	$1, -13$	$-1, 88$	$-2, -57$	$-3, 72$

a. Find all pure strategy Nash equilibria of this game.

b. Make a prediction in this game by iteratively eliminating (strongly or weakly) dominated strategies.

42. [from Spring 2002 midterm] Mother can ask either Sister or Brother to do the dishes while she goes out shopping. If Mother asks Sister, she can either do it or not do it. If Mother asks Brother, he can either do it or not do it. Since Sister does a better job, Mother prefers Sister doing the dishes over Brother doing the dishes. However, Mother prefers Brother doing the dishes over them not being done.
Both Sister's and Brother's preferences are like this: the best thing is for the other person to do the dishes; the second best thing is for Mother to ask the other person and have the other person not do it (since then the other person will get blamed). The third best thing is to do the dishes, and the worst thing is to be asked to do the dishes but then not do it (since you will get in trouble).

a. Represent this as a strategic form game.

b. Find all (pure strategy) Nash equilibria.

43. The simplest kind of game has two players, who each have two possible actions. We call these games "2×2 games."

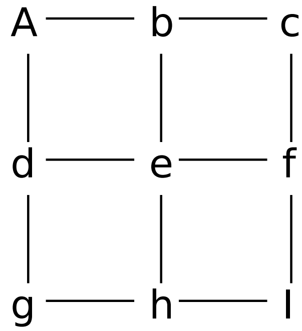
a. Write down a 2×2 game which has exactly one pure strategy Nash equilibrium and no mixed strategy Nash equilibrium. Solve for the equilibrium.

b. Write down a 2×2 game which has no pure strategy Nash equilibrium and exactly one mixed strategy Nash equilibrium. Solve for the equilibrium.

c. Write down a 2×2 game which has exactly three total (pure and mixed) Nash equilibria. Solve for the equilibria.

d. Write down a 2×2 game in which the total number of (pure and mixed) Nash equilibria is neither one nor three. Solve for the equilibria.

44. Say that country A and country I are at war. The two countries are separated by a system of rivers, as shown below.



Country I sends a naval fleet with just enough supplies to reach A. The fleet must stop for the night at intersections (for example, if the fleet takes the path IhebA, it must stop the first night at h, the second at e, and the third at b). If unhindered, on the fourth day the fleet will reach A and destroy country A. Country A can send a fleet to prevent this. Country A's fleet has enough supplies to visit three contiguous intersections, starting from A (for example Abcf). If it catches Country I's fleet (that is, if both countries stop for the night at the same intersection), it destroys the fleet and wins the war. Model this as a strategic form game, assuming that the winner gets payoff 1 and the loser gets payoff -1. Iteratively eliminate weakly dominated strategies and make some sort of prediction.

45. [from Spring 2002 final] There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability p and B with probability $1 - p$. Find p.

46. Say that Townsville is deciding how many coal-fired energy plants to build to supply its energy needs. Some people are more environmentally oriented and thus prefer fewer plants, and some people think that the jobs and electricity that the plants provide are more important. Hence people differ on how many plants they feel are necessary. An opinion poll is taken asking each person how many plants she or he prefers. The results are that 14 percent of the population prefer 0 plants, 16 percent prefer 1 plant, 18 percent prefer 2 plants, 6 percent prefer 3 plants, 30 percent prefer 4 plants, and 16 percent prefer 5 plants.

a. Say that there are two candidates running for office, and the only relevant issue is how many plants to build. Each candidate takes a position on how many plants to build, and then each voter votes for the candidate which is closest to her own position or "ideal point." For example, if candidate 1 is for 3 plants and candidate 2 is for 0 plants, a voter who prefers 2 plants will vote for candidate 1. If there is a "tie" (if two candidates are equally close to a voter's ideal point), then half of the votes go to each candidate. For example, if candidate

1 is for 2 plants and candidate 2 is for 0 plants, then half of the people who prefer 1 vote for candidate 1 and half vote for candidate 2. Each candidate wants to maximize the total number of votes she gets.
Model this as a strategic form game (the candidates move simultaneously) as in the Downsian model. Find the pure strategy Nash equilibrium. Predict what positions the candidates will take and how many plants the town will build.

b. Now say that there are three candidates. Is there a pure strategy Nash equilibrium which is similar to what you found in part a.?

c. Now say that opinions shift. A new poll is taken, and it is found that 4 percent of the population prefer 0 plants, 10 percent prefer 1 plant, 78 percent prefer 2 plants, 2 percent prefer 3 plants, 2 percent prefer 4 plants, and 4 percent prefer 5 plants. Say there are two candidates. Predict what positions the candidates will take and how many plants the town will build.

d. Now again say that there are three candidates. Is there a pure strategy Nash equilibrium which is similar to what you found in part c.?

47. Say that Gotham City is deciding how many skate parks and dog walks to build in the city. A survey is done and it is found that 40 percent of the population are cranky taxpayers who dislike public expenditures and prefer 0 skate parks and 0 dog walks; 22 percent are hard core skate punks who prefer 6 skate parks and 0 dog walks; 30 percent are yuppie golden retriever owners who prefer 0 skate parks and 6 dog walks; and 8 percent are consensus-minded Buddhists who prefer 2 skate parks and 2 dog walks.
Say that there are two candidates running for office who take positions on both issues. As in the Downsian model, each voter votes for the candidate which is closest to her own position or "ideal point." For example, if candidate 1 favors 1 skate park and 1 dog walk, and candidate 2 favors 4 skate parks and 2 dog walks, then candidate 1 gets 78 percent of the vote (the cranky taxpayers, the yuppies, and the Buddhists) and candidate 2 gets 22 percent (the skate punks). Each candidate wants to maximize the total number of votes she gets.

Math hint: in case you forgot, the distance between point (a, b) and point (c, d) is given by the Pythagorean Theorem: $\sqrt{(a-c)^2 + (b-d)^2}$. For example, if candidate 1 favors 1 skate park and 1 dog walk, her position is $(1, 1)$. If candidate 2 favors 4 skate parks and 2 dog walks, her position is $(4, 2)$. The Buddhists have ideal point $(2, 2)$. Thus candidate 1 is distance $\sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$ away, and candidate 2 is distance $\sqrt{(2-4)^2 + (2-2)^2} = \sqrt{4} = 2$ away. Since candidate 1 is closer, the Buddhists vote for candidate 1.

a. Let's try to make a prediction in this game by eliminating weakly and strongly dominated strategies. First, simplify the game a lot by considering only the following strategies: (0,0), (0,3), (0,6), (1,1), (2,2), (3,0), (3,3), (6,0). Here (3,0) means for example 3 skate parks and 0 dog walks. Each of the two candidates thus has eight possible strategies. Write this as a strategic form game and make a prediction by eliminating weakly and strongly dominated strategies.

b. Now say that most of the yuppies see Richard Gere movies and decide to become Buddhists (and take the Buddhist position of supporting 2 skate parks and 2 dog walks). Now there are 5 percent yuppies, 33 percent Buddhists, 40 percent cranky taxpayers, and 22 percent skate punks. Again, consider only the eight strategies above and make a prediction by eliminating weakly and strongly dominated strategies. As the electorate becomes more "moderate," do the candidates take more centrist positions?

48. The city council of Asbestosville wants to improve its image by bringing in the Palookaville Pirates, a minor league baseball franchise which is currently located in Palookaville. Asbestosville has built a brand new baseball field and is now trying to come up with other enticements for the Pirates, such as how much cash to give to the team. The city has already agreed to give the team $1 million, but some council members want to give the team more money. There are 11 council members. One member does not want to give the team any more money and prefers to give only $1 million total to the Pirates, one member prefers to give $2 million in total, one member prefers to give $3 million, one member prefers to give $4 million, and so forth; the eleventh council member wants to give the Pirates $11 million in total. As you can see, the median member of the council wants to give the Pirates $6 million.

a. The city council chairperson is a baseball fanatic and wants to give the Pirates $11 million in total. The chairperson controls the city agenda and thus decides what proposal to bring to the council. When a proposal is brought to the council, all council members simply vote yes or no (the chairperson also votes). If the vote fails, then the policy remains at the status quo (giving $1 million total). Like in the Downsian model, a council member wants the final policy to be as close to her own "ideal point" as possible. What proposal will the chairperson make? How much money will the Pirates receive?

b. Now say that the mayor can veto the city council decision. The mayor's ideal point is to give the Pirates a total of $2.6 million. If the council's decision is farther away than the status quo from the mayor's ideal point, then the mayor will veto the council's decision and the status quo will be implemented. The sequence of decisions is like this: the council chairperson first makes a proposal, then the council members vote, and then the mayor can veto. Now what proposal will the chair person make? How much money will the Pirates receive?

c. Now say that if the mayor vetoes the city council decision, the city council can override the veto with two thirds of the council vote (in this case, 8 of the 11 council members). If the council's proposal is vetoed, then if the council overrides the veto, it can implement its original proposal. So now the sequence of decisions is like this: the council chairperson first makes a proposal, then the council members vote, and then the mayor can veto; if the mayor vetoes, then the council can override. Now what proposal will the chair person make? How much money will the Pirates receive?

49. Say that there are three people and five candidates $\{a, b, c, d, e\}$. Say person 1's order of preference (from best to worst) is c, b, e, d, a. Person 2's order is d, c, a, b, e. Person 3's order is e, a, b, d, c.

a. Show that for each candidate, there is an order of voting (an "agenda") in which that candidate wins.

b. Say that there are three people and four candidates $\{a, b, c, d\}$. Say person 1's order of preference (from best to worst) is c, b, d, a. Person 2's order is b, a, d, c. Person 3's order is a, c, d, b. Show that there is no order of voting in which candidate d wins. Why is this?

50. Say that eleven people vote over four candidates $\{a, b, c, d\}$. Three people have preference order (from best to worst) b, a, c, d. Three people have preference order b, a, d, c. Three people have preference order a, c, d, b. Two people have preference order a, d, c, b.

a. Is there a candidate which beats all others by majority rule (a Condorcet winner)?

b. Say that instead of standard majority rule voting, that each person votes 3 points for their first choice, 2 points for their second choice, 1 point for their third choice, and no points for their last choice. This procedure is called the "Borda count." Which candidate wins now?

c. Which procedure (majority rule or Borda count) is more reasonable or more fair in your opinion?

51. Another system of voting is "approval voting," in which each voter can place vote for as many candidates as she wishes: for example, one person might put one vote on candidate a, and another person might put one vote on candidate a and one vote on candidate b. Say we have approval voting, and each person votes for his top two candidates.

a. Say that candidate a receives more votes by approval voting than the other two candidates. Is it possible for a to not be a Condorcet winner? If so, write down the preference orderings for each person which make this possible.

b. Say that a is a Condorcet winner. Is it possible for a to receive fewer approval votes than some other candidate? If so, write down the preference orderings for each person which make this possible.

52. Say that there are three people who are deciding over three alternatives, $\{a, b, c\}$. Is it possible for the Borda count winner to be different from the Condorcet winner? If so, write down the preference orderings for each person which make this possible.

53. Say that there are three groups in society. Group X's preferences (from best to worst) are a, b, c. Group Y's preferences (from best to worst) are c, b, a. Group Z's preferences (from best to worst) are b, c, a. There are 13 people in group Y and 7 people in group Z. There are x people in group X. Assume that x is an odd number (to help avoid ties).

a. Depending on the value of x, what is the Condorcet winner? Is there some value of x such that there is no Condorcet winner?

b. Say society uses the Borda count system. For what values of x does the outcome of the Borda count differ from what the society would choose if they chose the Condorcet winner?

c. Say society uses approval voting in which each person votes for her top two candidates. For what values of x does the outcome of this approval voting system differ from what the society would choose if they chose the Condorcet winner?

54. Say that there are three people deciding by majority rule over four candidates $\{a, b, c, d\}$. Person 1's preferences (from best to worst) are a, b, c, d. Person 2's preferences (from best to worst) are c, d, b, a. Person 3's preferences (from best to worst) are d, a, c, b. Consider voting agendas in which people vote on candidates sequentially.

a. Is there an agenda in which they decide on a? If there is, show it. If not, explain why.

b. Is there an agenda in which they decide on b? If there is, show it. If not, explain why.

c. Is there an agenda in which they decide on c? If there is, show it. If not, explain why.

d. Is there an agenda in which they decide on d? If there is, show it. If not, explain why.

55. Say that there are three people deciding by majority rule over eight candidates $\{a, b, c, d, e, f, g, h\}$. Person 1's preferences (from best to worst) are f, c, g, e, b, h, a, d. Person 2's preferences (from best to worst) are a, f, e, g, h, b, d, c. Person 3's preferences (from best to worst) are g, a, b, c, h, d, e, f. Consider voting agendas in which people vote on candidates sequentially.

a. Find the top cycle.

b. For every candidate in the top cycle, find a voting agenda in which that candidate wins.

56. Say that we have a threshold model in which there are 5 people. If the total number of other people who participate is greater or equal to a person's threshold, the person wants to participate also. If the total number of other people who are participating is less than a person's threshold, the person does not want to participate.

a. Say that one person has threshold 1, two people have threshold 2, and two people have threshold 4. Find all of the pure strategy Nash equilibria.

b. Now say that one of the threshold 2 people becomes a threshold 0 person. Find all of the pure strategy Nash equilibria. Does this change guarantee some level of participation?

57. Say that you have a group of 50 people who can either buy a color fax machine or not buy. No one wants to buy a color fax machine if no one else has one (because there would be no one to exchange color faxes with). In fact, each person will buy one only if at least 6 other people buy them. Thus each person has a threshold of 6.

a. Find the two pure strategy Nash equilibria.

b. Now say that you are a sales rep for the color fax machine company. You can offer discount coupons to potential customers. If you give 1 discount coupon to someone, that decreases their threshold by 1. For example, if you give 6 discount coupons to a single person, you can make that person have threshold 0. If you give 2 discount coupons to a single person, you can make that person have threshold 4. Obviously, you can guarantee that everyone will buy a color fax machine by giving all 50 people six coupons each, but that would be silly (the company would get no profits). Using the fewest possible number of coupons, how can you guarantee that everyone will buy a color fax machine?

58. [from Spring 2003 final] Say that we have 10 people. Each person is thinking about whether or not to join a revolt or not. Each person has a threshold: five people have threshold 4 and five people have threshold 6.

a. Find all pure strategy Nash equilibria of this game.

b. Say that you have some discount coupons which lower the cost of revolting and hence lower a person's threshold. For example, if I give 3 coupons to a person with threshold 4, she now has threshold 1. If I give 1 coupon to a person with threshold 6, he now has threshold 5. By giving out coupons, I can change the game so that the only Nash equilibrium is one in which everyone revolts. I want to do this by giving out the fewest number of coupons. How do I distribute the coupons (who gets coupons, and how many does each person get)?

c. Now say that you can give tickets which raise the cost of revolting and hence raise a person's threshold. For example, if I give 3 tickets to a person with threshold 4, she now has threshold 7. If I give 2 tickets to a person with threshold 6, he now has threshold 8. By giving out tickets, I can change the game so that the only Nash equilibrium is one in which no one revolts. I want to do this by giving out the fewest number of tickets. How do I distribute the tickets (who gets tickets, and how many does each person get)?

59. Say that there are four people: Alicia, Betsy, Carlos, and Davis. Each can choose whether to wear platform sandals or not. Alicia is very fashion-forward and will wear them even if no one else wears them; in fact, if more than one other person wears them, she won't wear them anymore because she hates being part of a crowd. Betsy also likes fashion but is not as cutting-edge: she will wear them if at least one other person wears them, but like Alicia, hates being "part of the crowd" and will not wear them if more than two other people wear them. Carlos thinks of himself as hip, but is kind of slow on the uptake and will wear them if at least two others wear them. Still, Carlos has at least some fashion pride and will not wear them if everyone else wears them. Finally, Davis gets his fashion tips from the JC Penney catalog and will wear them if everyone else wears them.

a. Say that at the beginning, no one wears platform sandals (they have just hit the market). Show that at first the sales of platform sandals steadily grow, but eventually sales "cycle" between high and low in a never-ending "fashion cycle."

b. Are there any pure strategy Nash equilibria of this game?

60. Say that there are three men A, B, and C and three women X, Y, and Z. Each of these six people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman X best, woman Y next, and woman Z least. Man B prefers woman Y best, woman Z next, and woman X least. Man C prefers woman Y best, woman X next, and woman Z least. Woman X prefers man B best, man A next, and man C least. Woman Y prefers man A best, man B next, and man C least. Woman Z prefers man A best, man C next, and man B least.

a. Say that man A is matched with woman X, man B is matched with woman Y, and man C is matched with woman Z. Is this matching stable?

b. Write down all possible matchings and determine which of them are stable and which are not stable.

c. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?

61. Say that there are four men, A, B, C, and D and four women W, X, Y, and Z. Each of these eight people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman W best, X next, Y, next, and Z least. Man B's preference ordering (from best to worst) is Y, X, W, Z. Man C's preference ordering is Y, Z, X, W. Man D's ordering is Y, Z, X, W. Woman W's preference ordering (from best to worst) is D, C, B, A. Woman X's preference ordering is C, B, A, D. Woman Y's ordering is D, A, C, B. Woman Z's ordering is B, C, D, A.

a. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?

62. Say that Person 1 and Person 2 each decide whether to go to the auto racing match or the ballet. If they go to different places, both get utility zero. If they both go to the auto racing match, person 1 gets a utility of \$4 and person 2 gets a utility of \$1. If they both go to the ballet, person 1 gets a utility of \$1 and person 2 gets a utility of \$4. This is a "battle of the sexes" game. Now say that before playing this battle of the sexes game, person 1 can either burn \$2 or not burn it. Person 2 can see whether person 1 burns the money or not.

a. Represent this as a strategic form game (each person has four strategies) and find all (pure strategy) Nash equilibria.

b. Show that by iteratively eliminating both strongly and weakly dominated strategies, one can predict that person 1 does not burn the money and that both go to the auto racing match.

63. Say that you and I are playing a game in which we both simultaneously yell out either a or b. If we say the same letter, then you get \$1 and I get nothing. If we say different letters, than I get \$1 and you get nothing.

a. Model this as a game and find the Nash equilibrium.

b. Now say that I just had my wisdom teeth pulled out and my mouth is still quite numb. So it's more difficult for me to say b: when I say b, I have to pay the cost r, where $r > 0$. Now we play the same game above (my payoffs have changed but yours have not). Model this as a game and find the Nash equilibrium.

c. How does my expected utility in the Nash equilibrium change as r changes? What happens when $r = 1$? When $r > 1$?

64. Say that person 1 and person 2 are playing a drinking game which goes like this. There are m beers in the refrigerator. Person 1 goes first by drinking either 1 or 2 beers. Then person 2 can drink either 1 or 2 beers. Then person 1 can drink either 1 or 2 beers, and so forth. In other words, when it is a person's turn to drink, she can drink either 1 or 2 beers. Whoever drinks the last beer wins the game. Winning the game yields a payoff of 1 and losing yields a payoff of 0. However, there is an additional feature to the game: there is a "magic number" x (which is greater than 0 and less than m). If after your turn, there are exactly x beers left, then you lose the game and also have to go out and buy more beer; this has a payoff of -3 for the loser and a payoff of 1 for the winner.

a. Say that $m = 6$ and $x = 4$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

b. Say that $m = 6$ and $x = 3$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

c. Now let m and x be any number. Find a subgame perfect Nash equilibrium. For what values of m and x can person 1 guarantee a win? For what values of m and x can person 2 guarantee a win?

65. There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability p and B with probability $1 - p$. Find p.

66. [from Spring 2003 final] Say that we have 10 people. Each person is thinking about whether or not to join a revolt or not. Each person has a threshold: five people have threshold 4 and five people have threshold 6.

a. Find all pure strategy Nash equilibria of this game.

b. Say that you have some discount coupons which lower the cost of revolting and hence lower a person's threshold. For example, if I give 3 coupons to a person with threshold 4, she now has threshold 1. If I give 1 coupon to a person with threshold 6, he now has threshold 5. By giving out coupons, I can change the game so that the only Nash equilibrium is one in which everyone revolts. I want to do this by giving out the fewest number of coupons. How do I distribute the coupons (who gets coupons, and how many does each person get)?

c. Now say that you can give tickets which raise the cost of revolting and hence raise a person's threshold. For example, if I give 3 tickets to a person with threshold 4, she now has threshold 7. If I give 2 tickets to a person with threshold 6, he now has threshold 8. By giving out tickets, I can change the game so that the only Nash equilibrium is one in which no one revolts. I want to do this by giving out the fewest number of tickets. How do I distribute the tickets (who gets tickets, and how many does each person get)?

67. Say that we have three men, A, B, and C, and three women X, Y, and Z. Each person has preferences about which member of the opposite sex they would like to be matched with. For example, say woman X likes A the best, B second best, and C worst. Assume that there are no ties (i.e. it cannot be that woman Y likes A the best and B and C are tied for worst). Note that there are six possible matchings.

a. Is it possible for people's preferences to be such that there exists exactly one stable matching? If so, write down the preferences which make this possible. If not, explain why not.

b. Is it possible for people's preferences to be such that there exists exactly two stable matchings? If so, write down the preferences which make this possible. If not, explain why not.

c. Is it possible for people's preferences to be such that there exists exactly three stable matchings? If so, write down the preferences which make this possible. If not, explain why not.

68. Say that there are five people choosing among three candidates x, y, z. Persons 1 and 2's preference ordering from best to worst is x, z, y. Persons 3 and 4 have preference ordering y, z, x. Person 5 has preference ordering z, x, y.

a. Say that they make their decision using a runoff procedure: first everyone votes for their first choice, and the two alternatives which get the most votes go to a runoff. In the runoff, each person votes for one of these two candidates, and whoever gets the most votes in the runoff wins. Which candidate wins in this procedure?

b. Is the candidate who wins the runoff the Condorcet winner? Is there a Condorcet winner?

69. Say that there are three people choosing among five candidates s, t, w, x, y. Person 1's preference ordering from best to worst is w, x, t, y, s. Person 2's preference ordering is y, w, x, t, s. Person 3's preference ordering is s, x, t, y, w.

a. Say that people decide using majority rule according to some agenda. For example, one agenda might be to vote on w first; if w loses, then vote on x; if x loses, then vote on s; if s loses, then vote on t versus y. Can you find an agenda in which t is chosen?

b. What is the top cycle? Remember that the candidates in the top cycle are those which win given some suitably chosen agenda. Candidates not in the top cycle are those which are never chosen regardless of the agenda.

70. Say that the town of Mollusk Beach is deciding how many oil refineries to build. Forty percent of voters prefer no refineries, 36 percent prefer 1 refinery, and 24 percent prefer two refineries. Say that there are two candidates A and B, and each has to take a position on this issue. Given their positions, each voter will vote for the candidate whose position is closest to their own (if there are two candidates who are equally far away, assume that the vote is split equally among the two candidates). Each candidate wants to maximize the total number of votes she receives.

a. Say that candidates A and B choose their positions simultaneously. Which positions will they take?

b. Say now that candidate A can choose her position first and then candidate B chooses her position. Which positions will they take?

c. Now say that there are three candidates, A, B, and C. Say that candidate A chooses her position first, then candidate B, and then finally candidate C. Which positions will they take?

71. Say that the city council of Bronx Beach is controlled by three political parties. Party A controls 4 seats in the council, party B controls 3 seats, and party C controls 1 seat. There are a total of 8 seats in the council, and passing a bill requires a majority. A majority means strictly greater than half; since there are 8 seats in total, a majority is 5 seats. The city council decides how to split one dollar. Since parties A and B together control 7 seats, they can create a majority, and we can write $v(\{A, B\}) = 1$. Similarly, party A and party C together have 5 seats and can create a majority, and we write $v(\{A, C\}) = 1$. But parties B and C together have only 4 seats and do not have a majority, and so we write $v\{B, C\} = 0$.

a. Find the Shapley value of this game. (3 points)

b. Now say that there are four parties. Party A controls 4 seats, party B controls 3 seats, party C controls 1 seat, and party D controls 1 seat. Since there are a total of 9 seats in the council, a majority requires 5 seats. Find the Shapley value of this game. (3 points)

c. Now say that party A controls 4 seats, party B controls 4 seats, party C controls 1 seat, and party D controls 1 seat. Since there are a total of 10 seats in the council, a majority requires 6 seats. Find the Shapley value of this game. (2 points)

d. Now say that there are five parties. Party A controls 4 seats, party B controls 3 seats, party C controls 1 seat, party D controls 1 seat, and party E controls 1 seat. Since there are a total of 10 seats in the council, a majority requires 6 seats. Find the Shapley value of this game. (2 points)

e. Now say that party A controls 4 seats, party B controls 4 seats, party C controls 1 seat, party D controls 1 seat, and party E controls 1 seat. Since there are a total of 11 seats in the council, a majority requires 6 seats. Find the Shapley value of this game. (2 points)

72. Say that a PTA council is composed of parents and teachers. The council approves a budget if at least one parent and one teacher vote for it. It does not matter how many total votes there are as long as at least one parent and one teacher vote for it.

a. Say that the council has three people: two parents (persons 1 and 2) and one teacher (person 4). So $v(\{1,4\}) = 1$ (a parent and teacher can pass a budget) but $v(\{1,2\}) = 0$ (two parents alone cannot pass a budget). We also have $v(\{1,2,4\}) = 1$ (as long as at least one parent and one teacher vote for it, the budget passes). Find the Shapley value of this game. (3 points)

b. Now say that another parent, person 3, is elected to the council. Now the council has four members: persons 1, 2, and 3 are parents and person 4 is a teacher. Like before, $v(\{1,4\}) = 1$ for example, but $v(\{1,3\}) = 0$ (two parents cannot pass a budget). Also, we still have $v(\{1,2,3,4\}) = 1$. Find the Shapley value of this game. (3 points)

c. Now say that person 5, another teacher, is elected to the council. Now the PTA council has five members: persons 1, 2, and 3 are parents and persons 4 and 5 are teachers. Again, the council approves a budget if at least one parent and one teacher vote for it. Like before, $v(\{1,4\}) = 1$ for example, but $v(\{1,3\}) = 0$ (two parents cannot pass a budget) and $v(\{4,5\}) = 0$ (two teachers cannot pass a budget). Also, we still have $v(\{1,2,3,4,5\}) = 1$. Find the Shapley value of this game. (3 points)

d. Finally, now say that the voting rule changes. Now it takes two parents and two teachers to approve a budget. Like before, persons 1, 2, and 3 are parents and persons 4 and 5 are teachers. So now $v(\{1,4\}) = 0$ and $v(\{1,2,4,5\}) = 1$. We also have $v(\{1,2,3,4\}) = 0$ (three parents and one teacher cannot pass a budget) and $v(\{1,2,3,4,5\}) = 1$. Find the Shapley value of this game. (3 points)