Last name	First name	Student ID number	TA

Part 1

(questions 1 and 2)

Answers to

Final exam PS 30 December 2013

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part contains two problems, and hence there are eight problems. Each question is weighted equally (12 points each). Each part is stapled separately (for ease in grading, since the parts are graded separately). Please make sure that your name and student ID number is on each part.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once. When out of the room, you cannot communicate with any other person in any manner.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1		5	
2		6	
3		7	
4		8	
		total	

Problem 1. Two UCLA students get into an argument over which frozen yogurt is better— Yogurtland or Pinkberry—in the middle of the street in Westwood Village. People start gathering around them, chanting "Fight!" The two students now have two choices: to leave or to fight. If only one leaves, that person looks like a coward and gets a payoff of -10, while the person who stays and fights gets a payoff of 20. If they both leave, they each get a payoff of 1. If they both fight, Student 1 gets a payoff of "x", and Student 2 gets a payoff of "y".

a. Model this situation as a strategic form game. (2 points)

	leave	fight
leave	1, 1	-10, 20
fight	20, -10	x, y

b. Suppose the two students are equally strong, and therefore x equals y. Circle the values of x below for which both students have a dominant strategy in the game in part a. Please explain your answer. (2 points)

x= -22	x= -17	x= -12	x= -7	x= -2	x= 2	x= 7	x= 12	x= 17	x= 22

fight s. dominates leave if $x > -10$.

c. Again, assume that x equals y. Circle the values of x below which make this game (the game in part a.) a Prisoner's Dilemma. Please explain your answer. (2 points)

x= -22	x= -17	x= -12	x= -7	x= -2	x= 2	x= 7	x= 12	x= 17	x= 22

In a prisoner's dilemma, both have a dominant strategy, but when both play it, they both get worse off than if they both played the other strategy. So $x > -10$ and $x < 1$.

d. Now say that x and y do not have to be equal. Sometimes in a game it is possible that after iteratively eliminating (strongly or weakly) dominated strategies, you are left with a single outcome. When this happens, we call the game "dominance solvable."

In other words, when a game is dominance solvable, by iteratively eliminating (strongly or weakly) dominated strategies, you can predict exactly what each person will do.

Circle the values of x and y below which make the game in part a. dominance solvable. Please explain your answer. (2 points)

x= -22 y= 22	x= -17 y= 22	x= -12 y= 22	x= -7 y= 22	x= -2 y= 22	x= 2 y= 22	x= 7 y= 22	x= 12 y= 22	x= 17 y= 22	x= 22 y= 22
x= -22 y= 17	x= -17 y= 17	x= -12 y= 17	x= -7 y= 17	x= -2 y= 17	x= 2 y= 17	x= 7 y= 17	x= 12 y= 17	x= 17 y= 17	x= 22 y= 17
x= -22 y= 12	x= -17 y= 12	x= -12 y= 12	x= -7 y= 12	x= -2 y= 12	x= 2 y= 12	x= 7 y= 12	x= 12 y= 12	x= 17 y= 12	x= 22 y= 12
x= -22 y= 7	x= -17 y= 7	x= -12 y= 7	x= -7 y= 7	x= -2 y= 7	x= 2 y= 7	x= 7 y= 7	x= 12 y= 7	x= 17 y= 7	x= 22 y= 7
x= -22 y= 2	x= -17 y= 2	x= -12 y= 2	x= -7 y= 2	x= -2 y= 2	x= 2 y= 2	x= 7 y= 2	x= 12 y= 2	x= 17 y= 2	x= 22 y= 2
x= -22 y= -2	x= -17 y= -2	x= -12 y= -2	x= -7 y= -2	x= -2 y= -2	x= 2 y= -2	x= 7 y= -2	x= 12 y= -2	x= 17 y= -2	x= 22 y= -2
x= -22 y= -7	x= -17 y= -7	x= -12 y= -7	x= -7 y= -7	x= -2 y= -7	x= 2 y= -7	x= 7 y= -7	x= 12 y= -7	x= 17 y= -7	x= 22 y= -7
x= -22 y= -12	x= -17 y= -12	x= -12 y= -12	x= -7 y= -12	x= -2 y= -12	x= 2 y= -12	x= 7 y= -12	x= 12 y= -12	x= 17 y= -12	x= 22 y= -12
x= -22 y= -17	x= -17 y= -17	x= -12 y= -17	x= -7 y= -17	x= -2 y= -17	x= 2 y= -17	x= 7 y= -17	x= 12 y= -17	x= 17 y= -17	x= 22 y= -17
x= -22 y= -22	x= -17 y= -22	x= -12 y= -22	x= -7 y= -22	x= -2 y= -22	x= 2 y= -22	x= 7 y= -22	x= 12 y= -22	x= 17 y= -22	x= 22 y= -22

	leave	fight
leave	1,1	-10,20
fight	20,-10	x,y

If fight s. dominates leave for player 1 $(x > -10)$, then 1 plays fight and then 2 plays leave or fight depending on the value of y. So the game is dominance solvable (as long as $y \neq -10$).

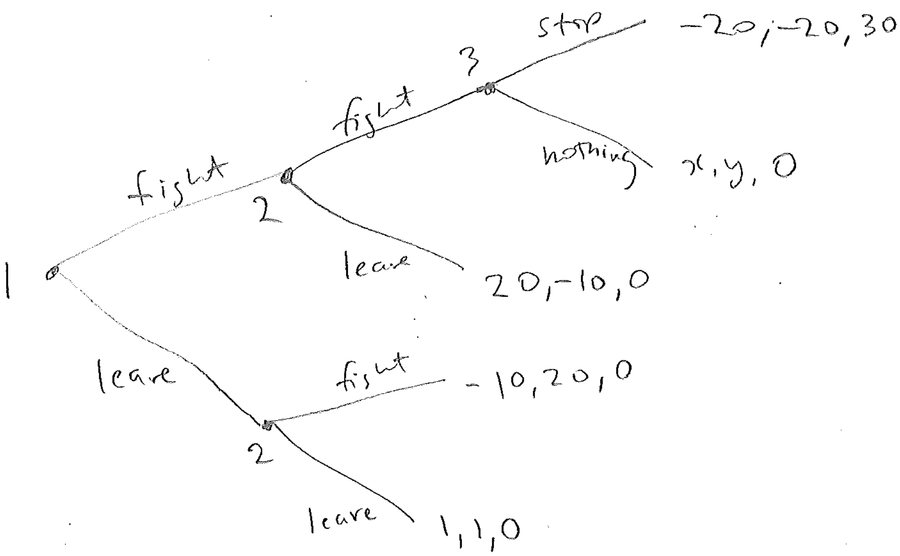
If fight s. dominates leave for player 2 $(y > -10)$, similarly the game is dominance solvable.

Hence the game is dominance solvable if $x > -10$ or $y > -10$.

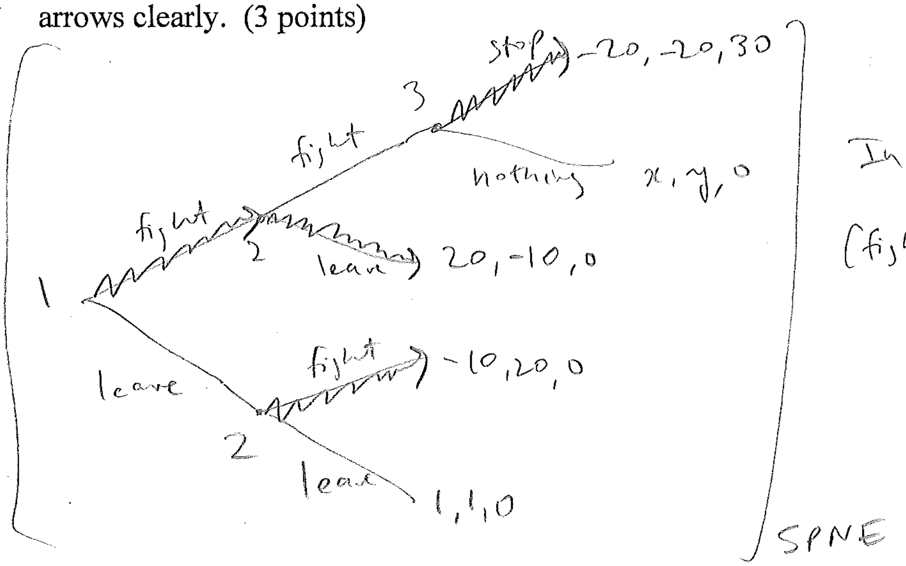
e. Now, assume that Student 1 gets to decide first whether to fight or leave. Then Student 2 decides to fight or leave. If they both choose to fight, then Student 3 can stop the fight or do nothing. If Student 3 stops the fight, she will become a hero and receive a payoff of 30 while the other two receive -20. If Student 3 does nothing when the other students fight, Student 3 gets a 0, and Student 1 again gets a payoff of x and Student 2 gets a payoff of y.

If either Student 1 or Student 2 leaves and the other stays to fight, the one who leaves gets -10 and the one who fights gets 20 (like before). If Student 1 and 2 both leave, they each get a payoff of 1 (like before). Student 3 only gets involved if Student 1 and Student 2 both decide to fight; for all the cases where Student 3 is not involved, she gets a 0.

Model this as an extensive form game. (3 points)



f. Find the subgame perfect Nash equilibria for the game in part e. Please mark your arrows clearly. (3 points)



In words, it is (fight, leave fight, stop)

Problem 2. Say we have the following game.

		q_a	q_b	$1-q_a-q_b$
		2a	2b	2c
p_a	1a	*7, 1	1, 7+	0, 0
p_b	1b	1, 7+	*7, 1	0, 0
$1-p_a-p_b$	1c	0, 0	0, 0	*4, 4+

a. Find all pure strategy Nash equilibria of this game. (2 points)

(1c, 2c)

b. Write down a mixed strategy Nash equilibrium of this game. (2 points)

set these all equal
$$\begin{cases} EU_1(a) = 7q_a + 1q_b \\ EU_1(b) = q_a + 7q_b \\ EU_1(c) = 4(1-q_a-q_b) \end{cases}$$

$$EU_2(a) = p_a + 7p_b$$
$$EU_2(b) = 7p_a + p_b$$
$$EU_2(c) = 4(1-p_a-p_b)$$

$\Rightarrow$ similarly find $p_a = \frac{1}{4}$, $p_b = \frac{1}{4}$

$EU_1(a) = EU_1(b)$

$7q_a + 1q_b = q_a + 7q_b$

$6q_a = 6q_b \Rightarrow q_a = q_b$

$EU_1(a) = EU_1(c)$

$7q_a + q_b = 4(1-q_a-q_b)$

Since $q_a = q_b$

$8q_a = 4(1-2q_a)$

$\Rightarrow 8q_a = 4 - 8q_a$

$16q_a = 4$

$q_a = \frac{1}{4}$

$q_b = \frac{1}{4}$

b. Can you find two (or more) mixed strategy Nash equilibria of this game? If so, please write two of them down. If not, please explain why not. (2 points)

$\Rightarrow$ So

(1 plays 1a with prob $\frac{1}{4}$, 1b $\frac{1}{4}$, 1c $\frac{1}{2}$, 2 plays 2a with prob $\frac{1}{4}$, 2b $\frac{1}{4}$, 2c $\frac{1}{2}$) is a mixed NE

There is another mixed NE:

(1 plays 1a with prob $\frac{1}{2}$, 1b $\frac{1}{2}$, 2 plays 2a with prob $\frac{1}{2}$, 2b $\frac{1}{2}$).

In this mixed NE, player 1 just randomizes between a and b (and never plays c) and player 2 just randomizes between a and b (and never plays c)

c. Now consider the following game.

	q_a	q_b	q_c	$1-q_a-q_b-q_c$
	2a	**2b**	**2c**	**2d**
p_a **1a**	*7, 1	1, 7+	0, 0	0, 0
p_b **1b**	1, 7+	*7, 1	0, 0	0, 0
p_c **1c**	0, 0	0, 0	*6, 6+	2, 2
$1-p_a-p_b-p_c$ **1d**	0, 0	0, 0	2, 2	*6, 6+

Find all pure strategy Nash equilibria of this game. (3 points)

(1c, 2c)
(1d, 2d)

d. Can you find three (or more) mixed strategy Nash equilibria of this game? If so, please write three of them down. If not, please explain why not. (3 points)

(1 plays 1a with prob $\frac{1}{2}$, 1b $\frac{1}{2}$, 2 plays 2a with prob $\frac{1}{2}$, 2b $\frac{1}{2}$) is a mixed NE

(1 plays 1c with prob $\frac{1}{2}$, 1d $\frac{1}{2}$, 2 plays 2c with prob $\frac{1}{2}$, 2d $\frac{1}{2}$) is a mixed NE

we find these in the usual way.

Is there another mixed NE?

set these all equal

$EU_1(a) = 7q_a + q_b$
$EU_1(b) = q_a + 7q_b$

$EU_1(a) = EU_1(b)$ so
$7q_a + q_b = q_a + 7q_b \Rightarrow 6q_a = 6q_b \Rightarrow \boxed{q_a = q_b}$

$EU_1(c) = 6q_c + 2(1-q_a-q_b-q_c)$
$EU_1(d) = 2q_c + 6(1-q_a-q_b-q_c)$

$EU_1(c) = EU_1(d)$ so
$6q_c + 2(1-q_a-q_b-q_c) = 2q_c + 6(1-q_a-q_b-q_c)$
$4q_c = 4(1-q_a-q_b-q_c)$
$q_c = 1-q_a-q_b-q_c \Rightarrow 2q_c = 1-q_a-q_b$
$\Rightarrow \boxed{2q_c = 1-2q_a}$ since $q_a = q_b$

$EU_1(b) = q_a + 7q_b = 8q_a$ since $q_a = q_b$

$EU_1(b) = EU_1(c)$ so
$8q_a = 6q_c + 2(1-q_a-q_b-q_c) = 4q_c + 2 - 2q_a - 2q_b = 4q_c + 2 - 4q_a$ since $q_a = q_b$

So $8q_a = 4q_c + 2 - 4q_a \Rightarrow 12q_a = 4q_c + 2$
since
$12q_a = 2(1-2q_a) + 2$ since $2q_c = 1-2q_a$
$16q_a = 4 \Rightarrow \boxed{q_a = \frac{1}{4}}$ so $\boxed{q_b = \frac{1}{4}}$ and $\boxed{q_c = \frac{1}{4}}$

Similarly, we find that $p_a = \frac{1}{4}$, $p_b = \frac{1}{4}$, $p_c = \frac{1}{4}$.

So

$$\left(\begin{array}{llll} 1 \text{ plays} & 1a \text{ with prob} & \frac{1}{4} & \\ & 1b & \frac{1}{4} & \\ & 1c & \frac{1}{4} & \\ & 1d & \frac{1}{4}, & \end{array} \quad \begin{array}{lll} 2 \text{ plays} & 2a \text{ with prob} & \frac{1}{4} \\ & 2b & \frac{1}{4} \\ & 2c & \frac{1}{4} \\ & 2d & \frac{1}{4} \end{array} \right)$$

is a mixed NE.

So there are at least 3 mixed NE.

Problem 3. Four voters try to choose between candidates Anita, Billie, Cecilia and Danielle. However, only some of the preferences of the voters are known. Note that since there are four voters, a majority requires three or more people (two people do not constitute a majority). Voter 1 likes Anita best of all, Voter 2 likes Billie second-best, and so on.

Voter 1	Voter 2	Voter 3	Voter 4
A	C	B	C
C	**B**	D	A
B	D	**C**	B
D	A	A	**D**

a. Is it possible to fill in the table so that Cecilia is the Condorcet winner? If so, fill in the table above in such a way as to make Cecilia the Condorcet winner. If not, explain why not. (3 points)

Here $C > A$ / 2,3,4 / 1; $C > B$ / 1,2,4 / 3; $C > D$ / 1,2,4 / 3

Note that A must go here (to make $C > A$)

b. Is it possible to fill in the table so that Danielle is the Condorcet winner? If so, fill in the table below in such a way as to make Danielle the Condorcet winner. If not, explain why not. (3 points)

Voter 1	Voter 2	Voter 3	Voter 4
A			
	B		
		C	
			D

For D to be the Condorcet winner, 3 people must like D over A.

But Voter 4 ranks D last and voter 1 likes A best.
So at most two people like D over A.

So it is not possible for D to be the Condorcet winner.

Borda count before filling in anything

A : 0+2+0+3 = 5
B : 2+0+3 = 5
C : 1+3+1 = 5
D : 3+1+2 = 6

c. Say voter preferences are as follows.

Voter 1	Voter 2	Voter 3	Voter 4	
A	D	C	B	3
C	B	A	D	2
B	C	D	C	1
D	A	B	A	0

Is it possible to fill in the table to make C the Borda count winner (in other words, C gets more Borda count points than all other candidates)? If so, fill in the table in such a way as to make C the Borda count winner. If not, explain why not. (2 points)

Before filling in blanks

A : 5
B : 5
C : 5
D : 6

Fill in blanks as above

A : 5
B : 5+1 = 6
C : 5+2 = 7 winner
D : 6+0 = 6

d. Say voter preferences are again as follows.

Voter 1	Voter 2	Voter 3	Voter 4
A	D	C	B
	B	A	D
	C	D	C
	A	B	A

Is it possible to fill in the table to make A in the top cycle? If so, fill in the table in such a way as to make A in the top cycle. If not, explain why not. (2 points)

It is not possible to make A in the top cycle because Voter 2 and Voter 4 like A last and so A cannot beat anything else by majority (a majority requires 3 people).

e. Say voter preferences are again as follows.

Voter 1	Voter 2	Voter 3	Voter 4
A	D	C	B
	B	A	D
	C	D	C
	A	B	A

Is it possible to fill in the table to make B in the top cycle? If so, fill in the table in such a way as to make B in the top cycle. If not, explain why not. (2 points)

B cannot beat D because 2 voters prefer D over B.

B can beat A but A cannot beat anything (as explained in part d. above).

So for B to be in the top cycle, B must beat C, in other words B > C.

Note that C > A but A does not beat anything.

So to construct a sequence starting from B, which includes all candidates the only possibility is B > C > D > A.

But it is not possible for C > D, because two voters prefer D over C.

So it is not possible to make B in the top cycle.

Problem 4. The country of Pacifica is composed of two states, Calivada and Orewash. Calivada has 100 voters and Orewash has 100 voters.

The distribution of voters in Calivada is

Far left	Left	Center	Right	Far right
25	40	10	10	15

median

The distribution of voters in Orewash is median

Far left	Left	Center	Right	Far right
7	10	4	40	39

a. Say that two candidates are running for the governorship of Calivada. Each candidate takes one of the five possible positions, and each voter votes for the candidate who is closest to his own position. If the two candidates are equally far away, then the voters split equally. Each candidate wants to maximize the number of votes she gets.

What positions will the candidates for governor of Calivada take? (2 points)

the median voter in Calivada is Left
So both candidates will take the Left position.

b. Two candidates are running for the governorship of Orewash. What positions will the candidates for governor of Orewash take? (2 points)

the median voter in Orewash is Right
So both candidates will take the Right position.

c. Now say that Pacifica elects its Speaker of the House by nationwide popular election. In other words, all 200 voters of Pacifica vote for Speaker in one big nationwide election (what state a voter lives in doesn't matter—all votes nationwide are tallied together). There are two candidates for Speaker of the House, and each candidate takes a nationwide position (a candidate can't take one position in Calivada and a different position in Orewash). What positions will the candidates for Speaker of the House take? (2 points)

Nationwide we have

median

FL	L	C	R	FR
32	50	14	50	54

the median voter is Right so both candidates will take Right.

d. Now let's go back to the governors' races in Calivada and Orewash. The Pacifica People's Party (PPP) wants to influence the governor's race in Calivada. At a cost of $100 per voter, the PPP can transport voters from Orewash into Calivada via its network of Green Tortoise buses. The PPP cannot change voters' preferences, but it can take any Orewash voter and make him a Calivada voter. The PPP is a leftist party and wants to move the governor of Calivada leftward, but at lowest cost. Can the PPP alter the governor's race in Calivada? If so, whom will the PPP transport? If not, explain why not. (2 points)

FL	L	C	R	FR	
7	10	4	40	39	Orewash
25	40	10	10	15	Calivada
32	40	10	10	15	all FL voters in Orewash moved to Calivada

median

109 Calivada voters now median is 54th

Since the Calivada governor takes the L position, the only way to make her take a FL position is to bring in more FL voters from Orewash. But even if all FL Orewash voters are transported, the median voter would still be L.

e. Now say that the PPP is defunct. The new party is the Stormfront Patriots (SP). The SP wants to influence the governor's race in Orewash. At a cost of $100 per voter, the SP can transport voters from Calivada into Orewash via its network of Chrome Rebel pickup trucks. The SP cannot change voters' preferences, but it can take any Calivada voter and make her an Orewash voter. The SP is a rightist party and wants to move the governor of Orewash rightward, but at lowest cost. Can the SP alter the governor's race in Orewash? If so, whom will the SP transport? If not, explain why not. (2 points)

FL	L	C	R	FR	
25	40	10	10	15	Calivada
7	10	4	40	39	Orewash
7	10	4	40	54	Orewash after all FR Calivada voters move to Orewash
			↑ median		

115 Orewash voters now
median is 58th

Since the Orewash governor takes the R position, the only way to make him move rightward is to bring in FR voters from Calivada. But even if all 15 Calivada FR voters move to Orewash, the median voter would still be at R.

f. Finally, say that the Stormfront Patriots try to influence the Calivada governor's race. At a cost of $100 per voter, the SP can transport voters from Orewash into Calivada. The SP cannot change voters' preferences, but it can take any Orewash voter and make her a Calivada voter. The SP is a rightist party and wants to move the governor of Calivada rightward, but at lowest cost. Can the SP alter the governor's race in Calivada without affecting the outcome of the governor's race in Orewash? If so, whom will the SP will transport? If not, explain why not. (2 points)

FL	L	C	R	FR	
7	10	4	40	39	Orewash
25	40	10	10	15	Calivada
25	40	10	10	15+x	

Originally the Calivada governor is at L.
Can the SP make the median voter C?

Say the SP moves x FR voters from Orewash to Calivada.

Then Calivada has 100 + x voters total and the median voter has (if x is odd) $\frac{100+x-1}{2}$ voters to her left and $\frac{100+x-1}{2}$ voters to her right.

If x is even, then there are two median voters and the "left" median voter has $\frac{100+x}{2} - 1$ voters to her left. The "right" median voter has $\frac{100+x}{2}$ voters to her left.

So to make the median voter at C, the median voter must have 65 (20+45) voters to her left.

x is odd
$$65 = \frac{100+x-1}{2}$$
$$130 = 99 + x$$
$$x = 31$$

x is even
$$65 = \frac{100+x}{2}$$
$$130 = 100 + x$$
$$x = 30$$

If the SP transports 31 people, the median is at C. If the SP transports 30 people, then the right median is at C and the left median is at L.

Here's a (much) simpler solution:

FL	L	C	R	FR	
25	40	10	10	15	Calivada

The original median voter in Calivada is at L.

To make the new median voter at C, note that there are 65 people to the left of C.

So to make C the median, you need to make 65 people to be at C or to the right of C.

Since there are 35 people at C or to the right of C (10+10+15), you need to move ~~move~~ 30 people at C or to the right of C.

FL	L	C	R	FR	
25	40	10	10	45	30 added
	↑ "left" median	↑ "right" median			

Here there are two median voters so (L,L) and (C,C) are both NE.

If you move 31 voters, you get

FL	L	C	R	FR	
25	40	10	10	46	31 added
		↑ median			

and (C,C) is the only NE.

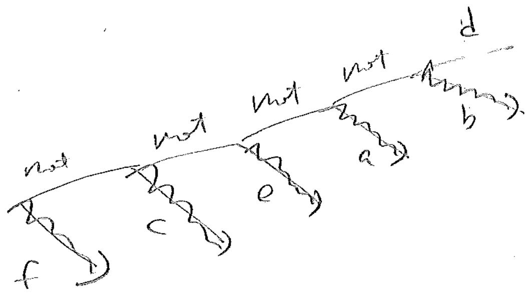
Problem 5. Say there are three voters choosing among six candidates a, b, c, d, e, f. The voters' preferences are as follows. For example, person 3 likes f best and a worst.

1	2	3
d	e	f
c	f	c
e	a	b
a	c	e
f	b	d
b	d	a

a > b
b > d
c > a, c > b, c > d, c > e,
d > a
e > a, e > b, e > d, e > f
f > a, f > b, f > c, f > d

a. Is it possible to create a voting agenda (an order of voting, in which all candidates are voted upon) so that candidate f wins? If so, write down the agenda. If not, explain why not. (2 points)

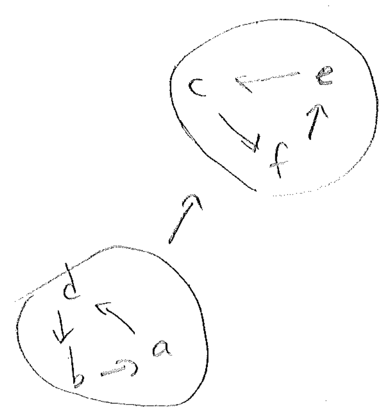
f > c > e > a > b > d for example



b. Say that person 1 controls the voting agenda. Obviously, person 1 would like to implement her favorite candidate, d. Is it possible to create a voting agenda so that candidate d wins? If so, write down the agenda. If not, explain why not. (2 points)

It is not possible. The only thing d beats is a, the only thing a beats is b, and the only thing b beats is d. It is not possible to "kill off" c, e, and f.

c. Write down the top cycle. (2 points)



top cycle is {c, e, f}

d. Now say that person 1 tries a different voting system. Person 1 can choose from five different voting systems: (1) the Borda count (2) approval voting when you vote for the top two (3) approval voting when you vote for the top three (4) approval voting when you vote for the top four, and (5) approval voting when you vote for the top five.

Can person 1 use one of these five voting systems to make her favorite candidate, d, win? If so, find that voting system. If not, explain why not. (3 points)

The voter preferences are below for your reference.

A5	A4	A3	A2	BC	1	2	3
1	1	1	1	5	d	e	f
1	1	1	1	4	c	f	c
1	1	1	0	3	e	a	b
1	1	0	0	2	a	c	e
1	0	0	0	1	f	b	d
0	0	0	0	0	b	d	a

(1) Borda count:

a	b	c	d	e	f
5	4	10	6	10	10

(2) Approval, top 2

a	b	c	d	e	f
0	0	2	1	1	2

(3) approval, top 3

a	b	c	d	e	f
1	1	2	1	2	2

(4) approval, top 4

a	b	c	d	e	f
2	1	3	1	3	2

(5) approval, top 5

a	b	c	d	e	f
2	2	3	2	3	3

In none of these voting systems does d win.

e. Say that voter preferences are given by

1	2	3
	e	f
e		
a	c	e
	b	
b		a

Is it possible to fill in the voter preferences so that the top cycle is {e, f}? If so, fill in the table and show that the top cycle is {e, f}. If not, please explain why not. (3 points)

The top cycle cannot be {e, f} because $e > f$ and $f > e$ is impossible. A cycle cannot composed of only two elements.

More precisely,
Say the top cycle is {e, f}. Then either $e > f$ or $f > e$ (since there are an odd number of voters).
Say $e > f$. Since f is in the top cycle, by definition of the top cycle, we can construct a sequence $f > \_ > \_ > \_ > \cdots > \_$ where all candidates other than f are placed in these blanks. e cannot be placed in the first blank since then $f > e$, which contradicts $e > f$. So some other candidate

must be in the first blank (say d). Then

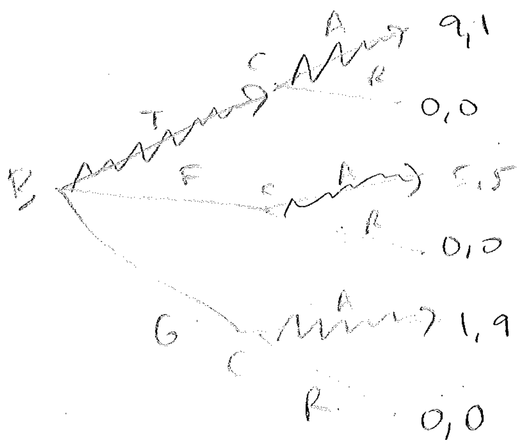
d is in the top cycle (because $d > \_ > \_ > \_ > e$

and e is in the top cycle)

and the top cycle can no longer be just $\{e, f\}$.

Problem 6. Person B and Person C are bargaining over 10 dollars. First, Person B can either make a tough offer (T), a fair offer (F), or a generous offer (G). Then, Person C can either accept (A) or reject (R) the offer. If Person C rejects an offer, then both people get 0. If Person C accepts the tough offer T, then person B gets 9 and person C gets 1. If Person C accepts the fair offer F, then both person B and person C get 5. If Person C accepts the generous offer G, then person B gets 1 and person C gets 9.

a. Model this as an extensive form game. (2 points)



b. Find a subgame perfect Nash equilibrium of this game. (1 point)

shown above

c. Represent this game as a strategic form game and find all (pure strategy) Nash equilibria of this strategic form game. (3 points)

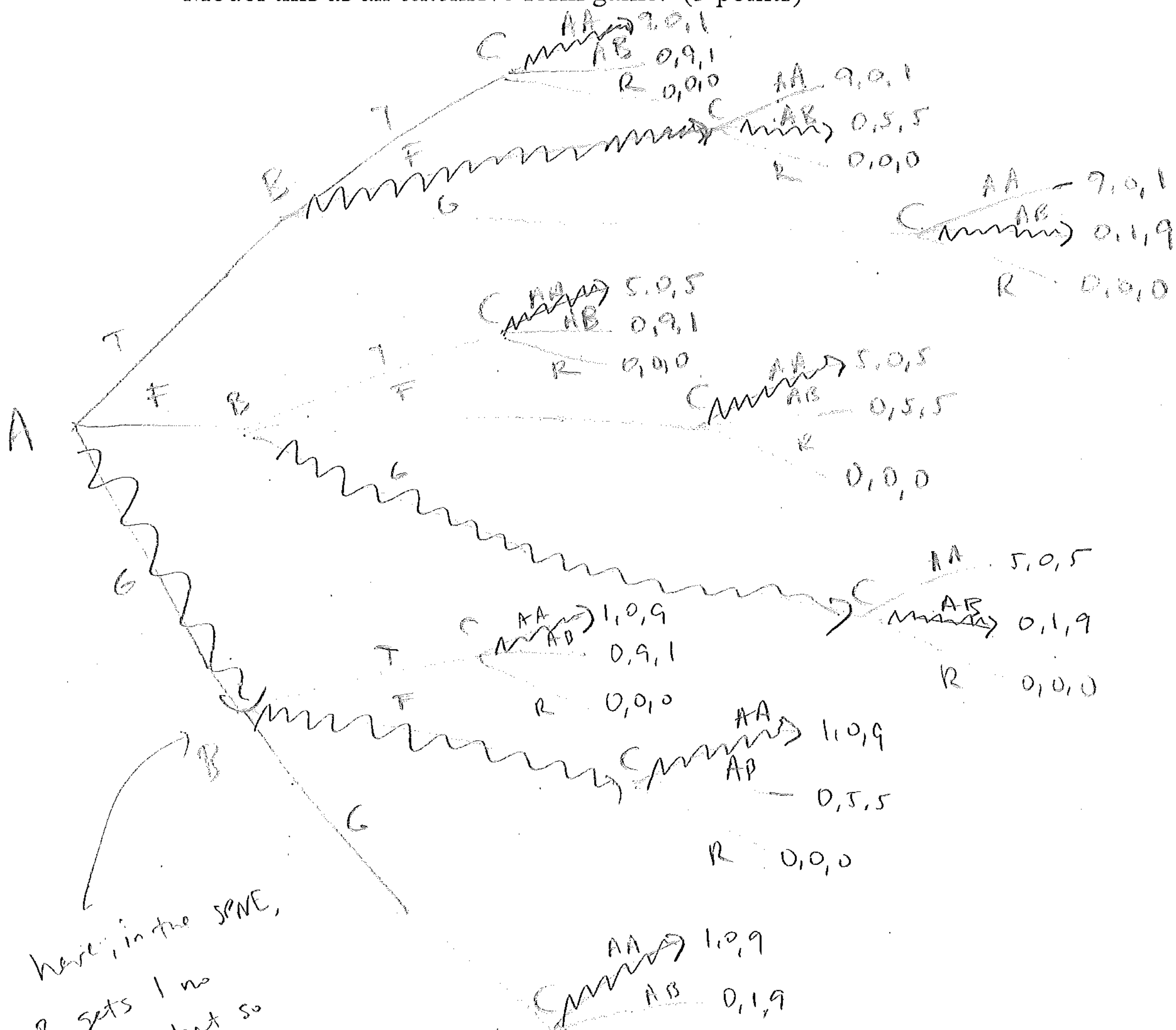
	AAA	AAR	ARA	ARR	RAA	RAR	RRA	RRR
T	*9,1+	*9,1+	*9,1+	*9,1+	0,0	0,0	0,0	*0,0
F	5,5+	5,5+	0,0	0,0	*5,5+	*5,5+	0,0	*0,0
G	1,9+	0,0	1,9+	0,0	1,9+	0,0	*1,9+	*0,0

NE (circled above): (T, AAA), (T, AAR), (T, ARA), (T, ARR)
(F, RAA), (F, RAR), (G, RRA)

d. Now say that Person A makes an offer to Person C first. Person A can make either a tough offer (T), a fair offer (F), or a generous offer (G). After Person A makes her offer, then Person B makes an offer to Person C. As before, Person B can make either a tough offer (T), a fair offer (F), or a generous offer (G).

After Person B makes his offer, then Person C can either accept Person A's offer (AA), accept Person B's offer (AB), or reject both offers (R). Person C cannot accept both offers. If Person C accepts Person A's offer, then Person B gets nothing. If Person C accepts Person B's offer, then Person A gets nothing. For example, if Person A makes a fair offer, and Person B makes a generous offer, and then Person C accepts Person A's offer, then Person A gets 5, Person B gets 0, and Person C gets 5. If Person C rejects both offers, everyone gets 0.

Model this as an extensive form game. (3 points)



here, in the SPNE, B gets 1 no matter what so we just pick one arbitrarily

e. Write down a subgame perfect Nash equilibrium of this game (you do not have to write down all of them). Note that if Person A and Person B make the same offer to Person C, then Person C is indifferent between accepting Person A's offer and Person B's offer. In this case, assume that Person C "breaks the tie" by accepting Person A's offer because Person A made it first. (2 points) SPNE indicated above

f. Which game does Person B prefer: the game in part a. or the game in part d.? Which game does person C prefer: the game in part a. or the game in part d.? (1 point)

In the game in part a., B gets 9 and C gets 1 in the SPNE

part d., B gets 0 and C gets 9 in the SPNE.

C prefers the game in d. C likes competition

B " a. B dislikes "

Problem 7. Say we have four women, A, B, C, and D, and four men, W, X, Y, and Z. Say that each woman wants to be matched with one man, and each man wants to be matched with one woman. Their preferences are as follows.

A	B	C	D
W	X	Y	Z
X	Y	Z	W
Y	Z	W	X
Z	W	X	Y

W	X	Y	Z
A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

In other words, woman A likes W best, X second-best, Y third-best, and Z least. All the men like A best, B second-best, C third-best, and D least.

a. Find all stable matchings. (2 points)

women - ask

(AW, BX, CY, DZ) stable

men - ask

(WA, XA, YA, ZA)

(WA, XB, YB, ZB)

(WA, XB, YC, ZC)

(WA, XB, YC, ZD) stable

since the woman-ask and the men-ask algorithms yield the same match, there is only one stable match.

b. Say that there is a flu epidemic which makes woman A and man Z really sick. So now there are three women (B, C, D) and three men (W, X, Y) who match together. Of these six remaining people, which men and women become worse off because of the flu epidemic? Does anyone among the six remaining people gain from the flu epidemic (if so, who)? (2 points)

B	C	D
X	Y	W
Y	W	X
W	X	Y

W	X	Y
B	B	B
C	C	C
D	D	D

women-ask

(BX, CY, DW)

men - ask

(WB, XB, YB)

(WC, XB, YC)

(WD, XB, YC)

same, so only one stable match.

Before epidemic

(AW, BX, CY, DZ)

after

(BX, CY, DW)

Result of epidemic:

B and X unchanged

C and Y "

D loses (goes from Z to W)

W loses big (goes from A to D)

Here are the preferences again for your reference.

A	B	C	D
W	X	Y	Z
X	Y	Z	W
Y	Z	W	X
Z	W	X	Y

W	X	Y	Z
A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

c. Now say that instead of a flu epidemic, Pope Francis makes an impassioned speech which makes woman B and man Y join the priesthood and remain single. So now there are three women (A, C, D) and three men (W, X, Z) remaining who match together. Of these six remaining people, which men and women become worse off because of Pope Francis's speech? Does anyone among the six remaining people gain from Pope Francis's speech (if so, who)? (2 points)

A	C	D
W	Z	Z
X	W	W
Z	X	X

W	X	Z
A	A	A
C	C	C
D	D	D

woman-ask

(AW, CZ, DZ)

(AW, CZ, DW)

(AW, CZ, DX)

man-ask

(WA, XA, ZA)

(WA, XB, ZB)

(WA, XB, ZC)

same so only one stable match.

Before : (AW, BX, CY, DZ)

After : (AW, CZ, DX)

Result of Pope Francis:

A, W remain unchanged

C loses (goes from Y to Z)

D loses (goes from Z to X)

X loses (goes from B to D)

Z gains (goes from D to C)

Z gains because his competition for C has gone away.

d. Now say that man Y has a devious relative who is trying to improve Y's marriage prospects. This devious relative, through bribery and trickery, can take out one woman and one man from the marriage market. For example, the devious relative could take out woman D and man Z, and then there would remain only women A, B, and C and men W, X, and Y.

Can this devious relative make it possible for man Y to get his first choice (woman A)? If so, please write down which woman and which man would have to be taken out. If not, please explain why not. (3 points)

For Y to get A, W would have to be taken out (because A is W's first choice and W is A's first choice).

Once W is out, then A's new first choice is X and X likes A best. So A and X will match.

So there's no way for Y to get A without taking out both W and X.

Here are the preferences again for your reference.

A	B	C	D
W	X	Y	Z
X	Y	Z	W
Y	Z	W	X
Z	W	X	Y

W	X	Y	Z
A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

e. Can this devious relative make it possible for man Y to get his second choice (woman B)? If so, please write down which woman and which man would have to be taken out. If not, please explain why not. (3 points)

take X out (his main competition for B)

and take D (for example) out.

A	B	C
W	Y	Y
Y	Z	Z
Z	W	W

W	Y	Z
A	A	A
B	B	B
C	C	C

woman-ask

(AW, BY, CY)

(AW, BY, CZ)

men-ask

(WA, YA, ZA)

(WA, YB, ZB)

= (WA, YB, ZC)

only one stable match

now Y gets B.

Problem 8. Recall that in the threshold model, a person participates if the total number of other participants is greater than or equal to her threshold. For example, a person with threshold 3 will participate if 3 or more other people participate.

a. Say that there are eight people with thresholds given in the table below. Find all Nash equilibria given these thresholds. You can write them in the table; please write "Y" if a person participates and "N" if a person doesn't participate. (2 points)

		NE	NE	NE					
threshold									
0		y	y	y					
2		n	y	y					
2		n	y	y					
3		n	y	y					
5		n	n	y					
5		n	n	y					
6		n	n	y					
9		n	n	n					

the 0 person always goes. The 2 people jump in together, and if they do,
9 never " . the 3 person goes too. The 5 people jump in together, and if they go, the 6 person goes.

b. Now say that thresholds are given in the table below. Find all Nash equilibria given these thresholds. You can write them in the table; please write "Y" when a person participates and "N" if a person doesn't participate. (2 points)

		NE	NE	NE					
threshold									
0		y	y	y					
3		n	y	y					
3		n	y	y					
3		n	y	y					
5		n	n	y					
5		n	n	y					
6		n	n	y					
9		n	n	n					

c. In parts a. and b., given thresholds, we found the set of Nash equilibria. Now we go in the opposite direction: given the set of Nash equilibria, we find thresholds. There are exactly four Nash equilibria, shown in the table below (the "Y"s are shaded for clarity). Write down thresholds such that these four are the set of Nash equilibria. There is more than one possible answer—just write down one of them. (2 points)

goes by herself so must be.

threshold	NE	NE	NE	NE					
0	Y	Y	Y	Y					
2 or 3	N	Y	Y	Y					
2 or 3	N	Y	Y	Y					
2 or 3	N	Y	Y	Y					
5	N	N	Y	Y					
5	N	N	Y	Y					
7	N	N	N	Y					
7	N	N	N	Y					

must be 7 (cannot be 6 or 8)

must be at least 2 (because if 1, then would not be a NE) and at most 3 (because willing to go with 3 others).

must be at least 5 (because if 4 or less, then would not be a NE) and at most 5 (because is a NE) So must be 5

d. Say that the "threshold total" is simply the sum of everyone's thresholds. For example, in part a., the threshold total is 0+2+2+3+5+5+6+9 = 32. To take another example, if everyone had threshold 2, then the threshold total would be 16 (since there are eight people).

Again, there are exactly four Nash equilibria, shown in the table below. There might be more than one possible set of thresholds that have these Nash equilibria. What is the maximum possible threshold total? What is the minimum possible threshold total? (2 points)

threshold	NE	NE	NE	NE	min thresholds		max thresholds		
	Y	Y	Y	Y	0		0		
	N	Y	Y	Y	2		3		
	N	Y	Y	Y	2		3		
	N	Y	Y	Y	2		3		
	N	N	Y	Y	5		5		
	N	N	Y	Y	5		5		
	N	N	N	Y	7		7		
	N	N	N	Y	7		7		
					30		33	total	

min total is 30

max total is 33

e. Say that there are four Nash equilibria, shown in the table below. Is it possible to write down thresholds which have these four Nash equilibria? If so, write down the thresholds. If not, explain why not. (2 points)

must be

threshold	NE	NE	NE	NE					
0	Y	Y	Y	Y					
2 or 3	N	Y	Y	Y					
2 or 3	N	Y	Y	Y					
2 or 3	N	Y	Y	Y					
	N	N	Y	Y					
	N	N	N	Y					
	N	N	N	Y					
	N	N	N	Y					

if this threshold is 4 or less, then would not be a NE.

But since is a NE, she is content to go with 4 partners, so her threshold must be 4 or less.

So it is not possible to write down thresholds which have these as NE.

f. Say there are exactly three Nash equilibria, shown in the table below. Is it possible to write down thresholds that have these three Nash equilibria? If so, what is the maximum possible threshold total and what is the minimum possible threshold total? If not, explain why not. (2 points)

must be

cannot be 1, cannot be greater than 4

threshold	NE	NE	NE		min thresholds		max thresholds	
0	Y	Y	Y		0		0	
	N	Y	Y		2		4	
	N	Y	Y		2		4	
	N	Y	Y		2		4	
	N	Y	Y		2		4	
	N	N	Y		6		7	
	N	N	Y		6		7	
	N	N	Y		6		7	

26. 37

cannot be 5 or lower, cannot be 8

min total is 26

max total is 37