Last name	First name	Student ID number	TA

Part 1

(questions 1 and 2)

Final exam PS 30 December 2013

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part contains two problems, and hence there are eight problems. Each question is weighted equally (12 points each). Each part is stapled separately (for ease in grading, since the parts are graded separately). Please make sure that your name and student ID number is on each part.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once. When out of the room, you cannot communicate with any other person in any manner.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1		5	
2		6	
3		7	
4		8	
		total	

Problem 1. Two UCLA students get into an argument over which frozen yogurt is better— Yogurtland or Pinkberry—in the middle of the street in Westwood Village. People start gathering around them, chanting “Fight!” The two students now have two choices: to leave or to fight. If only one leaves, that person looks like a coward and gets a payoff of -10, while the person who stays and fights gets a payoff of 20. If they both leave, they each get a payoff of 1. If they both fight, Student 1 gets a payoff of “x”, and Student 2 gets a payoff of “y”.

a. Model this situation as a strategic form game. (2 points)

b. Suppose the two students are equally strong, and therefore x equals y. Circle the values of x below for which both students have a dominant strategy in the game in part a. Please explain your answer. (2 points)

x= -22	x= -17	x= -12	x= -7	x= -2	x= 2	x= 7	x= 12	x= 17	x= 22

c. Again, assume that x equals y. Circle the values of x below which make this game (the game in part a.) a Prisoner’s Dilemma. Please explain your answer. (2 points)

x= -22	x= -17	x= -12	x= -7	x= -2	x= 2	x= 7	x= 12	x= 17	x= 22

d. Now say that x and y do not have to be equal. Sometimes in a game it is possible that after iteratively eliminating (strongly or weakly) dominated strategies, you are left with a single outcome. When this happens, we call the game "dominance solvable."

In other words, when a game is dominance solvable, by iteratively eliminating (strongly or weakly) dominated strategies, you can predict exactly what each person will do.

Circle the values of x and y below which make the game in part a. dominance solvable. Please explain your answer. (2 points)

x= -22 y= 22	x= -17 y= 22	x= -12 y= 22	x= -7 y= 22	x= -2 y= 22	x= 2 y= 22	x= 7 y= 22	x= 12 y= 22	x= 17 y= 22	x= 22 y= 22
x= -22 y= 17	x= -17 y= 17	x= -12 y= 17	x= -7 y= 17	x= -2 y= 17	x= 2 y= 17	x= 7 y= 17	x= 12 y= 17	x= 17 y= 17	x= 22 y= 17
x= -22 y= 12	x= -17 y= 12	x= -12 y= 12	x= -7 y= 12	x= -2 y= 12	x= 2 y= 12	x= 7 y= 12	x= 12 y= 12	x= 17 y= 12	x= 22 y= 12
x= -22 y= 7	x= -17 y= 7	x= -12 y= 7	x= -7 y= 7	x= -2 y= 7	x= 2 y= 7	x= 7 y= 7	x= 12 y= 7	x= 17 y= 7	x= 22 y= 7
x= -22 y= 2	x= -17 y= 2	x= -12 y= 2	x= -7 y= 2	x= -2 y= 2	x= 2 y= 2	x= 7 y= 2	x= 12 y= 2	x= 17 y= 2	x= 22 y= 2
x= -22 y= -2	x= -17 y= -2	x= -12 y= -2	x= -7 y= -2	x= -2 y= -2	x= 2 y= -2	x= 7 y= -2	x= 12 y= -2	x= 17 y= -2	x= 22 y= -2
x= -22 y= -7	x= -17 y= -7	x= -12 y= -7	x= -7 y= -7	x= -2 y= -7	x= 2 y= -7	x= 7 y= -7	x= 12 y= -7	x= 17 y= -7	x= 22 y= -7
x= -22 y= -12	x= -17 y= -12	x= -12 y= -12	x= -7 y= -12	x= -2 y= -12	x= 2 y= -12	x= 7 y= -12	x= 12 y= -12	x= 17 y= -12	x= 22 y= -12
x= -22 y= -17	x= -17 y= -17	x= -12 y= -17	x= -7 y= -17	x= -2 y= -17	x= 2 y= -17	x= 7 y= -17	x= 12 y= -17	x= 17 y= -17	x= 22 y= -17
x= -22 y= -22	x= -17 y= -22	x= -12 y= -22	x= -7 y= -22	x= -2 y= -22	x= 2 y= -22	x= 7 y= -22	x= 12 y= -22	x= 17 y= -22	x= 22 y= -22

e. Now, assume that Student 1 gets to decide first whether to fight or leave. Then Student 2 decides to fight or leave. If they both choose to fight, then Student 3 can stop the fight or do nothing. If Student 3 stops the fight, she will become a hero and receive a payoff of 30 while the other two receive -20. If Student 3 does nothing when the other students fight, Student 3 gets a 0, and Student 1 again gets a payoff of x and Student 2 gets a payoff of y.

If either Student 1 or Student 2 leaves and the other stays to fight, the one who leaves gets -10 and the one who fights gets 20 (like before). If Student 1 and 2 both leave, they each get a payoff of 1 (like before). Student 3 only gets involved if Student 1 and Student 2 both decide to fight; for all the cases where Student 3 is not involved, she gets a 0.

Model this as an extensive form game. (3 points)

f. Find the subgame perfect Nash equilibria for the game in part e. Please mark your arrows clearly. (3 points)

Problem 2. Say we have the following game.

	2a	2b	2c
1a	7, 1	1, 7	0, 0
1b	1, 7	7, 1	0, 0
1c	0, 0	0, 0	4, 4

a. Find all pure strategy Nash equilibria of this game. (2 points)

b. Write down a mixed strategy Nash equilibrium of this game. (2 points)

b. Can you find two (or more) mixed strategy Nash equilibria of this game? If so, please write two of them down. If not, please explain why not. (2 points)

c. Now consider the following game.

	2a	2b	2c	2d
1a	7, 1	1, 7	0, 0	0, 0
1b	1, 7	7, 1	0, 0	0, 0
1c	0, 0	0, 0	6, 6	2, 2
1d	0, 0	0, 0	2, 2	6, 6

Find all pure strategy Nash equilibria of this game. (3 points)

d. Can you find three (or more) mixed strategy Nash equilibria of this game? If so, please write three of them down. If not, please explain why not. (3 points)

Problem 3. Four voters try to choose between candidates Anita, Billie, Cecilia and Danielle. However, only some of the preferences of the voters are known. Note that since there are four voters, a majority requires three or more people (two people do not constitute a majority). Voter 1 likes Anita best of all, Voter 2 likes Billie second-best, and so on.

Voter 1	Voter 2	Voter 3	Voter 4
A			
	B		
		C	
			D

a. Is it possible to fill in the table so that Cecilia is the Condorcet winner? If so, fill in the table above in such a way as to make Cecilia the Condorcet winner. If not, explain why not. (3 points)

b. Is it possible to fill in the table so that Danielle is the Condorcet winner? If so, fill in the table below in such a way as to make Danielle the Condorcet winner. If not, explain why not. (3 points)

Voter 1	Voter 2	Voter 3	Voter 4
A			
	B		
		C	
			D

c. Say voter preferences are as follows.

Voter 1	Voter 2	Voter 3	Voter 4
A	**D**	**C**	**B**
	B	**A**	**D**
	C	**D**	**C**
	A	**B**	**A**

Is it possible to fill in the table to make C the Borda count winner (in other words, C gets more Borda count points than all other candidates)? If so, fill in the table in such a way as to make C the Borda count winner. If not, explain why not. (2 points)

d. Say voter preferences are again as follows.

Voter 1	Voter 2	Voter 3	Voter 4
A	**D**	**C**	**B**
	B	**A**	**D**
	C	**D**	**C**
	A	**B**	**A**

Is it possible to fill in the table to make A in the top cycle? If so, fill in the table in such a way as to make A in the top cycle. If not, explain why not. (2 points)

e. Say voter preferences are again as follows.

Voter 1	Voter 2	Voter 3	Voter 4
A	**D**	**C**	**B**
	B	**A**	**D**
	C	**D**	**C**
	A	**B**	**A**

Is it possible to fill in the table to make B in the top cycle? If so, fill in the table in such a way as to make B in the top cycle. If not, explain why not. (2 points)

Problem 4. The country of Pacifica is composed of two states, Calivada and Orewash. Calivada has 100 voters and Orewash has 100 voters.

The distribution of voters in Calivada is

Far left	Left	Center	Right	Far right
25	40	10	10	15

The distribution of voters in Orewash is

Far left	Left	Center	Right	Far right
7	10	4	40	39

a. Say that two candidates are running for the governorship of Calivada. Each candidate takes one of the five possible positions, and each voter votes for the candidate who is closest to his own position. If the two candidates are equally far away, then the voters split equally. Each candidate wants to maximize the number of votes she gets.

What positions will the candidates for governor of Calivada take? (2 points)

b. Two candidates are running for the governorship of Orewash. What positions will the candidates for governor of Orewash take? (2 points)

c. Now say that Pacifica elects its Speaker of the House by nationwide popular election. In other words, all 200 voters of Pacifica vote for Speaker in one big nationwide election (what state a voter lives in doesn't matter—all votes nationwide are tallied together). There are two candidates for Speaker of the House, and each candidate takes a nationwide position (a candidate can't take one position in Calivada and a different position in Orewash). What positions will the candidates for Speaker of the House take? (2 points)

d. Now let's go back to the governors' races in Calivada and Orewash. The Pacifica People's Party (PPP) wants to influence the governor's race in Calivada. At a cost of $100 per voter, the PPP can transport voters from Orewash into Calivada via its network of Green Tortoise buses. The PPP cannot change voters' preferences, but it can take any Orewash voter and make him a Calivada voter. The PPP is a leftist party and wants to move the governor of Calivada leftward, but at lowest cost. Can the PPP alter the governor's race in Calivada? If so, whom will the PPP transport? If not, explain why not. (2 points)

e. Now say that the PPP is defunct. The new party is the Stormfront Patriots (SP). The SP wants to influence the governor's race in Orewash. At a cost of $100 per voter, the SP can transport voters from Calivada into Orewash via its network of Chrome Rebel pickup trucks. The SP cannot change voters' preferences, but it can take any Calivada voter and make her an Orewash voter. The SP is a rightist party and wants to move the governor of Orewash rightward, but at lowest cost. Can the SP alter the governor's race in Orewash? If so, whom will the SP transport? If not, explain why not. (2 points)

f. Finally, say that the Stormfront Patriots try to influence the Calivada governor's race. At a cost of $100 per voter, the SP can transport voters from Orewash into Calivada. The SP cannot change voters' preferences, but it can take any Orewash voter and make her a Calivada voter. The SP is a rightist party and wants to move the governor of Calivada rightward, but at lowest cost. Can the SP alter the governor's race in Calivada without affecting the outcome of the governor's race in Orewash? If so, whom will the SP will transport? If not, explain why not. (2 points)

Problem 5. Say there are three voters choosing among six candidates a, b, c, d, e, f. The voters' preferences are as follows. For example, person 3 likes f best and a worst.

1	2	3
d	e	f
c	f	c
e	a	b
a	c	e
f	b	d
b	d	a

a. Is it possible to create a voting agenda (an order of voting, in which all candidates are voted upon) so that candidate f wins? If so, write down the agenda. If not, explain why not. (2 points)

b. Say that person 1 controls the voting agenda. Obviously, person 1 would like to implement her favorite candidate, d. Is it possible to create a voting agenda so that candidate d wins? If so, write down the agenda. If not, explain why not. (2 points)

c. Write down the top cycle. (2 points)

d. Now say that person 1 tries a different voting system. Person 1 can choose from five different voting systems: (1) the Borda count (2) approval voting when you vote for the top two (3) approval voting when you vote for the top three (4) approval voting when you vote for the top four, and (5) approval voting when you vote for the top five.

Can person 1 use one of these five voting systems to make her favorite candidate, d, win? If so, find that voting system. If not, explain why not. (3 points)

The voter preferences are below for your reference.

1	2	3
d	e	f
c	f	c
e	a	b
a	c	e
f	b	d
b	d	a

e. Say that voter preferences are given by

1	2	3
	e	f
e		
a	c	e
	b	
b		a

Is it possible to fill in the voter preferences so that the top cycle is {e, f}? If so, fill in the table and show that the top cycle is {e, f}. If not, please explain why not. (3 points)

Problem 6. Person B and Person C are bargaining over 10 dollars. First, Person B can either make a tough offer (T), a fair offer (F), or a generous offer (G). Then, Person C can either accept (A) or reject (R) the offer. If Person C rejects an offer, then both people get 0. If Person C accepts the tough offer T, then person B gets 9 and person C gets 1. If Person C accepts the fair offer F, then both person B and person C get 5. If Person C accepts the generous offer G, then person B gets 1 and person C gets 9.

a. Model this as an extensive form game. (2 points)

b. Find a subgame perfect Nash equilibrium of this game. (1 point)

c. Represent this game as a strategic form game and find all (pure strategy) Nash equilibria of this strategic form game. (3 points)

d. Now say that Person A makes an offer to Person C first. Person A can make either a tough offer (T), a fair offer (F), or a generous offer (G). After Person A makes her offer, then Person B makes an offer to Person C. As before, Person B can make either a tough offer (T), a fair offer (F), or a generous offer (G).

After Person B makes his offer, then Person C can either accept Person A's offer (AA), accept Person B's offer (AB), or reject both offers (R). Person C cannot accept both offers. If Person C accepts Person A's offer, then Person B gets nothing. If Person C accepts Person B's offer, then Person A gets nothing. For example, if Person A makes a fair offer, and Person B makes a generous offer, and then Person C accepts Person A's offer, then Person A gets 5, Person B gets 0, and Person C gets 5. If Person C rejects both offers, everyone gets 0.

Model this as an extensive form game. (3 points)

e. Write down a subgame perfect Nash equilibrium of this game (you do not have to write down all of them). Note that if Person A and Person B make the same offer to Person C, then Person C is indifferent between accepting Person A's offer and Person B's offer. In this case, assume that Person C "breaks the tie" by accepting Person A's offer because Person A made it first. (2 points)

f. Which game does Person B prefer: the game in part a. or the game in part d.? Which game does person C prefer: the game in part a. or the game in part d.? (1 point)

Problem 7. Say we have four women, A, B, C, and D, and four men, W, X, Y, and Z. Say that each woman wants to be matched with one man, and each man wants to be matched with one woman. Their preferences are as follows.

A	B	C	D
W	X	Y	Z
X	Y	Z	W
Y	Z	W	X
Z	W	X	Y

W	X	Y	Z
A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

In other words, woman A likes W best, X second-best, Y third-best, and Z least. All the men like A best, B second-best, C third-best, and D least.

a. Find all stable matchings. (2 points)

b. Say that there is a flu epidemic which makes woman A and man Z really sick. So now there are three women (B, C, D) and three men (W, X, Y) who match together. Of these six remaining people, which men and women become worse off because of the flu epidemic? Does anyone among the six remaining people gain from the flu epidemic (if so, who)? (2 points)

Here are the preferences again for your reference.

A	B	C	D
W	X	Y	Z
X	Y	Z	W
Y	Z	W	X
Z	W	X	Y

W	X	Y	Z
A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

c. Now say that instead of a flu epidemic, Pope Francis makes an impassioned speech which makes woman B and man Y join the priesthood and remain single. So now there are three women (A, C, D) and three men (W, X, Z) remaining who match together. Of these six remaining people, which men and women become worse off because of Pope Francis's speech? Does anyone among the six remaining people gain from Pope Francis's speech (if so, who)? (2 points)

d. Now say that man Y has a devious relative who is trying to improve Y's marriage prospects. This devious relative, through bribery and trickery, can take out one woman and one man from the marriage market. For example, the devious relative could take out woman D and man Z, and then there would remain only women A, B, and C and men W, X, and Y.

Can this devious relative make it possible for man Y to get his first choice (woman A)? If so, please write down which woman and which man would have to be taken out. If not, please explain why not. (3 points)

Here are the preferences again for your reference.

A	B	C	D
W	X	Y	Z
X	Y	Z	W
Y	Z	W	X
Z	W	X	Y

W	X	Y	Z
A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

e. Can this devious relative make it possible for man Y to get his second choice (woman B)? If so, please write down which woman and which man would have to be taken out. If not, please explain why not. (3 points)

Problem 8. Recall that in the threshold model, a person participates if the total number of other participants is greater than or equal to her threshold. For example, a person with threshold 3 will participate if 3 or more other people participate.

a. Say that there are eight people with thresholds given in the table below. Find all Nash equilibria given these thresholds. You can write them in the table; please write "Y" if a person participates and "N" if a person doesn't participate. (2 points)

threshold									
0									
2									
2									
3									
5									
5									
6									
9									

b. Now say that thresholds are given in the table below. Find all Nash equilibria given these thresholds. You can write them in the table; please write "Y" when a person participates and "N" if a person doesn't participate. (2 points)

threshold									
0									
3									
3									
3									
5									
5									
6									
9									

c. In parts a. and b., given thresholds, we found the set of Nash equilibria. Now we go in the opposite direction: given the set of Nash equilibria, we find thresholds. There are exactly four Nash equilibria, shown in the table below (the "Y"s are shaded for clarity). Write down thresholds such that these four are the set of Nash equilibria. There is more than one possible answer—just write down one of them. (2 points)

threshold	NE	NE	NE	NE					
	Y	Y	Y	Y					
	N	Y	Y	Y					
	N	Y	Y	Y					
	N	Y	Y	Y					
	N	N	Y	Y					
	N	N	Y	Y					
	N	N	N	Y					
	N	N	N	Y					

d. Say that the "threshold total" is simply the sum of everyone's thresholds. For example, in part a., the threshold total is 0+2+2+3+5+5+6+9 = 32. To take another example, if everyone had threshold 2, then the threshold total would be 16 (since there are eight people).

Again, there are exactly four Nash equilibria, shown in the table below. There might be more than one possible set of thresholds that have these Nash equilibria. What is the maximum possible threshold total? What is the minimum possible threshold total? (2 points)

threshold	NE	NE	NE	NE					
	Y	Y	Y	Y					
	N	Y	Y	Y					
	N	Y	Y	Y					
	N	Y	Y	Y					
	N	N	Y	Y					
	N	N	Y	Y					
	N	N	N	Y					
	N	N	N	Y					

e. Say that there are four Nash equilibria, shown in the table below. Is it possible to write down thresholds which have these four Nash equilibria? If so, write down the thresholds. If not, explain why not. (2 points)

threshold	NE	NE	NE	NE					
	Y	Y	Y	Y					
	N	Y	Y	Y					
	N	Y	Y	Y					
	N	Y	Y	Y					
	N	N	Y	Y					
	N	N	N	Y					
	N	N	N	Y					
	N	N	N	Y					

f. Say there are exactly three Nash equilibria, shown in the table below. Is it possible to write down thresholds that have these three Nash equilibria? If so, what is the maximum possible threshold total and what is the minimum possible threshold total? If not, explain why not. (2 points)

threshold	NE	NE	NE					
	Y	Y	Y					
	N	Y	Y					
	N	Y	Y					
	N	Y	Y					
	N	Y	Y					
	N	N	Y					
	N	N	Y					
	N	N	Y					