Last name	First name	Student ID number	TA

Answers to

Part 1
(problems 1 and 2)

Final exam PS 30 December 2012

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part contains two problems, and hence there are eight problems. Each question is weighted equally (12 points each). Each part is stapled separately (for ease in grading, since the parts are graded separately). Please make sure that your name and student ID number is on each part.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1		5	
2		6	
3		7	
4		8	
		total	

Problem 1. Consider the following strategic form game.

	W	X	Y	Z	
A	~~-1,1~~	~~3,-1~~	~~0,8~~	~~-2,4~~	(1)
B	~~4,3~~	~~4,6~~	~~1,0~~	~~0,1~~	(3)
C	~~-3,-2~~	~~5,0~~	8,2	4,5	
D	~~-5,0~~	~~1,1~~	4,4	6,2	
	(2)	(4)			

a. Iteratively eliminate any weakly or strongly dominated strategies. Be sure to list the order of deletion. (3 points)

(1) B s. dominates A

(2) X s. dominates W

(3) C s. dominates B

(4) Y s. dominates X

b. Find all pure strategy Nash equilibria (if any). (3 points)

remaining game:

	Y	Z
C	^{*8,2}	$4,5^+$
D	$4,4^+$	^{*6,2}

no pure strat NE

c. Find all mixed strategy Nash equilibria (if any). (3 points)

	$[q]$ Y	$[1-q]$ Z
$[p]$ C	8,2	4,5
$[1-p]$ D	4,4	6,2

$EU_1(C) = 8q + 4(1-q) = 4+4q$

$EU_1(D) = 4q + 6(1-q) = 6-2q$

$4+4q = 6-2q$

$6q = 2$

$\boxed{q = \frac{1}{3}}$

$EU_2(Y) = 2p + 4(1-p) = 4-2p$

$EU_2(Z) = 5p + 2(1-p) = 2+3p$

$4-2p = 2+3p$

$2 = 5p$

$\boxed{p = \frac{2}{5}}$

So mixed NE is

(1 plays C with prob $\frac{2}{5}$, D $\frac{3}{5}$, 2 plays Y with prob $\frac{1}{3}$, Z $\frac{2}{3}$)

d. Say we change the game slightly so that some payoffs depend on k.

	W	X	Y	Z	
A	~~-1,1~~	~~3,-1~~	~~0,k~~	~~-2,4~~	1
B	~~4,3~~	~~4,6~~	~~1,0~~	~~0,k~~	3
C	~~-3,-2~~	~~5,0~~	8,2	4,5	
D	~~k,0~~	~~1,1~~	4,4	6,2	
	2	4			

The k does not affect the iterative elimination of strongly dominated strategies earlier.
Hence the k does not affect the NE at all.

Please fill in the following table. (3 points).

If	then the (pure strategy and mixed strategy) NE are:
k = 1	no pure strat NE mixed NE: (1 plays C with prob $\frac{2}{5}$, D $\frac{3}{5}$, 2 plays Y with prob $\frac{1}{3}$, Z $\frac{2}{3}$)
k = 10	//
k = 100	//
k = 1000	//

Problem 2. Say that there are 101 people in a society which is deciding among three candidates, A, B, and C. The society is composed of two groups, Group 1 and Group 2. Group 1 has x members and Group 2 has $101 - x$ members. Assume throughout that x is a natural number (i.e. not a fraction—you can't have fractions of people). Assume that $1 \leq x$ and $x \leq 100$ (in other words, there is at least one person in each group). Their preferences are as follows.

	Group 1 (x people)	Group 2 ($101 - x$ people)
(Best)	A	C
	B	B
(Worst)	C	A

if $x = 38$

C > B
63 38

B > A
63 38

C > A
63 38

C is Condorcet winner

Borda count: A: 76
B: 101
$\boxed{C: 126}$

approval (top two):
A: 38
$\boxed{B: 101}$
C: 63

a. Say that $x = 38$. Please fill in the following table. Here approval voting means that each person votes for her top two candidates. (1 point)

If	then the Condorcet winner is	then the Borda count winner is	then the approval voting winner is
x = 38	C	C	B

b. Now say that x is unknown—it can be any number (as long as $1 \leq x$ and $x \leq 100$). Is it possible for the Condorcet winner to be *different* from the Borda count winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

Borda count A: $2x$
B: $x + 101 - x = 101$
C: $2(101-x) = 202 - 2x$

If $x \leq 50$, then C is Borda winner
$x \geq 51$, then A "

Condorcet:

A vs B
x $101-x$

A vs C
x $101-x$

B vs C
x $101-x$

If $x \leq 50$, C is the Condorcet winner
(group 2 is a majority)

If $x \geq 51$, A is the Condorcet winner
(group 1 is a majority)

So for any x, the Borda count winner and the Condorcet winner are the same.

Here are the preferences again.

	Group 1 (x people)	Group 2 ($101 - x$ people)
(Best)	A	C
	B	B
(Worst)	C	A

c. Again, x is unknown and can be any number (as long as $1 \leq x$ and $x \leq 100$). Is it possible for the Condorcet winner to be the *same* as the approval voting winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

Again, if $x \leq 50$, C is the Condorcet winner

$x \geq 51$, A "

approval voting (top two)

A : x

B : $x + 101 - x = 101$

C : $101 - x$

101 is always larger than x and $101 - x$.

B always wins approval voting (top two) because B is in everyone's top two.

So it is not possible for the Condorcet winner (either A or C) and the approval voting winner (always B) to be the same, regardless of what x is.

Now say that society is composed of three groups. Group 1 has x members, Group 2 has $96 - x$ members, and Group 3 has 5 members. Assume that $1 \le x$ and $x \le 95$ (in other words, there is at least one person in each group). Their preferences are as follows.

	Group 1 (x people)	Group 2 ($96 - x$ people)	Group 3 (5 people)
(Best)	A	C	B
	B	B	A
(Worst)	C	A	C

if $x = 38$

A vs B
38 63

B vs C
43 58

A vs C
43 58

C is Condorcet winner

Borda: A : 81
B : 106
$\boxed{C : 116}$

approval (top two)
A : 43
$\boxed{B : 101}$
C : 58

d. Again, say that $x = 38$. Please fill in the following table. Here approval voting means that each person votes for her top two candidates. (1 point)

If	then the Condorcet winner is	then the Borda count winner is	then the approval voting winner is
x = 38	C	C	B

e. Now say that x is unknown—it can be any number (as long as $1 \le x$ and $x \le 95$). Is it possible for the Condorcet winner to be *different* from the Borda count winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

Condorcet: A vs B
x $101-x$

A vs C
$5+x$ $96-x$

B vs C
$5+x$ $96-x$

A is condorcet winner when
$x > 101-x$ and $5+x > 96-x$
$2x > 101$ $2x > 91$
$x \geq 51$ and $x \geq 46$
$x \geq 51$

B is condorcet winner when
$101-x > x$ and $5+x > 96-x$
$101 > 2x$ $2x > 91$
$50 \geq x$ and $x \geq 46$
$46 \le x \le 50$

C is Condorcet winner when
$96-x > 5+x$ and $96-x > 5+x$
$91 > 2x$
$45 \geq x$

So when $x \le 45$, C is Condorcet winner
$46 \le x \le 50$, B "
$x \geq 51$, A "

Borda:
A: $2x+5$
B: $x+96-x+10 = 106$
C: $2(96-x) = 192-2x$

A is Borda count winner when
$2x+5 > 106$ and $2x+5 > 192-2x$
$2x > 101$ $4x > 187$
$x \geq 51$ and $x \geq 47$
$x \geq 51$

So when $x = 44$ or $x = 45$, C is the Condorcet winner But Borda winner is B

B is Borda count winner when
$106 > 2x+5$ and $106 > 192-2x$
$101 > 2x$ $2x > 86$
$50 \geq x$ $x \geq 44$

C is Borda count winner when
$192-2x > 2x+5$ and $192-2x > 106$
$187 > 4x$ $86 > 2x$
$46 \geq x$ and $43 \geq x \Rightarrow x \le 43$

Here are the preferences again.

	Group 1 (x people)	Group 2 ($96 - x$ people)	Group 3 (5 people)
(Best)	A	C	B
	B	B	A
(Worst)	C	A	C

f. Again, x is unknown and can be any number (as long as $1 \leq x$ and $x \leq 95$). Is it possible for the Condorcet winner to be the *same* as the approval voting winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

From before:

$x \leq 45 \Rightarrow$ C is Condorcet winner

$45 \leq x \leq 50 \Rightarrow$ B "

$x \geq 51 \Rightarrow$ A "

approval voting (top two)

A : $x+5$

B : 101 $\Rightarrow$ B is always the approval voting winner (everyone has B in their top two).

C : $96-x$

So for x such that $45 \leq x \leq 50$, the condorcet winner and approval voting winner are the same (B).

g. Again, x is unknown and can be any number (as long as $1 \leq x$ and $x \leq 95$). Is it possible for the approval voting winner to be the *same* as the Borda count winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

From before:

$x \leq 43 \Rightarrow$ C is Borda count winner

$44 \leq x \leq 50 \Rightarrow$ B "

$x \geq 51 \Rightarrow$ A "

From before:

B is always approval count winner.

So for x such that $44 \leq x \leq 50$,

the approval voting winner and the Borda count winner

are the same (B).

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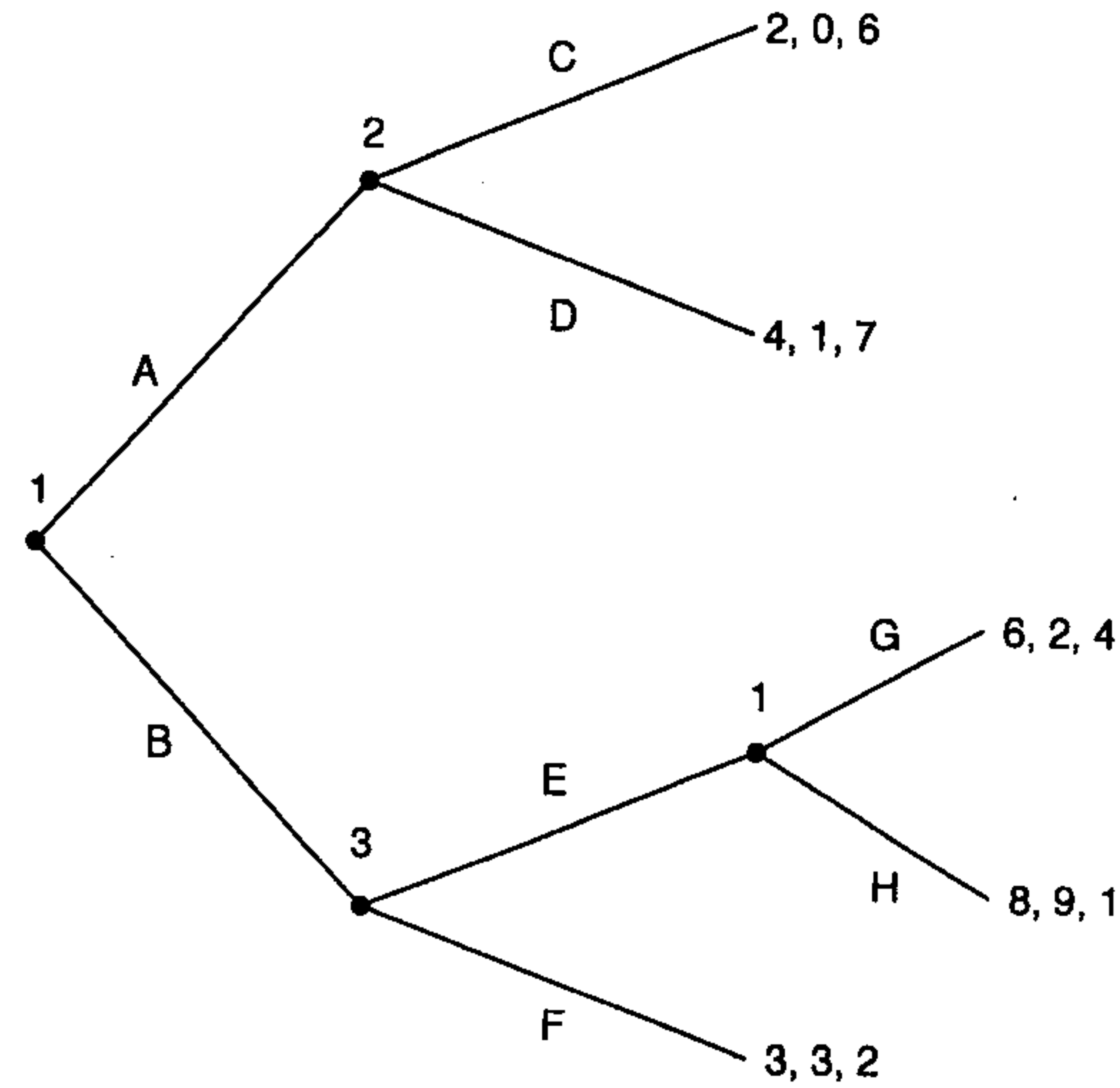
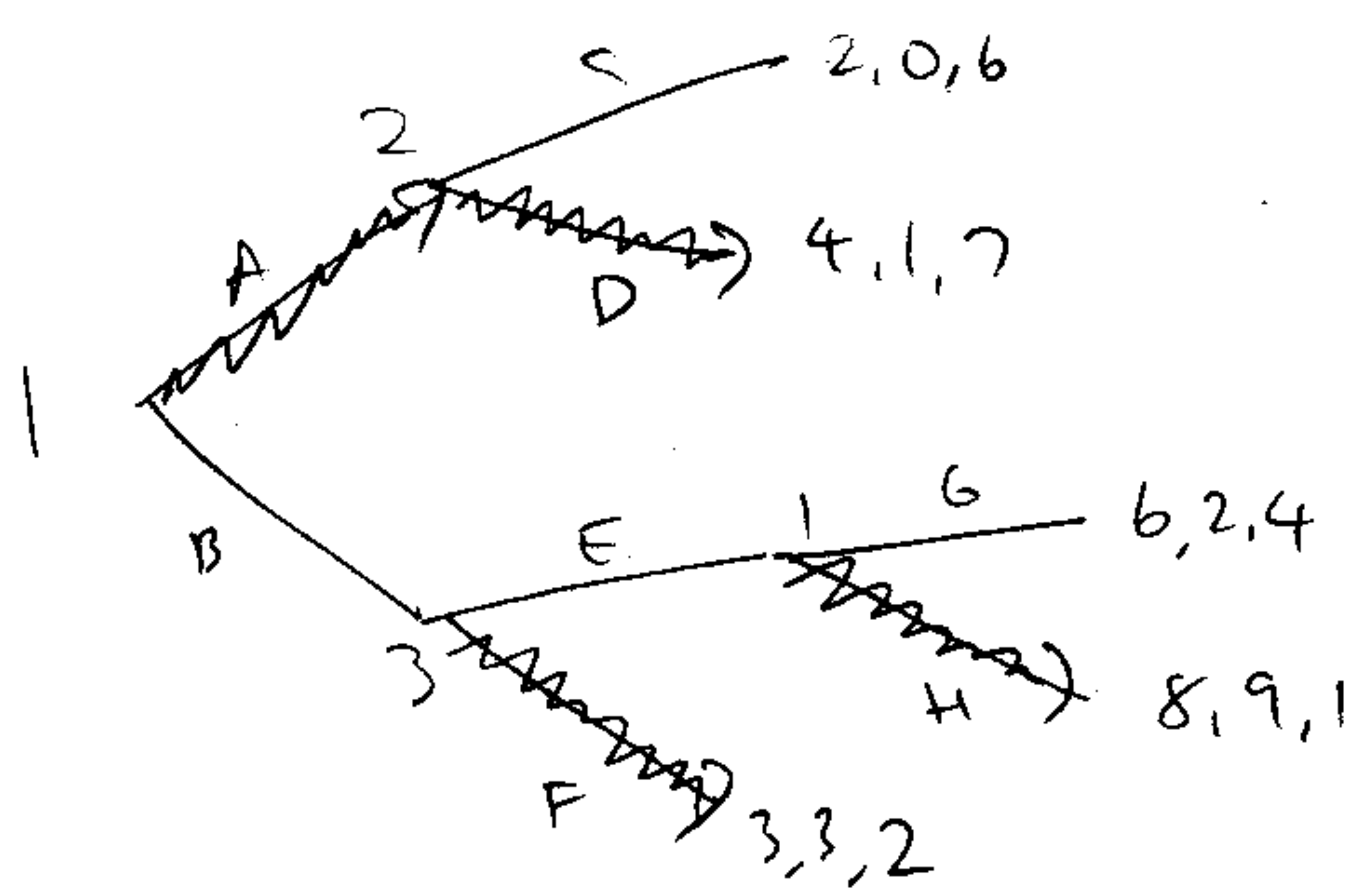
Part 2

(problems 3 and 4)

Problem 3. Consider the following extensive form game.



a. Find all subgame perfect Nash equilibria of this game. (4 points)

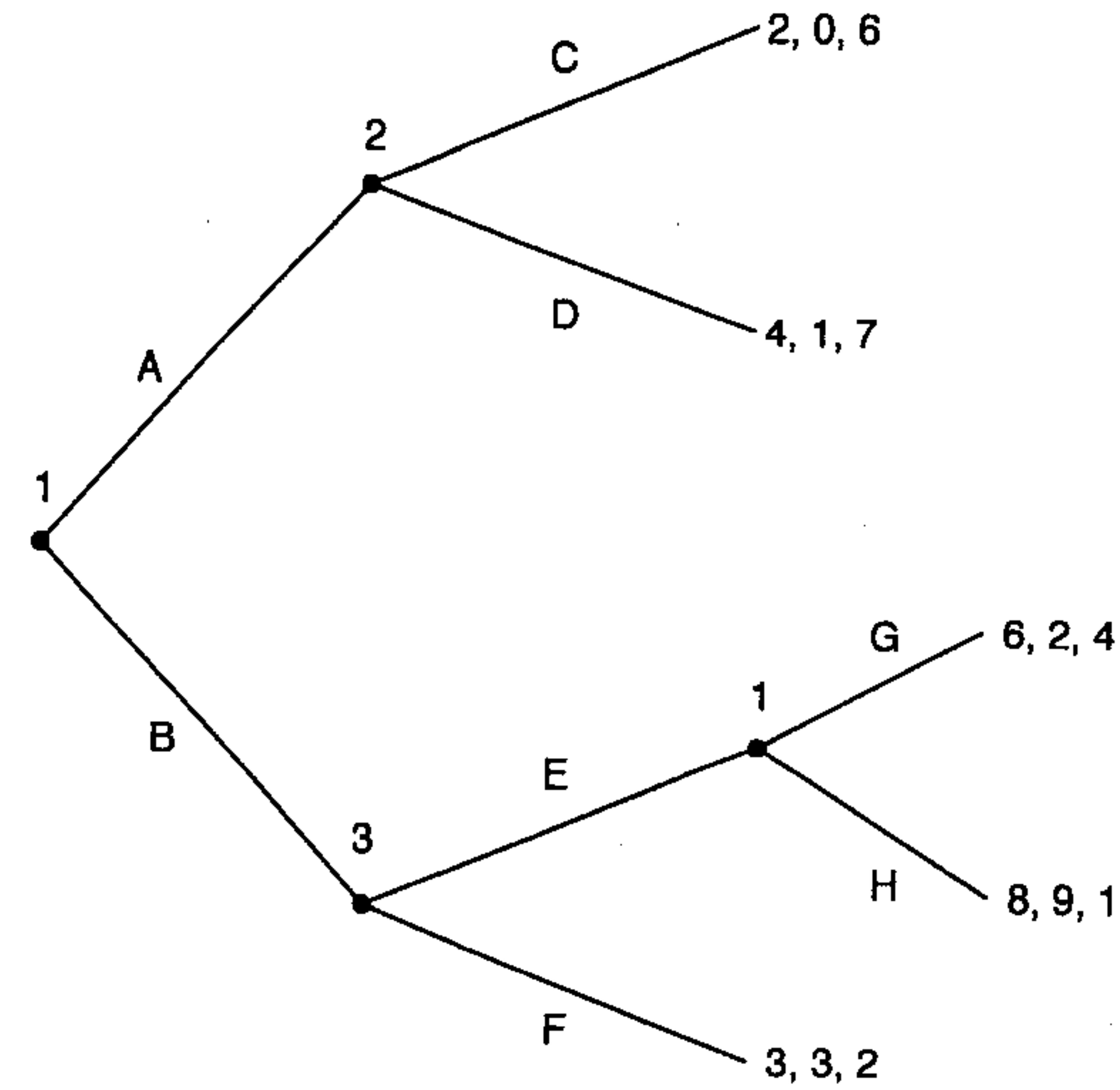


SPNE

or in other words

(A H, D, F)

Here is the game again.



b. Write this as a strategic form game. (4 points)

	C	D
AG	$2,0,\hat{6}$	$4,\overset{+}{1},\hat{7}$
AH	$2,0,\hat{6}$	$4,\overset{+}{1},\hat{7}$
BG	$6,\overset{+}{2},\hat{4}$	$6,\overset{+}{2},\hat{4}$
BH	$*8,\overset{+}{9},1$	$*8,\overset{+}{9},1$

E

	C	D
A G	$2,0,\hat{6}$	$*4,\overset{+}{1},\hat{7}$
A H	$2,0,\hat{6}$	$*4,\overset{+}{1},\hat{7}$
B G	$*3,\overset{+}{3},2$	$3,\overset{+}{3},2$
B H	$*3,\overset{+}{3},\hat{2}$	$3,\overset{+}{3},\hat{2}$

F

c. Find all (pure strategy) Nash equilibria of this game. (4 points)

NE: (BH, C, F)
(AG, D, F)
(AH, D, F)

Problem 4. The city council of Pleasantville has 11 members who choose among four alternatives: W, X, Y, and Z. The council is divided up into four groups, and their preferences are as follows:

	Group 1 (3 people)	Group 2 (3 people)	Group 3 (x people)	Group 4 ($5 - x$ people)
(Best)	W	X	W	Y
	Z	Y	Y	W
	X	W	X	Z
(Worst)	Y	Z	Z	X

For example, Group 1 is composed of 3 people who all like W best, Z second-best, X third-best, and Y worst. The number of people in Group 3 and Group 4 depends on x, where $1 \leq x$ and $x \leq 4$. In other words, Group 3 and Group 4 always have at least one member. Note that there is always a total of 11 people on the council.

The chairperson is an agenda setter. She proposes an alternative, and all 11 members (including himself) vote to agree or disagree. If the alternative is agreed by majority, the alternative is passed. If not, the chairperson proposes another alternative and all the members vote again. They keep doing this until an alternative is passed. Which alternative is chosen (and thus implemented) depends on the agenda (the order in which they vote on the alternatives).

a. Say that $x = 1$. Which of the four alternatives (W, X, Y, and Z) can be implemented by the council? For each alternative, write down an agenda which implements it. (3 points)

W > X   W > Z
8   3   11   0

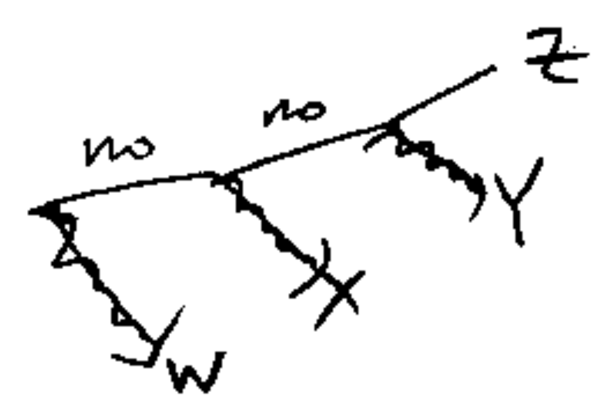
X > Y
6   5

Y > W   Y > Z
7   4   8   3

Z > X
7   4

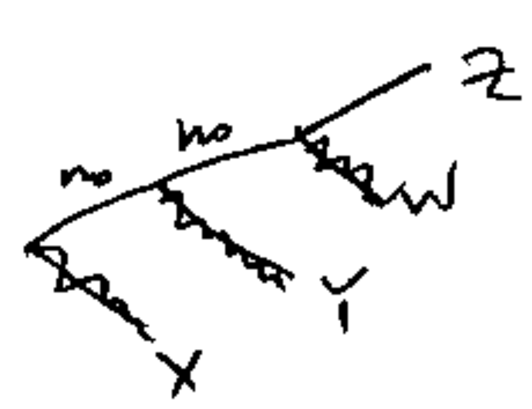
So we have W > X > Y > Z

To implement W, use agenda:



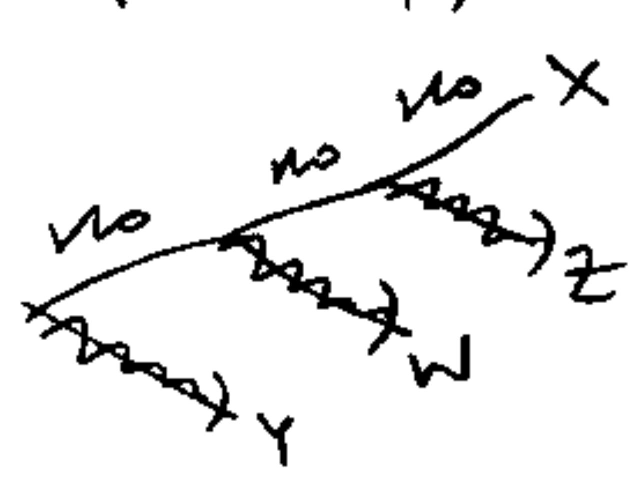
We have X > Y > W > Z.

to implement X, use agenda



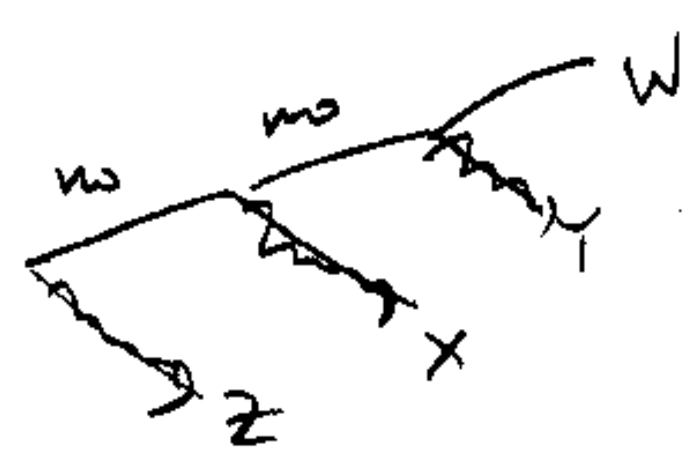
We have Y > W > Z > X

to implement Y, use agenda



We have Z > X > Y > W.

to implement Z, use agenda



Here are the groups' preferences again.

	Group 1 (3 people)	Group 2 (3 people)	Group 3 (x people)	Group 4 ($5-x$ people)
(Best)	W	X	W	Y
	Z	Y	Y	W
	X	W	X	Z
(Worst)	Y	Z	Z	X

b. Is there an alternative(s) that the council can implement regardless of the value of x? (Remember that $1 \leq x$ and $x \leq 4$.) If so, for each such alternative, write down the agenda which implements it. If not, explain why not. (3 points)

W vs X $\Rightarrow$ W > X always
10 3

W vs Y
$3+x$ $8-x$

W vs Z $\Rightarrow$ W > Z always
11 0

X vs Y $\Rightarrow$ X > Y always
6 5

X vs Z
$3+x$ $8-x$

Y vs Z $\Rightarrow$ Y > Z always
8 3

Since W > X > Y > Z always, the agenda

implements W.

c. Here is a list of the possible values of x:

$$x = 1 \quad x = 2 \quad x = 3 \quad x = 4$$

For some values of x, there does not exist an agenda which implements Z (in other words, Z is not chosen in any agenda). Please circle those values of x in the list above. Please explain your work. (3 points)

Since W > Z and Y > Z always, for Z to be implementable, we must have Z > X (otherwise Z would be beaten by all others).

Hence $8 - x > 3 + x$
$6 > 2x$
$3 > x$.

it is easy to check that if $x=3$ or $x=4$, then X > Z.

If $x = 1$,
Z > X > Y > W and thus Z is implementable.

If $x = 2$,
Z > X > Y > W and thus Z is implementable.

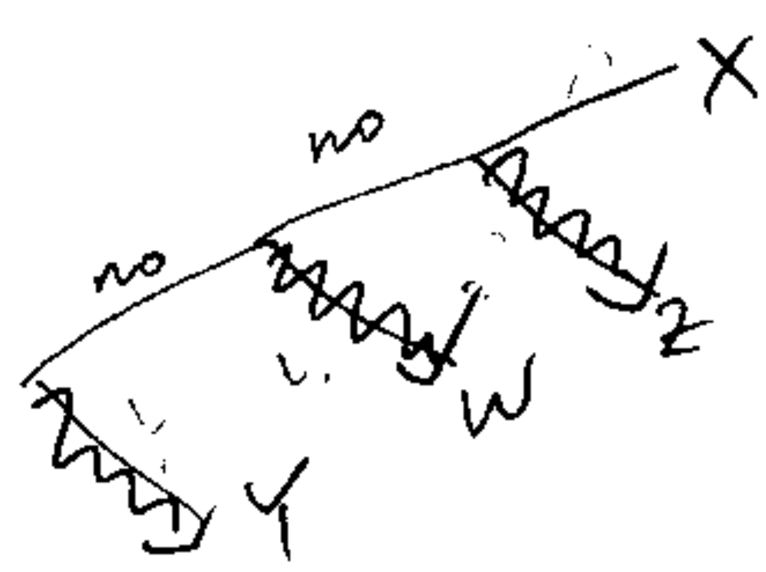
d. Now assume that $x = 2$. Here are the groups' preferences again.

	Group 1 (3 people)	Group 2 (3 people)	Group 3 (2 people)	Group 4 (3 people)
(Best)	W	X	W	Y
	Z	Y	Y	W
	X	W	X	Z
(Worst)	Y	Z	Z	X

The chairperson is a member of Group 3 and sets the agenda. However, the mayor of Pleasantville (who is not on the city council) likes either X or Y. After the city council decides, the mayor will veto any alternative other than X or Y. A mayor's veto is an outcome which all members of the city council very much dislike (it is worse than any of the four alternatives W, X, Y, and Z). Which agenda will the chairperson choose? (3 points)

The chairperson likes W best, but the mayor will veto it.

The chairperson can implement Y with this agenda:



$W > X$, $W > Z$
$X > Y$
$Y > W$, $Y > Z$
$Z > X$

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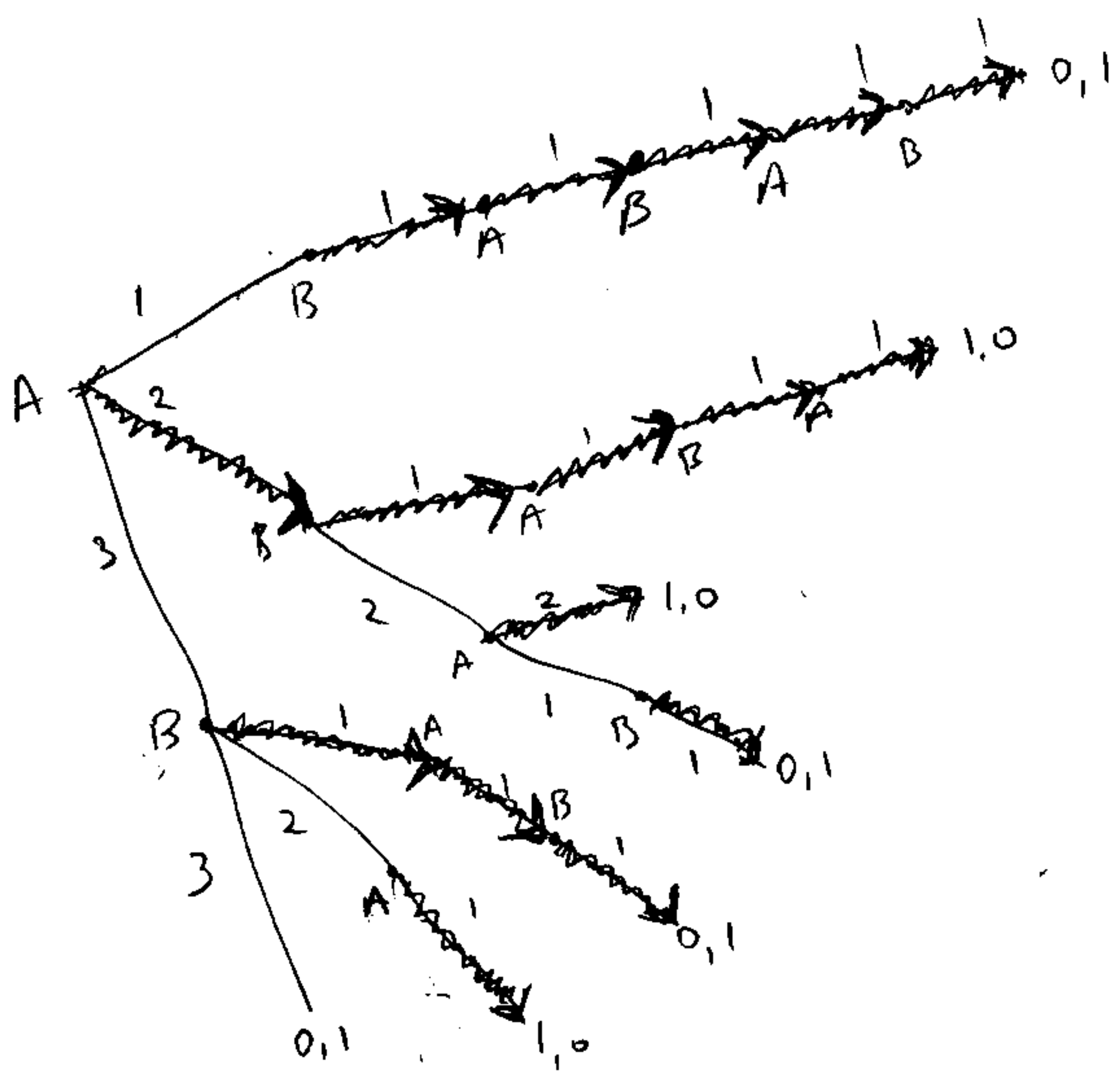
Part 3

(problems 5 and 6)

Problem 5. Alfonso and Beatrice are home for the holidays and are eating some holiday cookies. They decide to play a game. They take turns eating cookies. In each turn, a person can either take 1, 2, or 3 cookies. However, a person cannot take more cookies than what her opponent took last time. In other words, if your opponent just took 2 cookies, you can only take 1 or 2 cookies (you cannot take 3). Whoever eats the last cookie wins. Alfonso goes first.

a. Say that Alfonso and Beatrice start with a plate of 6 cookies. Model this as an extensive form game. Write payoffs as (Alfonso, Beatrice). (4 points)



b. Find a subgame perfect Nash equilibrium of this game by writing arrows in the tree you wrote above (i.e. you don't have to write down the SPNE in words). Please make your arrows nice and clear. If there is more than one SPNE, just write down one of them; you don't have to find all of them. (4 points)

one SPNE shown in arrows above

c. Now say that they start with x cookies, where x goes from 1 to 20, as shown in the table below. For each value of x, find out which person wins the game in an SPNE and write it in the table below. Remember that Alfonso always goes first. For example, when $x = 1$, there is only one cookie at the start, and Alfonso obviously wins by taking one cookie right at the start. So the table entry when $x = 1$ is already filled in for you as an example. It is crucial to explain your reasoning here; simply filling out the table is not sufficient without an explanation of where your answers come from. (4 points)

x	Who wins?
20	B
19	A
18	A
17	A
16	B
15	A
14	A
13	A
12	B
11	A
10	A
9	A
8	B
7	A
6	A
5	A
4	B
3	A
2	A
1	A

If the starting number of cookies is odd, Alfonso can always win by taking one cookie. Once he takes one cookie, henceforth every one must take just one cookie and thus he wins if the starting number of cookies is odd.

If the starting number of cookies is even, then if Alfonso takes 1, then he surely loses. (because from then on, everyone must take only 1). If Alfonso takes 3, then he leaves an odd number of cookies, and thus Beatrice can win by taking 1.

→ So Alfonso's only choice is to take 2 cookies if the starting number of cookies is even.

Note that if they start with 4 cookies, then Beatrice wins (if Alfonso takes 1, he loses
2, Beatrice takes 2
3 " " 1)

Similarly, if you leave four cookies, you win.

Hence if they start with 6 cookies, Alfonso can win by taking 2.

If they start with 8 cookies, Alfonso must take 2 (if he takes 1 or 3, he leaves an odd number and thus loses). Then Beatrice takes 2 and leaves 4 and thus wins.

Similarly, if they start with 10, Alfonso takes 2 and wins by leaving 8.
12, Alfonso " " Beatrice wins by taking 2 and leaving 8.

So Beatrice wins starting from any multiple of 4 and Alfonso wins otherwise.

Problem 6. Say that we have four women, A, B, C, and D, and four men, W, X, Y, and Z, who are being matched in heterosexual pairs. Their preferences are as follows.

	A	B	C	D		W	X	Y	Z
(Best)	W	Y	Y	X		C	C	D	D
	X	X	Z	W		A	A	A	A
	Y	W	X	Z		B	B	C	B
(Worst)	Z	Z	W	Y		D	D	B	C

For example, woman A likes man W best, X second-best, Y third-best, and Z least.

a. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

Woman ask:

(AW, BY, CY, DX)

(AW, BX, CY, DX)

(AW, BX, CY; DW)

(AW, BX, CY, DZ)

man ask:

(WC, XC, YD, ZD)

(WA, XC, YA, ZD)

(WA, XC, YC, ZD)

(WA, XA, YC, ZD)

(WA, XB, YC, ZD)

same

Since the best match for the women (the result from the woman-ask algorithm) is the same as the " " men (the ~~result from~~ the man-ask algorithm)

Then there is only one stable match,

(AW, BX, CY, DZ)

b. Now say their preferences are as follows. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

	A	B	C	D	W	X	Y	Z
	—	—	—	—	—	—	—	—
(Best)	W	W	X	X	A	B	A	B
	X	X	W	W	B	A	B	A
	Y	Z	Y	Z	C	D	D	C
(Worst)	Z	Y	Z	Y	D	C	C	D

woman - ask

(AW, BW, CX, DX)

(AW, BX, CW, DX)

(AW, BY, CY, DZ)

man - ask

(WA, XB, YA, ZB)

(WA, XB, YB, ZA)

(WA, XB, YD, ZC)

The best match for the women is (AW, BX, CY, DZ)

men (AW, BX, CZ, DY)

all stable matches must be "in between" these two, and thus A must be matched with W and B X in any stable match.

But the only possible such matches are (AW, BX, CY, DZ) and (AW, BX, CZ, DY).

So there are only two stable matches, (AW, BX, CY, DZ) and (AW, BX, CZ, DY).

Another way to see this is to notice that A and W are each others first choice. Hence they will always match. Hence B will get her second choice X (who likes B best). So the only possible variability is who C and D (and Y and Z) match with.

c. Now say their preferences are as follows. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

	A	B	C	D
(Best)	Y	Z	W	X
	Z	Y	X	W
	W	X	Y	Z
(Worst)	X	W	Z	Y

W	X	Y	Z
D	C	B	A
C	D	A	B
B	A	D	C
A	B	C	D

Here A and B care most about Y and Z (and vice-versa)

C and D " " " W and X (").

So the only variability is if we have (AY, BZ) or (AZ, BY)

and similarly if we have (CW, DX) or (CX, DW).

Hence there are ~~four~~ possible matches:

(AY, BZ, CW, DX) (AZ, BY, CW, DX)

(AY, BZ, CX, DW) (AZ, BY, CX, DW)

Another way to see this is that here the person who you like second-best likes you the most. So you will never match with your third-best or worst (your second-best will always accept you). So only the top two choices matter, as follows:

A	B	C	D
Y	Z	W	X
Z	Y	X	W

W	X	Y	Z
D	C	B	A
C	D	A	B

So it's as if we have ~~two~~ separate "worlds":

A	B
Y	Z
Z	Y

Y	Z
B	A
A	B

and

C	D
W	X
X	W

W	X
D	C
C	D

2 stable matches

2 stable matches

Hence there are four possible stable matches (2 times 2).

d. Now say their preferences are as follows. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

	A	B	C	D
(Best)	W	W	W	W
	X	Z	Y	X
	Y	Y	X	Z
(Worst)	Z	X	Z	Y

W	X	Y	Z
A	A	A	A
B	B	D	C
C	C	B	D
D	D	C	B

Here A and W are each other's first choice so they will always match.
So we can take A and W out of the picture and we have

B	C	D
Z	Y	X
Y	X	Z
X	Z	Y

X	Y	Z
B	D	C
C	B	D
D	C	B

this is the example we did in class in which we have
3 stable matches: (BZ, CY, DX) women get their first choice
(BX, CZ, DY) men "
(BY, CX, DZ) everyone gets their second choice

So there are <u>three</u> stable matches!

(AW, BZ, CY, DX)
(AW, BX, CZ, DY)
(AW, BY, CX, DZ)

Last name	First name	Student ID number	TA

Part 4

(problems 7 and 8)

Problem 7. There are two bicycle rental companies, Company 1 and Company 2, which each choose to set up a shop at one of four locations: the Santa Monica Pier (SM), the border between Santa Monica and Venice (B), Mid-Venice (MV), and South Venice (SV). It is OK for both companies to place their shops in the same location (both Company 1 and Company 2 can choose SV, for example). There are 100 tourists distributed at these locations:

SM	B	MV	SV
28	18	28	26

When the companies choose their locations, then customers choose to rent from the company which has the closest shop. If the customers at a given location find the two shops equally far apart, then the customers at that location split equally among the two shops. For example, if Company 1 chooses SM and Company 2 chooses MV, then Company 1 gets the customers from SM and half the customers from B, and Company 2 gets half the customers from B and all the customers from MV and SV. Each company makes one dollar from each customer.

a. Say that Company 1 has only a Santa Monica business license and can only place its shop in either SM or B. Say that Company 2 has only a Los Angeles business license and can only place its shop in either B, MV, or SV. Model this as a strategic form game and find all (pure strategy) Nash equilibria. (3 points)

	B	MV	SV
SM	28, 72+	37, 63	~~46~~46, 54+
B	*50, 50	*46, 54+	*60, 40

NE : (B, MV)

Here is the distribution of tourists again.

SM	B	MV	SV
28	18	28	26

b. Say Company 1 buys a Los Angeles business license and therefore can now place its shop in either SM, B, MV, or SV. As before, Company 2 can place its shop in either B, MV, or SV. Model this as a strategic form game and find all (pure strategy) Nash equilibria. How much is Company 1 willing to pay for such a license? (3 points)

	B	MV	SV
SM	28,72+	37,63	46,54
B	50,50	46,54+	60,40
MV	*54,46	*50,50+	*74,26
SV	40,60	26,74+	50,50

NE : (MV, MV)

Company 1 gets 50 and before it got 46, so it is willing to pay $4 for ~~the~~ the license.

c. Assume again that Company 1 can only place its shop in either SM or B. Say that Company 2 buys a Santa Monica business license and therefore can now place its shop in SM, B, MV, or SV. How much is Company 2 willing to pay for such a license? (3 points)

	SM	B	MV	SV
SM	50,50	28,72+	37,63	46,54
B	*72,28	*50,50	*46,54+	*60,40

NE : (B, MV)

Company 2 gets 54 and it got 54 before (in part a)

so it is willing to pay $0 for the license.

Here is the distribution of tourists again.

SM	B	MV	SV
28	18	28	26

d. Say that Company 1 has only a Santa Monica license and Company 2 has only a Los Angeles license. Now a third company, Company 3, enters. Company 3 has both a Santa Monica license and a Los Angeles license. Is there a Nash equilibrium in which all three companies choose the same location? Why or why not? (3 points)

If all 3 companies choose the same location,
each gets 33.3.

If all 3 are on SM, then company 3 can ~~get~~ get 72
by deviating to B

" B, then company 3 can get 54
by deviating to MV

" MV, then company 3 can get 46
by deviating to B

" SV, then company 3 can get 74
by deviating to MV.

So there is no NE in which all three companies
choose the same spot.

Problem 8. Simon, a concert promoter, is trying to organize a Dueling Divas event with as many as eight singers. However, each singer's participation depends on how many others participate. The singers' thresholds are as follows.

Singer	Lower Threshold	Upper Threshold	first night t=0	second night t=1	t=2	
Adele	1	2	y	n	n	n
Beyonce	1	2	y	n	n	n
Cher	1	2	y	n	n	n
Donna	4	7	y	y	n	n
Erykah	4	4	y	n	n	n
Florence	5	6	y	n	n	n
Gaga	6	7	y	y	n	n
Hyuna	6	7	y	y	n	n

A person participates if the total number of other participants is greater than or equal to her lower threshold and less than or equal to her upper threshold. For example, Donna participates if the total number of other participants is 4, 5, 6, or 7. Erykah participates only if the total number of other participants is 4.

a. Say that everyone participates in the first night of the event (the initial state). Who participates on the second night? Which singers will participate in the long run? (2 points)

On the second night, Donna, Gaga, and Hyuna participate.

In the long run, no one participates.

b. Find all (pure strategy) Nash equilibria of this game. (2 points)

$\boxed{(n, n, \ldots, n)}$, no one participating, is a NE, as shown above.

Adele, Beyonce, and Cher all have the same LT and UT and therefore are either all in or all out together.

If Adele, Beyonce, and Cher are out, Gaga and Hyuna are out, and thus no one participates.

If Adele, Beyonce, and Cher are in, we have $\boxed{(y, y, y, n, n, n, n, n)}$ this is a NE.

If A-B-C are in, G+H are in only if everyone is in. But then Erykah + Florence drop out. So G+H are not in.

So F will join only if ABCDE are in. But then E will drop out. So F is not in.

Thus D or E will go in only if the other does. But if they both join, then F will join and E will drop out.

So there are only two NE.

Here are the thresholds again.

Singer	Lower Threshold	Upper Threshold
Adele	1	2
Beyonce	1	2
Cher	1	2
Donna	4	7
Erykah	4	4
Florence	5	6
Gaga	6	7
Hyuna	6	7

c. Say that by paying $1000 to a singer, Simon can change a singer's upper or lower threshold by one (either decrease it by one or increase it by one). For example, by paying $1000, Simon can make Cher's lower threshold 2, or Gaga's upper threshold 6. What is the minimal amount that Simon has to pay in order to guarantee that there is at least one singer who participates in the long run regardless of the initial state? To whom should Simon make the payment(s) and which threshold(s) should be changed? (1 point)

By paying $1000 to Adele, her LT becomes zero and hence
(or Beyonce or Cher) she will always join.

Since Adele is in, Beyonce + Cher join in.

d. Go back to the original thresholds. Now Simon wants to ensure that there exists a Nash equilibrium in which Donna, Erykah, Florence, Gaga, and Hyuna participate. How can Simon do this by spending the least amount of money? To whom should Simon make the payment(s) and which threshold(s) should be changed? (2 points)

make	LT	UT
Florence	5 → 4	6
Gaga	6 → 4	7
Hyunah	6 → 4	7

By paying 1000 to F
2000 to G
2000 to H,
DEFGH all have LT ~~of~~ 4
and hence
(n,n,n, y,y,y,y,y) is a NE.

e. Go back to the original thresholds. Now Simon's rival Ryan has $6000 to spend. By spending this money, can Ryan ensure that no one will participate? To whom should Ryan make the payment(s) and which threshold(s) should be changed? (2 points)

make	LT	UT
Adele	1 → 3	2
Beyonce	1 → 3	2
Cher	1 → 3	2

now ABC never participate
hence GH " "
hence DEF " "

f. Now say that thresholds are as follows.

Singer	Lower Threshold	Upper Threshold
Adele	$y-4$	5
Beyonce	1	y
Cher	2	y
Donna	4	7
Erykah	y	y
Florence	5	6
Gaga	6	7
Hyuna	6	7

For what values of $y \in \{4,5,6,7,8,9\}$ does there exist a Nash equilibrium in which six or more people participate? (3 points)

Note that everyone participating is not a NE because Florence would drop out.

Among A–F, say all six participate. Then G+H jump in, and so everyone participates, but this is not a NE.

Among A–F, say exactly four participate. Then G+H do not want to participate, and hence we cannot have a NE in which at least six people in total participate.

Among A–F, if fewer than four participate, then we have fewer than six participating.

Thus the only possibility is that among A–F, exactly <u>five</u> people participate. Then G+H participate and a total of seven participate.

Thus Adele drops out and so everyone else must participate. The only way Erykah participates is if $y=6$.

When $y=6$, (n,y,y,y,y,y,y,y) is a NE.

So only when $y=6$ does there exist a NE in which at least six people participate.