Last name	First name	Student ID number	TA

Part 1

(problems 1 and 2)

Final exam PS 30 December 2012

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part contains two problems, and hence there are eight problems. Each question is weighted equally (12 points each). Each part is stapled separately (for ease in grading, since the parts are graded separately). Please make sure that your name and student ID number is on each part.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!

1		5	
2		6	
3		7	
4		8	
		total	

Problem 1. Consider the following strategic form game.

	W	X	Y	Z
A	-1,1	3,-1	0,8	-2,4
B	4,3	4,6	1,0	0,1
C	-3,-2	5,0	8,2	4,5
D	-5,0	1,1	4,4	6,2

a. Iteratively eliminate any weakly or strongly dominated strategies. Be sure to list the order of deletion. (3 points)

b. Find all pure strategy Nash equilibria (if any). (3 points)

c. Find all mixed strategy Nash equilibria (if any). (3 points)

d. Say we change the game slightly so that some payoffs depend on k.

	W	X	Y	Z
A	-1,1	3,-1	0,k	-2,4
B	4,3	4,6	1,0	0,k
C	-3,-2	5,0	8,2	4,5
D	k,0	1,1	4,4	6,2

Please fill in the following table. (3 points).

If	then the (pure strategy and mixed strategy) NE are:
k = 1	
k = 10	
k = 100	
k = 1000	

Problem 2. Say that there are 101 people in a society which is deciding among three candidates, A, B, and C. The society is composed of two groups, Group 1 and Group 2. Group 1 has x members and Group 2 has $101 - x$ members. Assume throughout that x is a natural number (i.e. not a fraction—you can't have fractions of people). Assume that $1 \leq x$ and $x \leq 100$ (in other words, there is at least one person in each group). Their preferences are as follows.

	Group 1 (x people)	Group 2 ($101 - x$ people)
(Best)	A	C
	B	B
(Worst)	C	A

a. Say that $x = 38$. Please fill in the following table. Here approval voting means that each person votes for her top two candidates. (1 point)

If	then the Condorcet winner is	then the Borda count winner is	then the approval voting winner is
x = 38			

b. Now say that x is unknown—it can be any number (as long as $1 \leq x$ and $x \leq 100$). Is it possible for the Condorcet winner to be *different* from the Borda count winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

Here are the preferences again.

	Group 1 (x people)	Group 2 ($101 - x$ people)
(Best)	A	C
	B	B
(Worst)	C	A

c. Again, x is unknown and can be any number (as long as $1 \leq x$ and $x \leq 100$). Is it possible for the Condorcet winner to be the *same* as the approval voting winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

Now say that society is composed of three groups. Group 1 has x members, Group 2 has $96 - x$ members, and Group 3 has 5 members. Assume that $1 \leq x$ and $x \leq 95$ (in other words, there is at least one person in each group). Their preferences are as follows.

	Group 1 (x people)	Group 2 ($96 - x$ people)	Group 3 (5 people)
(Best)	A	C	B
	B	B	A
(Worst)	C	A	C

d. Again, say that $x = 38$. Please fill in the following table. Here approval voting means that each person votes for her top two candidates. (1 point)

If	then the Condorcet winner is	then the Borda count winner is	then the approval voting winner is
x = 38			

e. Now say that x is unknown—it can be any number (as long as $1 \leq x$ and $x \leq 95$). Is it possible for the Condorcet winner to be *different* from the Borda count winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

Here are the preferences again.

	Group 1 (x people)	Group 2 ($96 - x$ people)	Group 3 (5 people)
(Best)	A	C	B
	B	B	A
(Worst)	C	A	C

f. Again, x is unknown and can be any number (as long as $1 \leq x$ and $x \leq 95$). Is it possible for the Condorcet winner to be the *same* as the approval voting winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

g. Again, x is unknown and can be any number (as long as $1 \leq x$ and $x \leq 95$). Is it possible for the approval voting winner to be the *same* as the Borda count winner? If so, give an example of x which makes it possible. If not, explain why not. (2 points)

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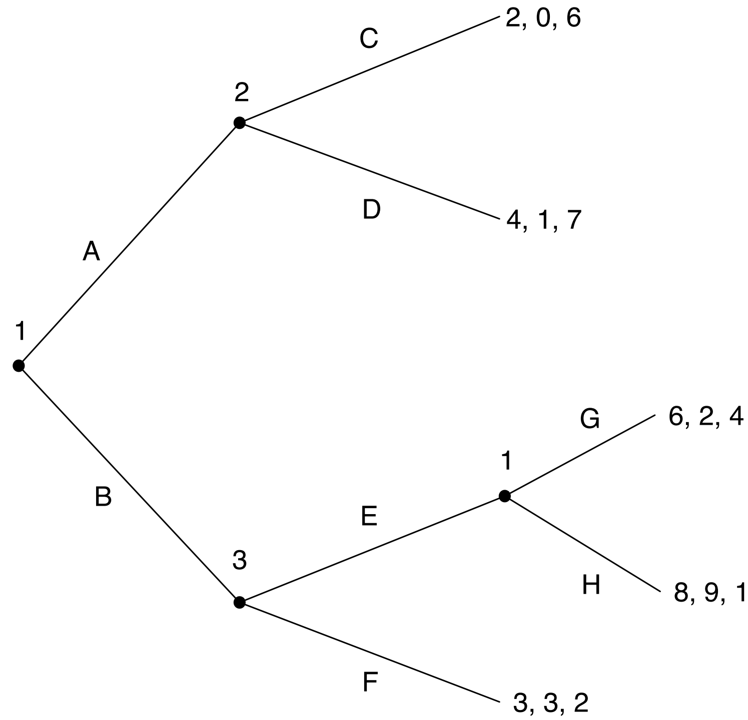
Part 2

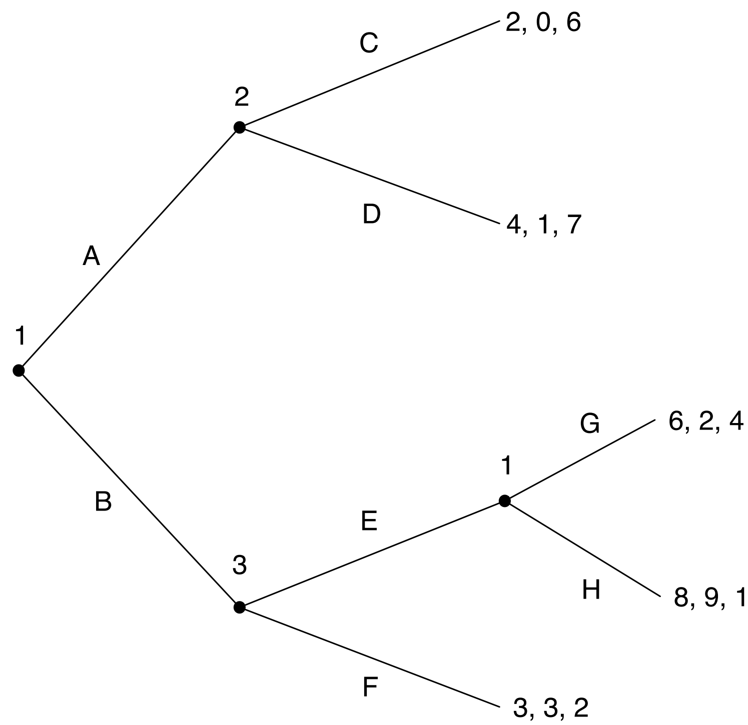
(problems 3 and 4)

Problem 3. Consider the following extensive form game.



a. Find all subgame perfect Nash equilibria of this game. (4 points)

Here is the game again.



b. Write this as a strategic form game. (4 points)

c. Find all (pure strategy) Nash equilibria of this game. (4 points)

Problem 4. The city council of Pleasantville has 11 members who choose among four alternatives: W, X, Y, and Z. The council is divided up into four groups, and their preferences are as follows:

	Group 1 (3 people)	Group 2 (3 people)	Group 3 (x people)	Group 4 ($5-x$ people)
(Best)	W	X	W	Y
	Z	Y	Y	W
	X	W	X	Z
(Worst)	Y	Z	Z	X

For example, Group 1 is composed of 3 people who all like W best, Z second-best, X third-best, and Y worst. The number of people in Group 3 and Group 4 depends on x, where $1 \leq x$ and $x \leq 4$. In other words, Group 3 and Group 4 always have at least one member. Note that there is always a total of 11 people on the council.

The chairperson is an agenda setter. She proposes an alternative, and all 11 members (including himself) vote to agree or disagree. If the alternative is agreed by majority, the alternative is passed. If not, the chairperson proposes another alternative and all the members vote again. They keep doing this until an alternative is passed. Which alternative is chosen (and thus implemented) depends on the agenda (the order in which they vote on the alternatives).

a. Say that $x = 1$. Which of the four alternatives (W, X, Y, and Z) can be implemented by the council? For each alternative, write down an agenda which implements it. (3 points)

Here are the groups' preferences again.

	Group 1 (3 people)	Group 2 (3 people)	Group 3 (x people)	Group 4 ($5 - x$ people)
(Best)	W	X	W	Y
	Z	Y	Y	W
	X	W	X	Z
(Worst)	Y	Z	Z	X

b. Is there an alternative(s) that the council can implement regardless of the value of x? (Remember that $1 \leq x$ and $x \leq 4$.) If so, for each such alternative, write down the agenda which implements it. If not, explain why not. (3 points)

c. Here is a list of the possible values of x:

$$x = 1 \quad x = 2 \quad x = 3 \quad x = 4$$

For some values of x, there does not exist an agenda which implements Z (in other words, Z is not chosen in any agenda). Please circle those values of x in the list above. Please explain your work. (3 points)

d. Now assume that $x = 2$. Here are the groups' preferences again.

	Group 1 (3 people)	Group 2 (3 people)	Group 3 (2 people)	Group 4 (3 people)
(Best)	W	X	W	Y
	Z	Y	Y	W
	X	W	X	Z
(Worst)	Y	Z	Z	X

The chairperson is a member of Group 3 and sets the agenda. However, the mayor of Pleasantville (who is not on the city council) likes either X or Y. After the city council decides, the mayor will veto any alternative other than X or Y. A mayor's veto is an outcome which all members of the city council very much dislike (it is worse than any of the four alternatives W, X, Y, and Z). Which agenda will the chairperson choose? (3 points)

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Part 3

(problems 5 and 6)

Problem 5. Alfonso and Beatrice are home for the holidays and are eating some holiday cookies. They decide to play a game. They take turns eating cookies. In each turn, a person can either take 1, 2, or 3 cookies. However, a person cannot take more cookies than what her opponent took last time. In other words, if your opponent just took 2 cookies, you can only take 1 or 2 cookies (you cannot take 3). Whoever eats the last cookie wins. Alfonso goes first.

a. Say that Alfonso and Beatrice start with a plate of 6 cookies. Model this as an extensive form game. Write payoffs as (Alfonso, Beatrice). (4 points)

b. Find a subgame perfect Nash equilibrium of this game by writing arrows in the tree you wrote above (i.e. you don't have to write down the SPNE in words). Please make your arrows nice and clear. If there is more than one SPNE, just write down one of them; you don't have to find all of them. (4 points)

c. Now say that they start with x cookies, where x goes from 1 to 20, as shown in the table below. For each value of x, find out which person wins the game in an SPNE and write it in the table below. Remember that Alfonso always goes first. For example, when $x = 1$, there is only one cookie at the start, and Alfonso obviously wins by taking one cookie right at the start. So the table entry when $x = 1$ is already filled in for you as an example. It is crucial to explain your reasoning here; simply filling out the table is not sufficient without an explanation of where your answers come from. (4 points)

x	Who wins?
20	
19	
18	
17	
16	
15	
14	
13	
12	
11	
10	
9	
8	
7	
6	
5	
4	
3	
2	
1	A

Problem 6. Say that we have four women, A, B, C, and D, and four men, W, X, Y, and Z, who are being matched in heterosexual pairs. Their preferences are as follows.

	A	B	C	D
	—	—	—	—
(Best)	W	Y	Y	X
	X	X	Z	W
	Y	W	X	Z
(Worst)	Z	Z	W	Y

W	X	Y	Z
—	—	—	—
C	C	D	D
A	A	A	A
B	B	C	B
D	D	B	C

For example, woman A likes man W best, X second-best, Y third-best, and Z least.

a. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

b. Now say their preferences are as follows. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

	A	B	C	D
	—	—	—	—
(Best)	W	W	X	X
	X	X	W	W
	Y	Z	Y	Z
(Worst)	Z	Y	Z	Y

W	X	Y	Z
—	—	—	—
A	B	A	B
B	A	B	A
C	D	D	C
D	C	C	D

c. Now say their preferences are as follows. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

	A	B	C	D
	—	—	—	—
(Best)	Y	Z	W	X
	Z	Y	X	W
	W	X	Y	Z
(Worst)	X	W	Z	Y

W	X	Y	Z
—	—	—	—
D	C	B	A
C	D	A	B
B	A	D	C
A	B	C	D

d. Now say their preferences are as follows. List all stable matchings. (You should be able to do this without considering all 24 possible matches!) Please explain your work (i.e. explain why the matchings which you do not list are not stable). (3 points)

	A	B	C	D
	—	—	—	—
(Best)	W	W	W	W
	X	Z	Y	X
	Y	Y	X	Z
(Worst)	Z	X	Z	Y

W	X	Y	Z
—	—	—	—
A	A	A	A
B	B	D	C
C	C	B	D
D	D	C	B

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Part 4

(problems 7 and 8)

Problem 7. There are two bicycle rental companies, Company 1 and Company 2, which each choose to set up a shop at one of four locations: the Santa Monica Pier (SM), the border between Santa Monica and Venice (B), Mid-Venice (MV), and South Venice (SV). It is OK for both companies to place their shops in the same location (both Company 1 and Company 2 can choose SV, for example). There are 100 tourists distributed at these locations:

SM	B	MV	SV
28	18	28	26

When the companies choose their locations, then customers choose to rent from the company which has the closest shop. If the customers at a given location find the two shops equally far apart, then the customers at that location split equally among the two shops. For example, if Company 1 chooses SM and Company 2 chooses MV, then Company 1 gets the customers from SM and half the customers from B, and Company 2 gets half the customers from B and all the customers from MV and SV. Each company makes one dollar from each customer.

a. Say that Company 1 has only a Santa Monica business license and can only place its shop in either SM or B. Say that Company 2 has only a Los Angeles business license and can only place its shop in either B, MV, or SV. Model this as a strategic form game and find all (pure strategy) Nash equilibria. (3 points)

Here is the distribution of tourists again.

SM	B	MV	SV
28	18	28	26

b. Say Company 1 buys a Los Angeles business license and therefore can now place its shop in either SM, B, MV, or SV. As before, Company 2 can place its shop in either B, MV, or SV. Model this as a strategic form game and find all (pure strategy) Nash equilibria. How much is Company 1 willing to pay for such a license? (3 points)

c. Assume again that Company 1 can only place its shop in either SM or B. Say that Company 2 buys a Santa Monica business license and therefore can now place its shop in SM, B, MV, or SV. How much is Company 2 willing to pay for such a license? (3 points)

Here is the distribution of tourists again.

SM	B	MV	SV
28	18	28	26

d. Say that Company 1 has only a Santa Monica license and Company 2 has only a Los Angeles license. Now a third company, Company 3, enters. Company 3 has both a Santa Monica license and a Los Angeles license. Is there a Nash equilibrium in which all three companies choose the same location? Why or why not? (3 points)

Problem 8. Simon, a concert promoter, is trying to organize a Dueling Divas event with as many as eight singers. However, each singer's participation depends on how many others participate. The singers' thresholds are as follows.

Singer	Lower Threshold	Upper Threshold
Adele	1	2
Beyonce	1	2
Cher	1	2
Donna	4	7
Erykah	4	4
Florence	5	6
Gaga	6	7
Hyuna	6	7

A person participates if the total number of other participants is greater than or equal to her lower threshold and less than or equal to her upper threshold. For example, Donna participates if the total number of other participants is 4, 5, 6, or 7. Erykah participates only if the total number of other participants is 4.

a. Say that everyone participates in the first night of the event (the initial state). Who participates on the second night? Which singers will participate in the long run? (2 points)

b. Find all (pure strategy) Nash equilibria of this game. (2 points)

Here are the thresholds again.

Singer	Lower Threshold	Upper Threshold
Adele	1	2
Beyonce	1	2
Cher	1	2
Donna	4	7
Erykah	4	4
Florence	5	6
Gaga	6	7
Hyuna	6	7

c. Say that by paying $1000 to a singer, Simon can change a singer's upper or lower threshold by one (either decrease it by one or increase it by one). For example, by paying $1000, Simon can make Cher's lower threshold 2, or Gaga's upper threshold 6. What is the minimal amount that Simon has to pay in order to guarantee that there is at least one singer who participates in the long run regardless of the initial state? To whom should Simon make the payment(s) and which threshold(s) should be changed? (1 point)

d. Go back to the original thresholds. Now Simon wants to ensure that there exists a Nash equilibrium in which Donna, Erykah, Florence, Gaga, and Hyuna participate. How can Simon do this by spending the least amount of money? To whom should Simon make the payment(s) and which threshold(s) should be changed? (2 points)

e. Go back to the original thresholds. Now Simon's rival Ryan has $6000 to spend. By spending this money, can Ryan ensure that no one will participate? To whom should Ryan make the payment(s) and which threshold(s) should be changed? (2 points)

f. Now say that thresholds are as follows.

Singer	Lower Threshold	Upper Threshold
Adele	$y-4$	5
Beyonce	1	y
Cher	2	y
Donna	4	7
Erykah	y	y
Florence	5	6
Gaga	6	7
Hyuna	6	7

For what values of $y \in \{4, 5, 6, 7, 8, 9\}$ does there exist a Nash equilibrium in which six or more people participate? (3 points)