

Name:

UID:

TA and section number:

*This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are eight parts in this exam. Each part is weighted equally (12 points for each part). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.*

*If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If the TA responsible for a given question is not in the room at the time, work on other parts of the exam and hold the question until that TA rotates to your exam location. When the end of the exam is announced, please stop working immediately. People who continue working after the end of the exam is announced will have their grades penalized by 25 percent. If you need to leave the room to use the bathroom during the exam, please write your name down on the bathroom log before you leave. A person cannot leave the room more than once during the exam (a person who leaves for a second time will be considered to have completed his or her exam).*

*Please turn in your exam to one of the TAs. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Please turn off and put away all cell phones and other electronic gadgets. Please put away all notes and close all bags. Before you hand in your exam, make sure you flip through the exam and at least look at all questions—sometimes two pages get stuck to each other and you can miss an entire section of the exam. Good luck!*

1	
2	
3	
4	
5	
6	
7	
8	
total	

Part 1. Say that there are three men, A, B, and C, and three women X, Y, and Z. They each have preferences over the others, as follows.

	A	B	C
best	X	Y	Y
	Y	Z	Z
worst	Z	X	X

	X	Y	Z
best	A	C	A
	B	A	C
worst	C	B	B

a. Find the stable match that the men most prefer. (2 points)

Men-ask algorithm: (AX, BY, CY) B rejected  
 (AX, BZ, CY) stable

(AX, BZ, CY) is the <sup>stable</sup> match that men most prefer.

b. Find the stable match that the women most prefer. (2 points)

Women-ask algorithm: (XA, YC, ZA) Z rejected  
 (XA, YC, ZC) Z rejected  
 (XA, YC, ZB) stable

(XA, YC, ZB) is the stable match that women most prefer.

c. Find all stable matches. (2 points)

Since the <sup>stable</sup> match which the men most prefer (AX, BZ, CY) is the same as the stable match which the women most prefer (XA, YC, ZB)

there is only one stable match, (AX, BZ, CY)

d. Say that all we know about people's preferences is below.

	A	B	C
best			
worst	X	Z	Y

	X	Y	Z
best			
worst			

In other words, all we know is that A likes X least, B likes Z least, and C likes Y least. Say that we are told that (AX, BZ, CY) is a stable match. What can we conclude about the women's preferences? Why? (3 points)

We can conclude that women's preferences are this:

X	Y	Z
A	C	B
-	-	-
-	-	-

In other words, X likes A best, Y likes C best, and Z likes B best.

If for example X likes B best 

X
B
-
-

 then (AX, BZ, CY) would not be stable since X would call B and B would surely accept (since B is matched with his worst choice Z).

Since all the men are matched with their worst choice, any call to any man will be gladly accepted.

Hence for (AX, BZ, CY) to be stable, the women must all be getting their first choice.

e. Now say that all we know about people's preferences are below.

	A	B	C
best			Y
worst			

	X	Y	Z
best		C	
worst			

In other words, all we know <sup>is</sup> if that C likes Y best and Y likes C best. How many possible stable matchings are there at the most? Why? (3 points)

Since C likes Y best and Y likes C best,  
they must be together in any stable match.

Hence there are at most two stable matchings:

(AX, BZ, CY)  
and (AZ, BX, CY)

Part 2. The Supreme Court will soon consider the constitutionality of the Affordable Care Act (ACA), the health care law introduced by President Obama. Three of the Supreme Court justices are under pressure to recuse themselves (not participate) in this case: Justices Scalia, Thomas, and Kagan. Critics argue that these Justices cannot rule impartially because Justices Scalia and Thomas recently attended a dinner sponsored in part by pharmaceutical companies and Justice Kagan served as counsel to the Obama administration in the drafting of the ACA. Justices Scalia and Thomas prefer the ACA to be ruled unconstitutional, while Justice Kagan prefers it to be ruled constitutional.

a. Assume that Kagan is forced to recuse herself because of public pressure from talk radio host Laura Graham. Thus we have a two person game with players Scalia and Thomas. Each can either recuse himself or not. Because Kagan is out, the two justices feel confident that the ruling will go their way as long as one of them remains. The best thing for either justice is if he recuses himself and the other justice doesn't, because then his integrity will be preserved without changing the Court's ruling. The second best thing is for neither justice to recuse himself. The worst thing, however, is if they both recuse themselves, because then the ACA might be found constitutional.

Write this as a strategic form game in the matrix below. For each person's payoffs, use the numbers 1, 2, 3, and 4. (2 points)

		[2] Thomas [1-2]	
		recuse	not
[P] Scalia [1-p]	recuse	1, 1	4, 2
	not	2, 4	3, 3

b. Find all pure strategy and mixed strategy Nash equilibria of this game. (2 points)

pure strategy NE: (not, recuse) and (recuse, not)

mixed NE:  $1q + 4(1-q) = 2q + 3(1-q)$        $1p + 4(1-p) = 2p + 3(1-p)$

$$4 - 3q = 2q + 3 - 3q$$

$$4 - 3p = 2p + 3 - 3p$$

$$1 = 2q$$

$$2 = \frac{1}{2}$$

$$1 = 2p$$

$$p = \frac{1}{2}$$

(recuse with prob  $\frac{1}{2}$ , not with prob  $\frac{1}{2}$ )

c. Can you make a prediction using iterative elimination of (strongly or weakly) dominated strategies? If so, show the order of elimination. If not, explain why not. (2 points)

No strategies are weakly or strongly dominated  
 no prediction possible

d. Now say that Laura Graham has been found to be on the payroll of GlaxoSmithKline, a pharmaceutical company, and thus her attacks on Justice Kagan no longer resonate with the public. Now Kagan is a third player who can freely choose whether to recuse herself or not.

In this three-player game, when Kagan recuses herself, the payoffs of Scalia and Thomas are the same as before in part a. However, when Kagan does not recuse herself, the payoffs of Scalia and Thomas change: if either Scalia or Thomas (or both) recuses himself, then the payoffs of both are 2 lower than what they were before (for example, if Scalia's and Thomas's payoffs were (4, 2) before, now they are (2, 0)). If neither Scalia nor Thomas recuses himself, then their payoffs remain the same as before.

Kagan's payoffs are as follows. For her, the best thing is if Scalia and Thomas both recuse themselves and she does not, because then the ACA will surely be found constitutional. The second best is if Scalia and Thomas both recuse themselves and she also recuses herself; it is still very likely that the ACA will be found constitutional and her integrity is preserved. The worst is if she recuses herself and Scalia and Thomas do not. The second worst is if all three justices do not recuse themselves. Finally, if only one of the two opposing justices (Scalia or Thomas) recuses himself, then Kagan is indifferent about whether she recuses herself or not (she gets the same payoff either way).

Write this as a strategic form game in the matrix below. For Kagan's payoffs, use the numbers 5, 6, 7, 8, 9. (2 points)

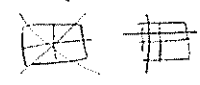
		Thomas		Thomas	
		recuse	not	recuse	not
Scalia	recuse	1, 1, 8	4, 2, 7	-1, -1, 9	2, 0, 7
	not	2, 4, 7	3, 3, 5	0, 2, 7	3, 3, 6
		Kagan		Kagan	
		recuse	not	recuse	not

e. Find all pure strategy Nash equilibria of this game. (2 points)

- (recuse, not, recuse)
- (not, recuse, recuse)
- (not, not, not)

f. Can you make a prediction using iterative elimination of (strongly or weakly) dominated strategies? If so, show the order of elimination. If not, explain why not. (2 points)

- ① For Kagan, not w. dominates recuse
- ② For Scalia, not s. dominates recuse
- For Thomas " "



Prediction: everyone chooses not to recuse

Part 3. There are 200 legislators in the parliament of Kandyland. Each legislator belongs to one of the five political parties: V, W, X, Y, and Z. Party V members make up 35%, Party W members make up 25%, Party X members make up 20%, Party Y members make up 10%, and Party Z members make up 10%. The parliament is considering four different bills for a free trade agreement: A, B, C, or D.

Members of Party V prefer A to B to C to D (in other words, Party V members like A best, B second-best, C third-best, and D worst). Members of Party W prefer C to A to B to D. Members of Party X prefer B to C to A to D. Members of Party Y prefer B to A to C to D. Members of Party Z prefer D to C to A to B.

a. Is there a Condorcet winner? If so, which bill is the Condorcet winner? (2 points)

	V	W	X	Y	Z
A	C	B	B	D	
B	A	C	A	C	
C	B	A	C	A	
D	D	D	D	B	

70	A	vs	30	B	so	A > B	no Condorcet winner every bill is beaten by some other
45	A	vs	55	C	so	C > A	
90	A	vs	10	D	so	A > D	
65	B	vs	35	C	so	B > C	
90	B	vs	10	D	so	B > D	
90	C	vs	10	D	so	C > D	

b. Which bill will win if the runoff procedure is used? Please show your work. (2 points)

First vote:

A	35	} so A and B are in runoff.
B	30	
C	25	
D	10	

Between A and B, A wins.

c. Which bill will win if the Borda count is used? Explain why by showing how many votes each bill receives under the Borda Count. (2 points)

A:  $3(35) + 2(25) + 1(20) + 2(10) + 1(10) = 205$  ← Borda count winner

B:  $2(35) + 1(25) + 3(20) + 3(10) + 0(10) = 185$

C:  $1(35) + 3(25) + 2(20) + 1(10) + 2(10) = 180$

D:  $0(35) + 0(25) + 0(20) + 0(10) + 3(10) = 30$

d. Which bill will win if approval voting (for top two) is used? Explain why by showing how many votes each bill receives under approval voting. (2 points)

A:  $35 + 25 + 10 = 70$  ← approval voting (for top two) winner.

B:  $35 + 20 + 10 = 65$

C:  $25 + 20 + 10 = 55$

D:  $10 = 10$

e. Say that the status quo voting method is the runoff procedure as in part b above.

Now say that the parliament tests a new voting system called "instant runoff." Each legislator votes for her first choice. If a bill gets a majority, that bill wins. If not, then the bill with the fewest votes is eliminated. A new round of voting takes place, with each legislator voting for his first choice among the bills that have not been eliminated (for instance, if bill C is removed after the first round, Party W members will vote for bill A in the second round). At each round of voting, the bill with the lowest number of votes is eliminated and is never considered again.

This process continues until one bill gets a majority of the votes. This bill wins.

Which bill wins if instant runoff is adopted? Explain why by showing the results of each voting round. (3 points)

Round 1:      A      B      C      D      → D is eliminated.  
                 35      30      25      10

Round 2:      A      B      C      → B is eliminated  
                 35      30      35

Round 3:      A      C      → A is eliminated  
                 45      55

C is chosen.

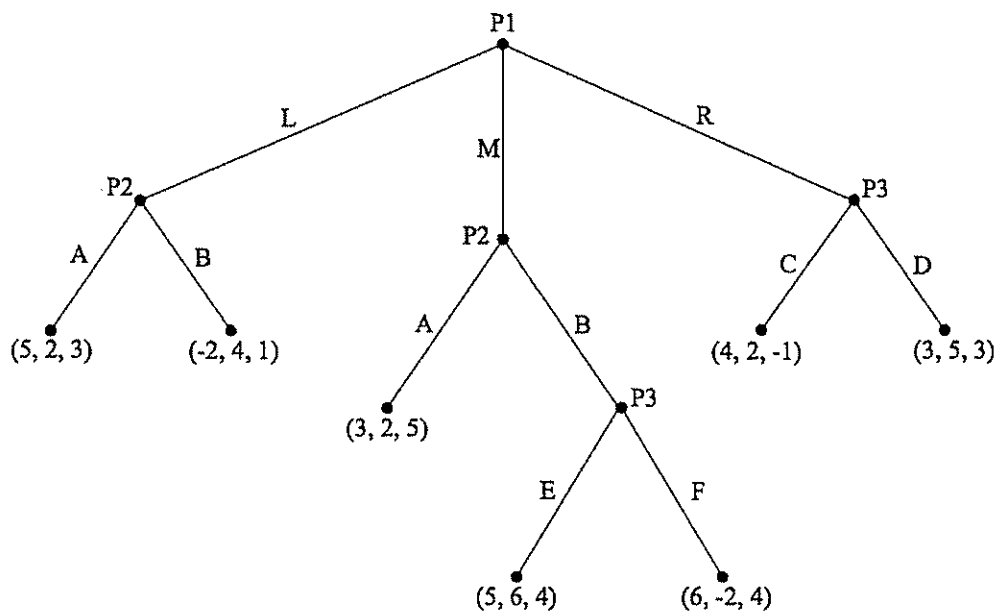
f. Remember that the status quo voting system is the traditional runoff procedure as in part b. above. Which party would be happiest with the new instant runoff system? Will a new bill on whether to adopt the instant runoff system (as opposed to the status quo) pass the parliament by majority rule? (1 point)

Party W would be happiest with the instant runoff system  
since C is Party W's first choice

C beats A by majority rule, so a bill to adopt instant runoff will pass.

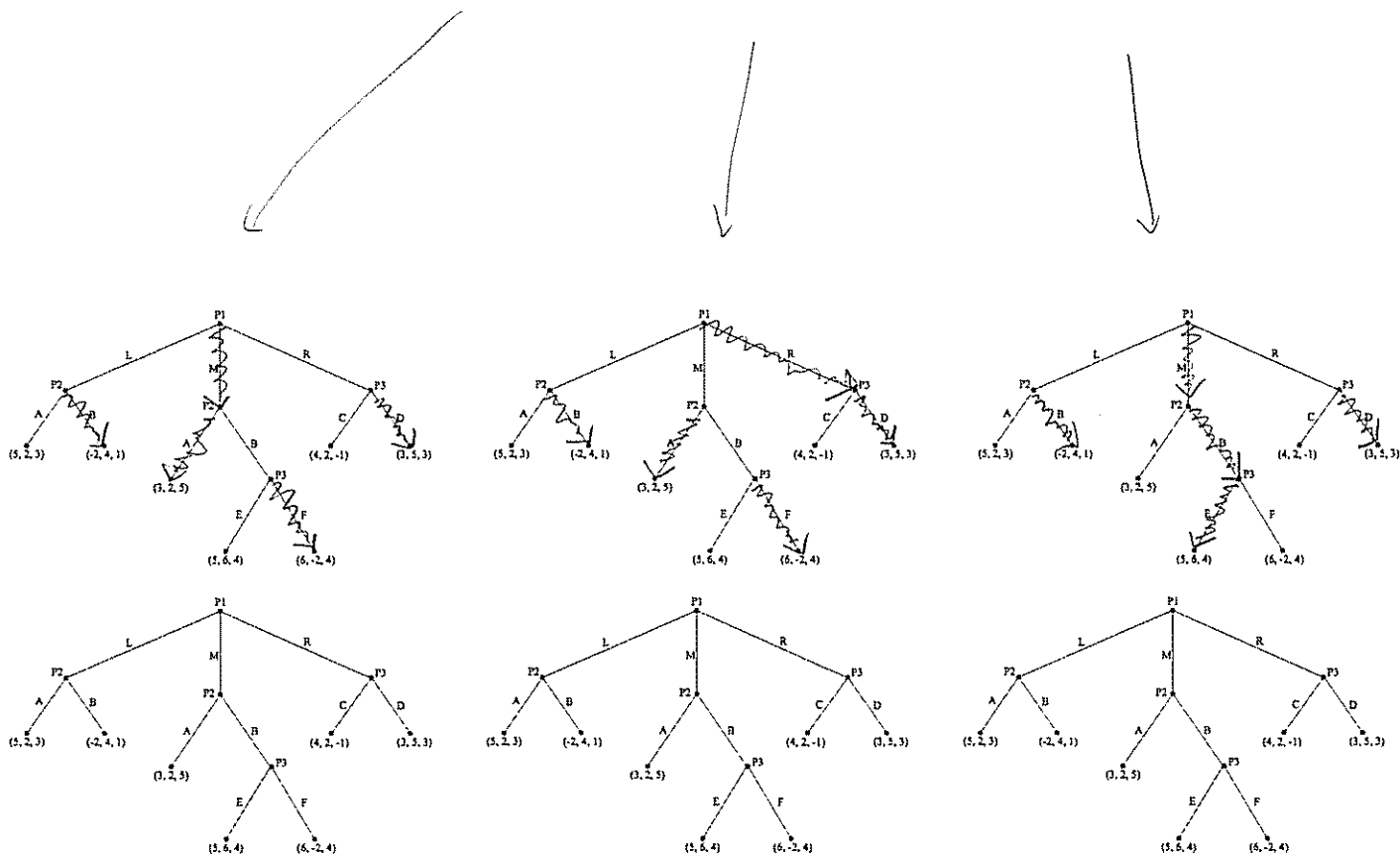


Part 4. Consider the following extensive form game.

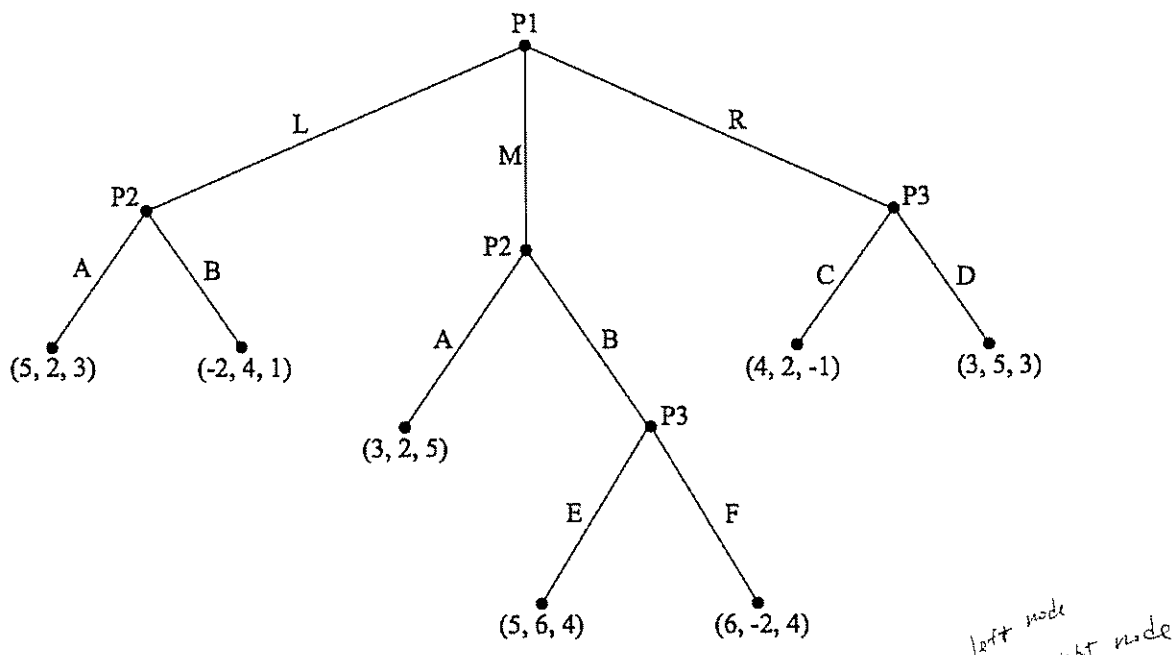


a. Find all subgame perfect Nash equilibria of this game. If you want to do this by writing arrows, some game trees are below for convenience. (4 points)

SPNE :  $(M, BA, DE)$   $(R, BA, DF)$   $(M, BB, DE)$



Here is the game again for your reference.



b. Write this game as a strategic form game. (4 points)

	AA	AB	BA	BB
L	$5, 2, 3$	$5, 2, 3$	$-2, 4, 1$	$-2, 4, 1$
M	$3, 2, 5$	$5, 6, 4$	$3, 2, 5$	$5, 6, 4$
R	$4, 2, -1$	$4, 2, -1$	$4, 2, -1$	$4, 2, -1$

CE

	AA	AB	BA	BB
L	$5, 2, 3$	$5, 2, 3$	$-2, 4, 1$	$-2, 4, 1$
M	$3, 2, 5$	$5, 6, 4$	$3, 2, 5$	$5, 6, 4$
R	$3, 5, 3$	$3, 5, 3$	$3, 5, 3$	$3, 5, 3$

DE

	AA	AB	BA	BB
L	$5, 2, 3$	$5, 2, 3$	$-2, 4, 1$	$-2, 4, 1$
M	$3, 2, 5$	$6, -2, 4$	$3, 2, 5$	$6, -2, 4$
R	$4, 2, -1$	$4, 2, -1$	$4, 2, -1$	$4, 2, -1$

CF

	AA	AB	BA	BB
L	$5, 2, 3$	$5, 2, 3$	$-2, 4, 1$	$-2, 4, 1$
M	$3, 2, 5$	$6, -2, 4$	$3, 2, 5$	$6, -2, 4$
R	$3, 5, 3$	$3, 5, 3$	$3, 5, 3$	$3, 5, 3$

DF

c. Find all (pure strategy) Nash equilibria. (4 points)

$(M, AB, CE)$   $(M, BB, CE)$   $(M, AB, DE)$   $(M, BB, DE)$

$(R, BA, DE)$

$(M, BA, DF)$

$(R, BA, DF)$

Part 5. Say that there are ten girls and ten boys who are thinking about going to the 8th grade dance. Each girl only cares about how many other girls are going, and each boy only cares about how many other boys are going (this is still 8th grade). The girls have thresholds 0, 1, 2, 4, 4, 5, 8, 8, 9, 9, as shown in the table below. For example, the girl with threshold 5 will go to the dance as long as at least 5 other girls go. The girl with threshold 0 doesn't mind going alone.

a. Say that initially (at time  $t=0$ ), none of the girls go. Which girls will go at time  $t=10$ ? (2 points)

The threshold 0, 1, and 2 girls go at  $t=1$  and from then nothing changes.

Girls' thresholds	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	...	$t=10$			NE	NE	NE		
0	n	y	y	y	y		y			y	y	y		
1	n	n	y	y	y		y			y	y	y		
2	n		n	y	y		y			y	y	y		
4	n			n	n		n			n	y	y		
4	n										y	y		
5	n										y	y		
8	n										n	y		
8	n											y		
9	n												y	
9	n	n	n	n	n		n			n	n	y		

b. Find all Nash equilibria. Please write your answer as something like  $(n, y, y, y, n, n, n, n, n, n)$ . For example,  $(n, y, y, y, n, n, n, n, n, n)$  means that the girl with threshold 1 goes, the girl with threshold 2 goes, and one of the girls with threshold 4 goes. If it is more convenient, you can write your answer in the table above and indicate clearly which are the Nash equilibria. (2 points)

In any NE, the threshold 0 (and hence 1, and then 2) go.

So we have  $(y, y, y, n, \dots, n)$  is a NE

If we add the threshold 4 girls, then the threshold 5 girl goes too

So  $(y, y, y, y, y, y, n, \dots, n)$  is a NE.

The threshold 8 girls wait jump in by themselves, but if the threshold 8 and 9 go, we have

$(y, y, y, \dots, y)$  is a NE (everyone goes).

The NE are shown in the table above.

c. The boys have thresholds 2, 2, 2, 4, 4, 5, 6, 7, 9, 10, as shown in the table below. For example, the boy with threshold 9 will go to the dance as long as at least 9 other boys go. Remember that boys only care about other boys.

Say that initially (at time  $t=0$ ), all of the boys go. Which boys will go at time  $t=10$ ? (2 points)

Boys' thresholds	$t=0$	$t=1$	$t=2$	$t=3$				NE	NE	NE			
2	y	y	y	y				y	y	n			
2	y	.	.	.				y	y	n			
2	y	.	.	.				.	y	.			
4	y	.	.	.				.	n	.			
4	y	.	.	.				.	.	.			
5	y	.	.	.				.	.	.			
6	y	.	.	.				.	.	.			
7	y	.	y	y				y	.	.			
9	y	y	n	n				n	.	.			
10	y	n	n	n				n	n	n			

d. Find all Nash equilibria. Please write your answer as something like  $(n,n,n,y,y,n,n,n,n,n)$ . For example,  $(n,n,n,y,y,n,n,n,n,n)$  means that the boys with threshold 4 go. If it is more convenient, you can write your answer in the table above and indicate clearly which are the Nash equilibria. (2 points)

The threshold 9 and 10 boys never go.

NE:

$(y, n, \dots, y, n, n)$

$(y, y, y, n, \dots, n)$

only threshold 2 boys go

If the threshold 4 boys go, then the 5, 6, and 7 threshold boys jump in.

Finally

$(n, n, \dots, n)$  is also a NE.

e. Now make one small change. Now the girl with threshold zero starts to care about boys. She is the only girl who cares about boys, and still none of the boys cares about girls. Everyone except this girl is exactly the same as before. This girl now wants to attend the dance only if at least eight boys show up.

Girls' thresholds	NE	NE	NE	NE	NE
0*	n	n	y	y	y
1	n	n	y	y	y
2	n	n	y	y	y
4	n	n	y	y	y
4	n	n	n	y	y
5	n	n	n	y	y
8	n	n	y	n	y
8	n	n	n	n	y
9	n	n	n	n	y
9	n	n	n	n	y

\* the threshold 0 girl goes only if at least 8 boys show up

Boys' thresholds	NE	NE	NE	NE	NE
2	n	y	y	y	y
2	n	y	y	y	y
2	n	y	y	y	y
4	n	n	y	y	y
4	n	n	y	y	y
5	n	n	y	y	y
6	n	n	y	y	y
7	n	n	y	y	y
9	n	n	n	n	n
10	n	n	n	n	n

Find all Nash equilibria. Now that the girls and boys are interacting with each other and are all in one big game. So write your answer as something like  $((n,n,n,n,n,n,n,n,y,y), (n,n,n,n,n,n,n,n,y))$ . In  $((n,n,n,n,n,n,n,n,y,y), (n,n,n,n,n,n,n,n,y))$  for example, the girls with threshold 9 go and the boy with threshold 10 goes. If it is more convenient, you can write your answer in the table above and indicate clearly which are the Nash equilibria.

(4 points)

The boys don't care about girls.

If the boys do  $(n, \dots, n)$ , then the threshold 0 girl doesn't go and hence the girls play  $(n, \dots, n)$ .

Similarly, if the boys do  $(y,y,y,n, \dots, n)$ , the girls play  $(n, \dots, n)$ .

If the boys play  $(y,y,y, \dots, y, n, n)$ , then the girls play either  $(y,y,y, n, \dots, n)$ ,  $(y,y,y,y, y, y, n, \dots, n)$

So NEs are:

$((n, \dots, n), (n, \dots, n))$  or  $(y,y, \dots, y)$   
 $((y,y,y,n, \dots, n), (y,y,y, \dots, y, n, n))$   
 $((n, \dots, n), (y,y,y,n, \dots, n))$   $((y,y,y,y,y,y,n, \dots, n), (y,y,y, \dots, y, n, n))$   
 $((y,y, \dots, y), (y,y,y, \dots, y, n, n))$

as shown above in the table.

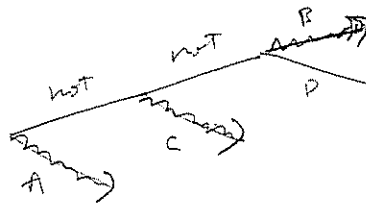
Part 6. There are five city councilpeople (1, 2, 3, 4, and 5), who make decisions by majority rule. They have four alternatives to choose from (A, B, C, and D). Their preferences are the following:

	1	2	3	4	5
<b>Best</b>	A	D	B	C	B
	D	C	A	B	A
	C	A	C	D	C
<b>Worst</b>	B	B	D	A	D

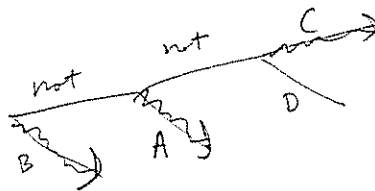
Say the councilpeople vote sequentially. For example, first they vote on A or NOT A, if NOT A wins then they vote on B or NOT B, and so on.

a. Write an agenda in which alternative A wins. If no such agenda exists, explain why. (1 point)

$A > C, A > D$   
 $B > A, B > D$   
 $C > B, C > D$



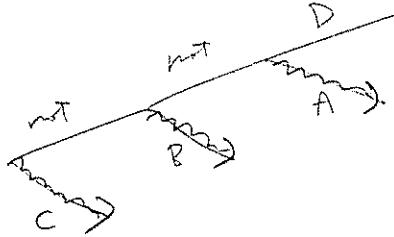
b. Write an agenda in which alternative B wins. If no such agenda exists, explain why. (1 point)



Here are the councilpersons' preferences again for your reference.

	1	2	3	4	5
<b>Best</b>	A	D	B	C	B
	D	C	A	B	A
	C	A	C	D	C
<b>Worst</b>	B	B	D	A	D

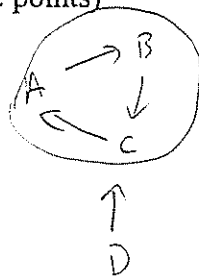
c. Write an agenda in which alternative C wins. If no such agenda exists, explain why. (1 point)



d. Write an agenda in which alternative D wins. If no such agenda exists, explain why. (1 point)

there is no agenda in which D wins,  
because D beats nothing by majority.

e. What is the top cycle? (2 points)



top cycle =  $\{A, B, C\}$

f. Now say an interest group wants the city council to pass a different proposal, E, which was not originally taken into consideration. The interest group has to spend time and money to lobby the councilpeople and convince them to consider E. In particular, you need to spend \$1 on each councilperson to get that person to consider E as her worst alternative; if you spend \$2 on that person, E will end up ranking second-worst; and so on until you spend \$5, which will make E the best choice for that councilperson.

If you spend...on a given councilperson	E will rank...in that councilperson's preferences
\$ 1	5 <sup>th</sup>
\$ 2	4 <sup>th</sup>
\$ 3	3 <sup>rd</sup>
\$ 4	2 <sup>nd</sup>
\$ 5	1 <sup>st</sup>

The interest group cannot change the order of the existing alternatives; it can only “insert” the new alternative E while preserving the original ranking. For example, if the interest group spends \$3 on councilperson 1, his preferences change from A, D, C, B (A is best, B is worst) to A, D, E, C, B.

The interest group obviously must spend at least \$1 per councilperson = \$5. The interest group wants to make sure that E wins, regardless of which agenda is adopted, but at the same time it wants to spend as little money as possible. For example, it could make E everyone's first choice by spending \$25, but that would be a waste of money.

How much money will the interest group spend on each councilperson so that E wins regardless of the voting agenda?

Insert E into the original ranking of alternatives so that E wins in any voting agenda and the interest group spends as little money as possible. How much money do they need to spend?

Please show your work on the next page. (6 points)



Here are the councilpersons' preferences again for your reference.

	1	2	3	4	5
Best	A	D	B	C	B
				E	E
	D	C	A	B	A
	E	E	E		
	C	A	C	D	C
Worst	B	B	D	A	D

Here  $E \succ A, E \succ B, E \succ C$  and  $E \succ D$  and so E wins regardless of the agenda. The interest group spends \$17.

Other solutions exist, such as

1	2	3	4	5
A	D	B	C	B
D	C	<b>E</b>	<b>E</b>	A
<b>E</b>	<b>E</b>	A	B	<b>E</b>
C	A	C	D	C
B	B	D	A	D

or

1	2	3	4	5
A	D	B	C	B
D	C	<b>E</b>	<b>E</b>	A
<b>E</b>	<b>E</b>	A	B	<b>E</b>
C	A	C	D	C
B	B	D	A	D

but in any solution, you have to spend at least \$17.

Why?

For E to win regardless of the agenda, it must be a Condorcet winner: it must beat all others by majority.

For it to beat all others by majority, it has to be above A for at least 3 people in the rankings

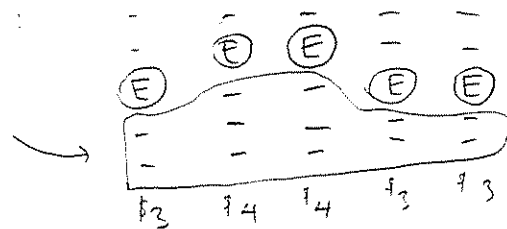
" B "

" C "

and " D "

Hence E must be like this:

There must be at least 12 entries "below" E.



To do this, you

have to spend at least \$17.

Part 7. Amy and Barbara are playing a card game. Each player has four cards: one, two, three, and a Joker. Each player chooses a card simultaneously. Amy wins the game if they either both play Jokers or if they both play numbers (one, two, or three) and the numbers are different. In all other circumstances, Barbara wins the game. For example, if Amy plays 1 and Barbara plays Joker, then Barbara wins. If Amy plays 2 and Barbara plays 2, then Barbara wins. If Amy plays Joker and Barbara plays 1, then Barbara wins. The winner receives \$1 and the loser gets \$0.

a. Model this as a strategic form game in the matrix below. Find all pure strategy Nash equilibria. (3 points)

Barbara

		1	2	3	J
Amy	1	0,1	1,0	1,0	0,1
	2	1,0	0,1	1,0	0,1
	3	1,0	1,0	0,1	0,1
	J	0,1	0,1	0,1	1,0

no pure strategy NE

b. Now we will find mixed strategy Nash equilibria of this game. For simplicity, assume that Amy plays each number 1, 2, or 3 with the same probability  $p$  and that Barbara plays each number 1, 2, or 3 with the same probability  $q$ , as shown below. Find the mixed strategy Nash equilibria of this game. (3 points)

		$[q]$ 1	$[q]$ 2	$[q]$ 3	$[1-3q]$ J
$[p]$	1	0,1	1,0	1,0	0,1
$[p]$	2	1,0	0,1	1,0	0,1
$[p]$	3	1,0	1,0	0,1	0,1
$[1-3p]$	J	0,1	0,1	0,1	1,0

Amy's  
exp. utility

$$q + q$$

$$q + q$$

$$q + q$$

$$1 - 3q$$

Barb's  
exp. utility:  $p + 1 - 3p$      $p + 1 - 3p$      $p + 1 - 3p$      $3p$

$$p + 1 - 3p = 3p$$

$$1 - 2p = 3p$$

$$1 = 5p$$

$$p = \frac{1}{5}$$

$$2q = 1 - 3q$$

$$5q = 1$$

$$q = \frac{1}{5}$$

So

$\left( \begin{array}{l} \text{Amy plays } 1 \text{ with prob } \frac{1}{5} \\ \text{Amy plays } 2 \text{ with prob } \frac{1}{5} \\ \text{Amy plays } 3 \text{ with prob } \frac{1}{5} \\ \text{Amy plays } J \text{ with prob } \frac{2}{5} \end{array} \right) \text{ Barbara plays } \left( \begin{array}{l} 1 \text{ with prob } \frac{1}{5} \\ 2 \text{ with prob } \frac{1}{5} \\ 3 \text{ with prob } \frac{1}{5} \\ J \text{ with prob } \frac{2}{5} \end{array} \right)$  is the mixed NE

c. Now say the game changes. Now Amy and Barbara each have five cards: one, two, three, four, and a Joker. Just like before, Amy wins if either she and Barbara play different numbers or if they both play Jokers. In all other circumstances, Barbara wins the game. In other words, the game is just like before except that now both Amy and Barbara have four numbered cards instead of three. Does this game have any pure strategy Nash equilibria? Find mixed strategy equilibria of this game using the approach you used in part b (in other words, assume that each numbered card is played with the same probability). You can write out the entire 5 x 5 game but it should not be necessary. (3 points)

		[q]	[q]	[q]	[q]	[1-4q]
		1	2	3	4	J
[p]	1	0,1	1,0	1,0	1,0	0,1
[p]	2		1,0			0,1
[p]	3		1,0			0,1
[p]	4		1,0			0,1
[1-4p]	J	0,1	0,1	0,1	0,1	1,0

For Amy, we must have  $q + q + q = 1 - 4q$

$$3q = 1 - 4q$$

$$7q = 1$$

$$q = \frac{1}{7}$$

For Barbara, we must have  $p + 1 - 4p = 4p$

$$1 - 3p = 4p$$

$$1 = 7p$$

$$p = \frac{1}{7}$$

So the mixed NE is

$$\left( \begin{array}{c|c|c|c|c} \text{Amy plays} & 1 & \text{with prob} & \frac{1}{7} & \\ & 2 & \text{"} & \frac{1}{7} & \\ & 3 & \text{"} & \frac{1}{7} & \\ & 4 & \text{"} & \frac{1}{7} & \\ & J & \text{"} & \frac{3}{7} & \end{array} \right) \quad \left( \begin{array}{c|c|c|c|c} \text{Barbara plays} & 1 & \text{with prob} & \frac{1}{7} & \\ & 2 & \text{"} & \frac{1}{7} & \\ & 3 & \text{"} & \frac{1}{7} & \\ & 4 & \text{"} & \frac{1}{7} & \\ & J & \text{"} & \frac{3}{7} & \end{array} \right)$$

d. Now say the game changes again. Now Amy and Barbara each have six cards: one, two, three, four, a Red Joker, and a Black Joker. Now Amy wins if either she and Barbara play different numbers or if they both play the same color Joker (both play Red Jokers or both play Black Jokers). In all other circumstances, Barbara wins the game. For example, if Amy plays a Black Joker and Barbara plays a Red Joker, then Barbara wins. If Amy plays a number and Barbara plays either Joker, then Barbara wins. If Amy plays either Joker and Barbara plays a number, then Barbara wins.

Does this game have any pure strategy Nash equilibria? Find mixed strategy equilibria of this game using the approach you used above. You can write out the entire 6 x 6 game but it should not be necessary. (3 points)

		$[q]$	$[q]$	$[q]$	$[q]$	$[\frac{1-4q}{2}]$	$[\frac{1-4q}{2}]$
		1	2	3	4	RJ	BJ
$[p]$	1	0,1	1,0	1,0	1,0	0,1	0,1
$[p]$	2	1,0				0,1	0,1
$[p]$	3	1,0				0,1	0,1
$[p]$	4	1,0				0,1	0,1
$[\frac{1-4p}{2}]$	RJ	0,1	0,1	0,1	0,1	1,0	0,1
$[\frac{1-4p}{2}]$	BJ	0,1	0,1	0,1	0,1	0,1	1,0

Assume that both Amy and Barbara play the Red and Black Joker with the same probability.

We thus have

$$3q = \frac{1-4q}{2}$$

$$6q = 1-4q$$

$$10q = 1$$

$$q = \frac{1}{10}$$

We also have

$$p + \frac{1-4p}{2} = \frac{1-4p}{2} = 4p + \frac{1-4p}{2}$$

$$p + \frac{1-4p}{2} = 4p$$

$$2p + 1 - 4p = 8p$$

$$1 = 10p$$

$$p = \frac{1}{10}$$

So mixed NE is

Amy plays	1	with prob	$\frac{1}{10}$	Barbara plays	1	with prob	$\frac{1}{10}$
	2		$\frac{1}{10}$		2	"	$\frac{1}{10}$
	3		$\frac{1}{10}$		3	"	$\frac{1}{10}$
	4		$\frac{1}{10}$		4	"	$\frac{1}{10}$
	RJ		$\frac{3}{10}$		RJ	"	$\frac{3}{10}$
	BJ		$\frac{3}{10}$		BJ	"	$\frac{3}{10}$

Part 8. Suppose two candidates run for president in a country which uses an electoral college. This country has a population of 300 people and is comprised of three states, Alazona, Delafornia, and Virwaii.

Alazona has 8 electoral votes and its voters are distributed as follows:

HL	ML	C	MR	HR
10	10	30	30	20

Delafornia has 4 electoral votes and its voters are distributed as follows:

HL	ML	C	MR	HR
4	18	30	38	10

Virwaii has  $x$  electoral votes and its voters are distributed as follows:

HL	ML	C	MR	HR
10	60	6	14	10

Each candidate chooses a single position nationwide—a candidate can't be conservative in one state and liberal in another. Each voter votes for the candidate who is closest to her own position. If two candidates are equally far away, then the voters split 50-50; in other words, if candidate 1 is at HL and candidate 2 is at C, half of the ML voters vote for each candidate. If a candidate wins a majority of votes in a state, she gets all of that state's electoral votes. If the two candidates tie and each get 50 percent of the votes in a state, then that state's electoral votes are split equally. Each candidate's payoff is the total number of electoral votes she receives.

a. Model this as a strategic form game. (4 points)

	HL	ML	C	MR	HR
HL	$6 + \frac{x}{2}, 6 + \frac{x}{2}$	0, $12 + x$	0, $12 + x^+$	$x, 12$	$x, 12$
ML	$^y 12 + x, 0$	<u><math>6 + \frac{x}{2}, 6 + \frac{x}{2}</math></u>	<u><math>x, 12</math></u>	$x, 12$	$x + 8, 4$
C	$^+ 12 + x, 0$	<u><math>12, x</math></u>	<u><math>6 + \frac{x}{2}, 6 + \frac{x}{2}</math></u>	$^* x + 8, 4$	$^y x + 12, 0$
MR	$12, x$	<u><math>12, x</math></u>	$4, x + 8^+$	$6 + \frac{x}{2}, 6 + \frac{x}{2}$	$^* x + 12, 0$
HR	$12, x$	$4, x + 8$	$0, x + 12^+$	$0, x + 12^+$	$6 + \frac{x}{2}, 6 + \frac{x}{2}$

possible best responses are underlined.

Possible NE (depending on  $x$ ): (ML, ML) (ML, C)  
(C, ML) (C, C)

b. How does the set of pure strategy Nash equilibria depend on  $x$ ? (Your answer should be a set of statements like "If  $x$  is less than 7, then the NE are (HL, HL) and (C, C); if  $x$  is equal to 7, then the NE is (HR, HR); if  $x$  is greater than 7 and less than 10, then the NE are (ML, ML) and (HL, HL)" and so forth.) (4 points)

If  $x < 12$ , then  $6 + \frac{x}{2} < 12$  and  $6 + \frac{x}{2} > x$

Hence the NE is (C, C).

If  $x > 12$ , then  $6 + \frac{x}{2} > 12$  and  $6 + \frac{x}{2} < x$

Hence the NE is (ML, ML).

If  $x = 12$ , then there are four NE:

(ML, ML) (ML, C)

(C, ML) (C, C)

c. Now say that the country gets rid of its electoral college system and chooses its president by nationwide majority vote. In other words, states don't matter anymore—all that matters is a candidate's nationwide total of votes. Find the (pure strategy) Nash equilibria of this new game. You can write out the new 5 x 5 game but it should not be necessary. (4 points)

You can just find the nationwide median voter.

Nationwide distribution:

HL	ML	C	MR	HR	(300 total)
24	88	66	82	40	

↑  
median voter

So the NE is (C, C).