Answers to

Final exam PS 30 December 2009

Name:

UID:

TA and section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are eight parts in this exam. Each part is weighted equally (12 points for each part). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If the TA responsible for a given question is not in the room at the time, work on other parts of the exam and hold the question until that TA rotates to your exam location. When the end of the exam is announced, please stop working immediately. People who continue working after the end of the exam is announced will have their grades penalized by 25 percent. If you need to leave the room to use the bathroom during the exam, please write your name down on the bathroom log before you leave. A person cannot leave the room more than once during the exam (a person who leaves for a second time will be considered to have completed his or her exam).

Please turn in your exam to one of the TAs. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Please turn off and put away all cell phones and other electronic gadgets. Please put away all notes and close all bags. Before you hand in your exam, make sure you flip through the exam and at least look at all questions—sometimes two pages get stuck to each other and you can miss an entire section of the exam. Good luck!

1	
2	
3	
4	
5	
6	
7	
8	
total	

Part 1. Consider the following two-person game. 2 3

	2a	2b	2c	2d	
1a	*8, 5+	1, 0	2, -1	0, 4	
~~1b~~	~~4, 8~~	~~3, 6~~	~~1, 9+~~	~~1, 5~~	1
1c	7, -1	5, 2	3, 0	*2, 3+	
~~1d~~	~~0, 7+~~	~~*6, 1~~	~~*7, -2~~	~~1, 2~~	4

a. Iteratively eliminate strongly and weakly dominated strategies. Eliminate as many as possible. Show the order of deletion. (4 points).

1. 1c strongly dominates 1b
2. 2d " " 2b
3. 2d " " 2c
4. 1c " " 1d

b. Find all pure strategy Nash equilibria. (4 points)

(1a, 2a)
(1c, 2d)

c. Find all mixed strategy Nash equilibria. Please write your answer out in words (writing "p=1/7, q=5/8" is not sufficient). (4 points)

	[q] 2a	[1-q] 2d
[p] 1a	8, 5	0, 4
[1-p] 1c	7, -1	2, 3

$8q + 0(1-q) = 7q + 2(1-q)$

$8q = 7q + 2 - 2q$

$3q = 2$ $\quad q = \frac{2}{3}$

$5p + -1(1-p) = 4p + 3(1-p)$

$5p - 1 + p = 4p + 3 - 3p$

$6p - 1 = p + 3$

$5p = 4$ $\quad p = \frac{4}{5}$

$\boxed{\text{mixed NE: (1 plays 1a with prob } \tfrac{4}{5}\text{, 1c " " } \tfrac{1}{5}\text{, 2 plays 2a with prob } \tfrac{2}{3}\text{, 2d " " } \tfrac{1}{3})}$

Part 2. Consider a world with two states who are players 1 and 2. Player 2 has an oil field in its territory that player 1 wants. Player 1 starts off with three choices: it can do nothing (N) and allow player 2 to keep the oilfield, it can threaten (T) player 2 with force, or it can launch a surprise attack (S) at player 2.

If player 1 chooses to do nothing, the game ends.

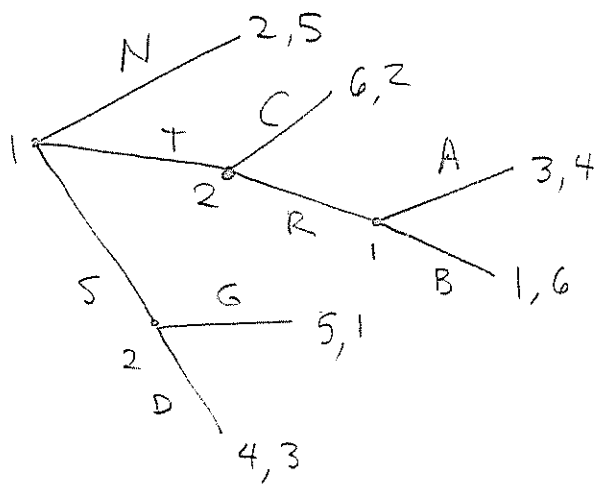
If player 1 threatens, player 2 can either capitulate (C) and give up the oil field (in which case the game ends), or reject (R) player 1's threat and prepare for war. If player 2 rejects player 1's threat, then player 1 can follow up by either attacking (A) player 2, which starts a war, or backing down (B) from its threat and allowing player 2 to keep the oil field. Both of these actions end the game.

If player 1 instead launches a surprise attack, player 2 can either give up (G) or try to defend (D) itself.

Player 1 ideally wants player 2 to capitulate and give up the oil field without a fight, but getting player 2 to give up the oil field after a surprise attack is a second best option. If player 2 will not capitulate or give up the oil field, player 1 prefers to attack player 2. However, if he does attack player 2, player 1 would prefer to launch a surprise attack rather than attack after player 2 has had a chance to prepare for war. Player 1's worst option is to back down from the use of force after making a threat, and only slightly better than this is to lose the oil field by doing nothing.

Player 2 ideally wants to keep the oil field without a fight. It can do this if player 1 either does nothing or backs down, but player 2 would prefer that player 1 back down after making a threat because it makes player 2 look strong in the eyes of the international community. If player 2 cannot keep the oil field without a fight, it prefers to fight for the oil field. However, if player 2 must fight, it prefers to fight player 1 after it has had a chance to reject player 1's threat and prepare for war, instead of defending itself from a surprise attack. If player 2 does not fight for the oil field, then it either capitulates to player 1's threats or gives up after a surprise attack from player 1. However, given a choice between the two, it would prefer to capitulate to threats alone, since this avoids any conflict taking place on player 2's territory.

a. Write out this game in extensive form. For actions, use the choices in the game indicated in brackets (i.e. N, T, S, A, and B for player 1, and C, R, G, D for player 2). For each player payoffs, you can use the payoff values {1, 2, 3, 4, 5, 6}. You will end up using all 6 values for both players. (4 points)



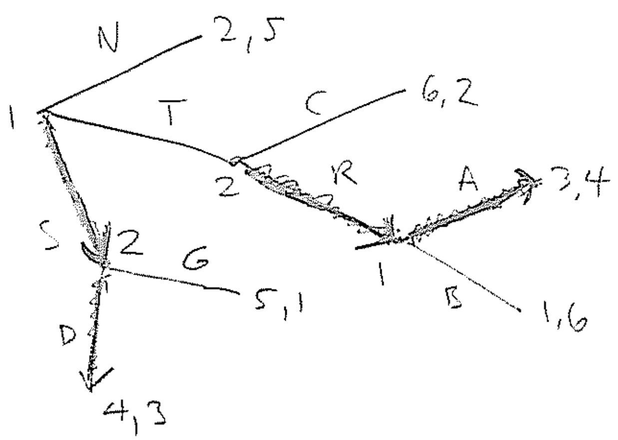
b. Write this game in strategic form. (4 points)

	CG	CD	RG	RD
NA	2, 5+	2, 5+	2, 5+	2, 5+
NB	2, 5+	2, 5+	2, 5+	2, 5+
TA	* 6, 2	* 6, 2	3, 4+	3, 4+
TB	* 6, 2	* 6, 2	1, 6+	1, 6+
SA	5, 1	4, 3+	* 5, 1	* 4, 3+
SB	5, 1	4, 3+	* 5, 1	* 4, 3+

c. Find all pure strategy Nash equilibria of this game. (2 points)

(SA, RD)
(SB, RD)

d. Find all Subgame Perfect Nash Equilibria of this game. (2 points)



SPNE: (SA, RD)

Part 3. Say that there are five people V, W, X, Y, and Z, who choose among four candidates a, b, c, and d. Their preferences are shown in the table below, where the most preferred is listed first and the least-preferred is listed last. For instance, person V likes a best and d worst.

	V	W	X	Y	Z
3	a	c	b	c	a
2	b	d	c	d	b
1	c	b	a	b	c
0	d	a	d	a	d

a. Is there a Condorcet winner? If so, who? (1 point)

$a > d$

$b > a, \; b > c, \; b > d$

$c > a, \; c > d$

b is the Condorcet winner (it beats all others by majority)

b. Is there an agenda in which they decide on c? If there is, show one. If not, explain why not. (1 point)

No: b is a Condorcet winner and since nothing beats it, regardless of the agenda, b will be chosen

c. Who is the Borda count winner? (1 point)

$a: 3+0+1+0+3 = 7$

$b: 2+1+3+1+2 = 9$

$c: 1+3+2+3+1 = 10$

$d: 0+2+0+2+0 = 4$

(Part 3 continued) Now suppose there are three groups in society with preference orders as shown below (the best on top and worst at the bottom). Group X has 5 people, group Y has y people, where y is assumed to be **odd**, and group Z has 3 people.

	5	y	3
	X	Y	Z
2	a	c	a
1	b	b	c
0	c	a	b

d. What is the smallest value of y for which c is a Condorcet winner? (3 points)

a vs. b: 8, y

b vs. c: 5, $y+3$

a vs. c: 8, y

For c to be a Condorcet winner, we must have

$y+3 > 5$ (c beats b)

and $y > 8$ (c beats a)

Since y is assumed odd, these conditions are true when $y \geq 9$. So the smallest value of y is 9

e. Suppose this society uses the Borda count system. List all the possible values of y for which a is the Borda count winner. Remember that y is assumed to be odd. (3 points)

a: $5 \cdot 2 + 3 \cdot 2 = 16$

b: $5 \cdot 1 + y \cdot 1 = 5 + y$

c: $2 \cdot y + 3 = 2y + 3$

For a to be the Borda count winner, we must have

$$\left.\begin{array}{l} 16 > 5 + y \\ \text{and } 16 > 2y + 3 \end{array}\right\} \Rightarrow \begin{array}{l} 11 > y \\ 13 > 2y \end{array}$$

Since y is odd, these conditions hold for

$y = 1, 3, 5.$

f. Depending on the value of y, can b be a Borda count winner? If so, find the smallest value of y for which b is a Borda count winner. Remember that y is assumed to be odd. If not, explain why not. (3 points)

For b to be a Borda count winner, we must have

$$\left.\begin{array}{l} 5+y>16 \\ \text{and } 5+y>2y+3 \end{array}\right\} \Rightarrow \begin{array}{l} y>11 \\ 2>y \end{array}$$

But it is impossible to have both $y>11$ and $2>y$.

So b cannot be a Borda count winner regardless of the value of y.

Part 4. Say that there are three city councilpeople, Trudy, Ursula, and Victor, who make decisions by majority rule. They choose among four alternatives, c, d, e, and f. Their preferences are given by the following table.

	Trudy	Ursula	Victor
Best	c	d	e
	d	e	f
	e	f	c
Worst	f	c	d

For example, Trudy likes c best, d second-best, e third-best, and f worst.

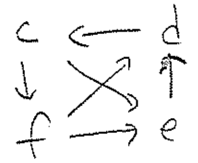
a. What is the top cycle? (3 points)

$c > d$

$d > e, \ d > f$

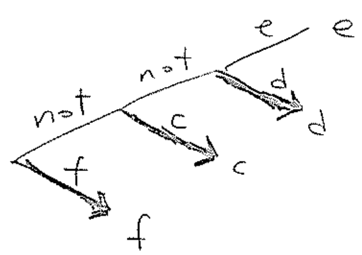
$e > c, \ e > f$

$f > c$



We have $c > d > e > f > c$. So $\{c, d, e, f\}$ is the top cycle.

b. Is it possible to write an agenda in which f wins? If so, write one down. If not, explain why not. (3 points)



f wins in this agenda.

(Part 4 continued) Now say that you are an interest group. By contributing to a councilperson's campaign, you make that councilperson's preferences reverse completely. For example, if you contribute to Victor, he now likes d best, c second-best, f third-best, and e worst. You can contribute to just one councilperson, two councilpersons, or all three, but to save cash you would rather contribute to as few of them as possible.

c. By making contributions, is it possible to change councilpersons' preferences so that c is chosen no matter what the agenda is? If so, which councilperson(s) should you contribute to, given that you want to contribute to as few as possible? If not, explain why not. (3 points)

For an alternative to be chosen regardless of what the agenda is, it has to be a Condorcet winner. If we contribute to Urusula, the preferences are now

	Trudy	Ursula	Victor
Best	c	c	e
	d	f	f
	e	e	c
Worst	f	d	d

and hence c is obviously a Condorcet winner, since a majority (Trudy and Ursula) have c as their first choice.

d. By making contributions, is it possible to change councilpersons' preferences so that f is chosen no matter what the agenda is? If so, which councilperson(s) should you contribute to, given that you want to contribute to as few as possible? If not, explain why not. (3 points)

Note that everyone prefers e over f. Thus if we contribute to only one person, e will still be preferred by a majority over f (e > f), and thus f will not be a Condorcet winner. To make f a Condorcet winner, we thus need to contribute to at least two councilpersons. If we contribute to both Ursula and Victor, we have c > f. If we contribute to both Trudy and Victor, we have d > f. If we contribute to both Trudy and Ursula, we have

	Trudy	Ursula	Victor
Best	f	c	e
	e	f	f
	d	e	c
Worst	c	d	d

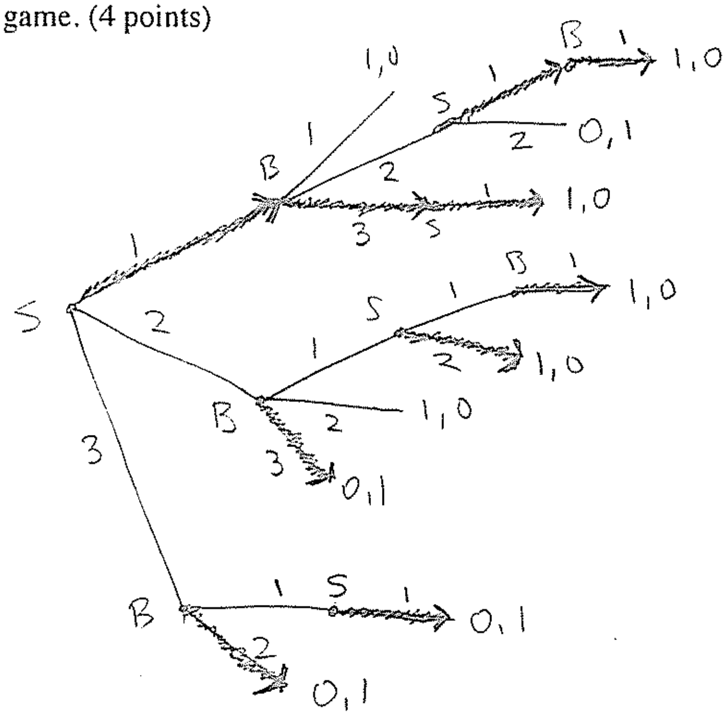
and it is easy to see that f is a Condorcet winner.

Part 5. Sister (S) and Brother (B) are sitting around the fireplace enjoying their holiday when their mother serves them a plate of m cookies. Since they just finished taking a game theory class, they decide to make a game out of it. They take turns eating the cookies. Each person can eat 1, 2, or 3 cookies. Whoever eats the last cookie wins. However, if you choose the same number that your opponent chose last time, then the game is immediately over and you lose. This is true even if you eat the last cookie.

For example, say they start with four cookies (m=4). If Sister eats 1 cookie, then if Brother eats 1 cookie he immediately loses because he chose the same number (1) as Sister. If Sister eats 1 cookie, then Brother can win by eating 3 cookies. If Sister eats 2 cookies, then if Brother eats 2 cookies, he loses because he chose the same number as Sister, even though he ate the last cookie.

Here each person gets a payoff of 1 from winning, and a payoff of 0 from losing. For convenience, write payoffs as (Sister, Brother). Sister always gets to go first.

a. Say m=5, in other words they start with 5 cookies. Write down this game as an extensive form game. (4 points)



b. Write down a subgame perfect Nash equilibrium (SPNE) of this game by writing arrows in the tree you wrote above (i.e. you don't have to write down the SPNE in words). Please make your arrows nice and clear. If there is more than one SPNE, just write down one of them; you don't have to write down all of them. (4 points)

c. Now let m, the starting number of cookies, go from 1 to 20, as shown in the table below. For each value of m, find out which person wins the game in an SPNE and write it in the table below. Remember that Sister (S) always goes first. For example, when m=1, there is only one cookie at the start, and Sister obviously wins by taking one cookie right at the start. So the table entry when m=1 is already filled in for you as an example. It is crucial to explain your reasoning here; simply filling out the table is not sufficient without an explanation of where your answers come from. (4 points)

m	Who wins?
20	**B**
19	**S**
18	**S**
17	**S**
16	**B**
15	**S**
14	**S**
13	**S**
12	**B**
11	**S**
10	**S**
9	**S**
8	**B**
7	**S**
6	**S**
5	**S**
4	**B**
3	**S**
2	**S**
1	**S**

Note that if there were no "no-repeats" rule (you lose if you choose the same number as your opponent did last time), then we have a standard Nim game and it is always a winning strategy to leave a multiple of 4. For example, if there are 6 cookies, you should eat 2, leaving 4. Thus if your opponent chooses 3, you choose 1 and win; if your opponent chooses 2, you choose 2 and win; if your opponent chooses 1, you choose 3 and win. Leaving 4 cookies is as good as winning, and thus leaving 8 cookies is as good as winning (because no matter what the other person does, you can leave 4), and so forth. Thus if the starting number of cookies is not a multiple of 4, Sister wins because she can leave a multiple of 4. If the starting number of cookies is a multiple of 4, Brother wins because regardless of what Sister chooses at the start, Brother can leave a multiple of 4.

It turns out that with the no-repeats rule, nothing changes. Consider when m=4. If Sister takes 1 or 3 cookies, then obviously Brother wins. If Sister takes 2 cookies, Brother can take 1 cookie, leaving 1 cookie left, but then Sister loses because she is forced to choose 1, the same number as what Brother just chose.

It turns out that it is still a winning strategy to leave a multiple of 4. This is illustrated in part a. when m=5, and Sister wins by taking 1 at the start. Say I leave 16 cookies, a multiple of 4. If you take 1 cookie, I take 3, and if you take 3 cookies, I take 1; either way I leave 12, a multiple of 4. The only thing which changes with the no-repeat rule is what happens if you take 2 cookies. If you take 2 cookies, leaving 14, I cannot take 2 cookies because of the no-repeat rule. But if I take 1 cookie, there are 13 cookies left and you cannot take 1 cookie because of the no-repeat rule. Hence you can either take 2 cookies or 3 cookies. If you take 2 cookies, leaving 11, then I take 3 and leave 8, a multiple of 4. If you take 3 cookies, leaving 10, then I take 2 and leave 8, a multiple of 4. Hence if I leave 16 cookies, no matter what you do, I can leave a multiple of 4.

Hence Sister wins when m is not a multiple of 4, because she can always leave a multiple of 4. Brother wins when m is a multiple of 4, because he can always leave a multiple of 4.

Part 6. Mike and Sarah are competing in an election to be their party's nominee for president. The only issue in the election is how many troops to send to a peacekeeping mission in another country. Their options (in army divisions) are to send 1, 2, 3, 4 or 5.

Assume that 32% of the population wants to send 1 division, 18% wants to send 2 divisions, 4% wants to send 3 divisions, 8% wants to send 4 divisions, and 38% wants to send 5 divisions.

Mike and Sarah each choose to take a position on this issue: either 1, 2, 3, 4, or 5 divisions. Once they choose their positions, each voter votes for the candidate whose position is closest to their own. If the two candidates are equally far away from a voter's position, then half of the voters at that position vote for one candidate, and half vote for the other candidate. For example, if Mike takes position 1 and Sarah takes position 3, then their positions are equally far away from the 18% of voters at position 2. Thus half of these voters (9%) vote for Mike and half (9%) vote for Sarah.

Each candidate's payoff is the percentage of votes he or she receives.

a. Model this as a strategic form game and find all pure strategy Nash equilibria. (6 points)

32% 18% 4% 8% 38%
1 2 3 4 5

The game looks like the following.

	1	2	3	4	5
1	50, 50	32, 68+	41, 59	50, 50	52, 48
2	*68, 32	*50, 50+	*50, 50+	52, 48	54, 46
3	59, 41	*50, 50+	*50, 50+	*54, 46	58, 42
4	50, 50	48, 52	46, 54+	50, 50	*62, 38
5	48, 52	46, 54	42, 58	38, 62+	50, 50

The pure strategy Nash equilibria are (2, 2), (2, 3), (3, 2), and (3, 3).

b. Now there is a third player, Rush. Rush has three possible actions: he can either endorse Mike, endorse Sarah, or not make an endorsement. If he chooses to endorse a candidate, then the voters act the same as before except for one change. Now if two candidates are equally far away from a voter's position, then all voters vote for the candidate whom Rush endorses.

For example, if Mike takes position 1 and Sarah takes position 3, then their positions are equally far away from the 18% of voters at position 2. If Rush endorses Mike, all of these voters (18%) vote for Mike and none (0%) vote for Sarah. Of course, voters at position 1 still vote for Mike and voters at positions 3, 4, and 5 still vote for Sarah. Rush's endorsement affects only those voters who would otherwise be indifferent between Mike and Sarah.

As before, Mike and Sarah's payoffs are the percentage of votes they get. Rush's payoffs are as follows. If he endorses a candidate, he gets a payoff of 1 if he endorses a candidate who gets 50% of the vote or more; otherwise he gets a payoff of 0. If he does not endorse a candidate, then he gets a payoff of 1 if Mike and Sarah tie and get the same percentage of votes (because he looks like a statesman) but he gets a payoff of 0 if there is a clear winner.

Find all pure strategy Nash equilibria of this game. (Hint: one can answer this question without writing down the whole game, although of course writing down the whole game is one way to do it.) (3 points)

Say that Rush endorses a candidate, say Mike. Then Mike can always get a payoff of 100 by taking the same position as Sarah, because for all voters, Mike and Sarah are equally far away, and hence all voters will pay attention to Rush and vote for Mike. Thus the only possible Nash equilibria when Rush endorses Mike are those in which Mike gets 100 percent of the vote. But in all of those situations, Sarah gets zero percent of the vote, and she can always get greater than zero votes by deviating and taking a different position than Mike. Hence there are no pure strategy Nash equilibria when Rush endorses a candidate.

So the only possible pure strategy Nash equilibria are when Rush does not make an endorsement. Here the payoffs are as in a. above, and thus (2, 2, No endorsement), (2, 3, No endorsement), (3, 2, No endorsement), and (3, 3, No endorsement) are the only possible Nash equilibria. We already know from a. above that Sarah and Mike cannot gain from deviating from these strategy profiles. The only thing we have to check is that Rush cannot gain by deviating. But notice that in all of these four strategy profiles, Rush gets a payoff of 1, and since 1 is his highest possible payoff, he cannot gain by deviating.

Hence the pure strategy Nash equilibria are (2, 2, No endorsement), (2, 3, No endorsement), (3, 2, No endorsement), and (3, 3, No endorsement).

c. Again, Rush has three possible actions: he can either endorse Mike, endorse Sarah, or not make an endorsement. But now only the 38% of voters at position 5 care about Rush's endorsement. All other voters do not care.

For example, say Rush endorses Sarah. If Mike takes position 2 and Sarah takes position 2, then their positions are equally far away from the 4% of voters at position 3. Since the voters at position 3 could care less about Rush, half of these voters (2%) vote for Mike and half (2%) vote for Sarah. But the 38% of voters at position 5 care about Rush, and all 38% vote for Sarah. Rush's endorsement affects only those voters at position 5, and only when they would otherwise be indifferent between Mike and Sarah.

Mike and Sarah's payoffs, and Rush's payoffs, are the same as before.

Find all pure strategy Nash equilibria of this game. (Hint: again, one can answer this question without writing down the whole game, although of course writing down the whole game is one way to do it.) (3 points)

Say that Rush endorses a candidate, say Sarah. The only case in which his endorsement affects anything is when Mike and Sarah have the same position (because Mike and Sarah are equally distant from the voters at position 5 only when Mike and Sarah have the same position). So Mike and Sarah's payoffs are the same as in a. except for when Mike and Sarah take the same position (on the "main diagonal"). When Mike and Sarah take the same position, Mike gets a payoff of 31 (half of the 62% of voters who are at positions 1, 2, 3, and 4, and who don't care about Rush) and Sarah gets a payoff of 69 (half of the 62% of voters who don't care about Rush plus the 38% of voters at position 5). Note that all of Sarah's payoffs "off the main diagonal" are less than 69, and hence Sarah can always do better by taking the same position as Mike. Hence the only possible Nash equilibria when Rush endorses Sarah are those in which Mike and Sarah have the same position (and thus Mike has a payoff of 31). In (1, 1, Sarah), Mike can get a payoff of 68 by deviating to 2. In (2, 2, Sarah), Mike can get a payoff of 50 by deviating to 3. In (3, 3, Sarah), Mike can get a payoff of 50 by deviating to 2. In (4, 4, Sarah), Mike can get a payoff of 54 by deviating to 3. In (5, 5, Sarah), Mike has can get a payoff of ~~54~~ by deviating to 3. Hence there are no pure strategy Nash equilibria when Rush endorses a candidate. 58.

So the only possible pure strategy Nash equilibria are when Rush does not make an endorsement. Thus we have (2, 2, No endorsement), (2, 3, No endorsement), (3, 2, No endorsement), and (3, 3, No endorsement) again. Again, the only thing we have to check is that Rush cannot gain by deviating. But notice that in all of these four strategy profiles, Rush gets a payoff of 1, and since 1 is his highest possible payoff, he cannot gain by deviating.

Hence the pure strategy Nash equilibria are again (2, 2, No endorsement), (2, 3, No endorsement), (3, 2, No endorsement), and (3, 3, No endorsement).

Part 7. Say there are two men A and B and two women X and Y. Each person wants to match up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Each person ranks his or her potential partners from best to worst.

a. (2 points) Say that their preferences are given by the following tables.

	A	B
Best	X	Y
Worst	Y	X

	X	Y
Best	**A**	A
Worst	**B**	B

In other words, man A likes woman X best and woman Y worst and man B likes woman Y best and woman X least. Woman Y likes man A best and man B least.

Note that woman X's preferences are left blank. In other words, X could either like A best and B worst or he could like B best and A worst.

Is it possible to fill in woman X's preferences so that there exists exactly one stable match? If so, fill in her preferences in the table above and show that there exists exactly one stable match. If not, explain why not.

Say we fill out Woman X's preferences as indicated by the bold entries above (she prefers A over B). When there are two women and two men, there are only two possible matches: (AX, BY) and (AY, BX). The match (AX, BY) is stable because both men have their first choice and never want to change. The match (AY, BX) is not stable because A and X would rather dump their current partners and be together instead. Thus there exists exactly one stable match.

b. (2 points) Say that their preferences are again given by the following tables.

	A	B
Best	X	Y
Worst	Y	X

	X	Y
Best	**B**	A
Worst	**A**	B

Is it possible to fill in woman X's preferences so that there exist exactly two stable matches? If so, fill in her preferences in the table above and show that there exist exactly two stable matches. If not, explain why not.

Say we fill out Woman X's preferences as indicated by the bold entries above (she prefers B over A). When there are two women and two men, there are only two possible matches: (AX, BY) and (AY, BX). The match (AX, BY) is stable because both men have their first choice and never want to change. The match (AY, BX) is stable because both women have their first choice and never want to change. Thus there exist exactly two stable matches.

c. (4 points) Now say there are three men A, B, and C, and three women X, Y, and Z. Their preferences are given by the following tables.

	A	B	C
Best	X	Y	Z
	Y	Z	X
Worst	Z	X	Y

	X	Y	Z
Best	**B**	C	A
	C	A	B
Worst	**A**	B	C

Note that woman X's preferences are left blank.

Is it possible to fill in woman X's preferences so that there exist exactly three stable matches? If so, fill in her preferences in the table above and show that there exist exactly three stable matches. If not, explain why not.

First, note that (AX, BY, CZ) is stable regardless of the women's preferences, because all the men are getting their first choice.

Say we fill out Woman X's preferences as indicated by the bold entries above (she prefers B over C over A). Note that with these preferences, if you like someone, then they hate you. However, if someone is your second choice, you are their second choice.

Now (AZ, BX, CY) is also stable because all the women are getting their first choice. There are only four other possible matches to consider.

(AX, BZ, CY) is not stable because C and X would like to get together.
(AY, BX, CZ) is not stable because B and Z would like to get together.
(AY, BZ, CX) is stable because no couple can form in which both the man and woman are happier than in their current relationships. Here everyone is matched with their second choice.
(AZ, BY, CX) is not stable because A and Y would like to get together.

Hence there are exactly three stable matches. We've shown that if we fill out the table above with the bold entries, then there are exactly three matches. We haven't shown that this is the only way of filling out the table so that there are exactly three matches, so another correct answer might be possible.

d. (4 points) Again, say that their preferences are given by the following tables.

	A	B	C
Best	X	Y	Z
	Y	Z	X
Worst	Z	X	Y

	X	Y	Z
Best		C	A
		A	B
Worst		B	C

Note that woman X's preferences are left blank.

Is it possible to fill in woman X's preferences so that there exists exactly one stable match? If so, fill in her preferences in the table above and show that there exists exactly one stable match. If not, explain why not.

Again, (AX, BY, CZ) is stable regardless of the women's preferences, because all the men are getting their first choice. So the question is if there exists another stable match.

If Woman X's first choice is A, then in the woman-ask algorithm, we start with (XA, YC, ZA). A prefers X and rejects Z, and so we have (XA, YC, ZB). Since all men get exactly one invitation, this is stable. This is another stable match, so there will be at least two stable matches.

If Woman X's first choice is B, then in the woman-ask algorithm, we start with (XB, YC, ZA). Since all men get exactly one invitation, this is stable. This is another stable match, so there will be at least two stable matches.

If Woman X's first choice is C, then in the woman-ask algorithm, we start with (XC, YC, ZA). C prefers X and rejects Y, and so we have (XC, YA, ZA). A prefers Y and rejects Z, and so we have (XC, YA, ZB). Since all men get exactly one invitation, this is stable. This is another stable match, so there will be at least two stable matches.

So no matter what Woman X's first choice is, there will be at least two stable matches. So it is not possible to fill in Woman X's preferences so that there exists exactly one stable match.

Part 8. A revolutionary group is trying to start an insurgency to topple the government. They need to gather enough soldiers to constitute a serious threat. However, they face two hurdles: first they need to convince people to participate and second they need to keep them in the ranks once they've joined.

Sacrificing yourself for the good of others is noble but having someone else die for your country is easier on the wardrobe. Thus, even if you support the revolution, there is a great temptation to say, "Why do they need me? I can drop out and 'free ride' on the insurgents' success."

These dynamics are depicted below in a threshold model. There are two groups in the population, the Sympathizers (S) and the Moderates (M), and even within these groups there is a good mix of thresholds, as shown below.

Lower threshold		Upper threshold	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
S	0	9	n								
S	1	2	n								
S	1	2	n								
S	2	2	n								
M	3	3	n								
M	3	4	n								
M	4	6	n								
M	6	6	n								
M	7	9	n								

A person can either participate (p) in the insurgency or not participate (n). As explained in class, a person participates if the total number of other people who participate is greater than or equal to her lower threshold and less than or equal to her upper threshold.

a. Say that they start from a situation (t=0) in which no one participates. Fill in the table above to show how participation changes over time. How many people will participate in the insurgency at t=3? How many people will participate in the insurgency at t=5? (4 points)

We have the following table.

Lower threshold		Upper threshold	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
S	0	9	n	p	p	p	p	p	p	p	p
S	1	2	n		p	p			p		
S	1	2	n		p	p			p		
S	2	2	n						p		
M	3	3	n			p		p			p
M	3	4	n			p	p			p	
M	4	6	n				p			p	
M	6	6	n								
M	7	9	n								

The people who participate are shown above (everything which is left blank should be an "n"; I didn't fill these out for the sake of clarity). Five people participate at t=3 and 2 at t=5.

(Part 8 continued) The insurgents can win if they get 7 or more soldiers. However, they will lose and be crushed by the government if they ever drop below 3 soldiers after t=1. The revolutionary leaders have two strategies they can follow to try to win: they can either resort to coercion to keep soldiers or give out benefits to encourage recruits.

If they resort to coercion they will raise the upper thresholds of the sympathizers by 2 but raise the lower thresholds of the moderates by 2.

If they decide to give out benefits they lower the lower thresholds of the sympathizers by 2 and lower the upper thresholds of the moderates by 2.

b. Say that they resort to coercion. Again, assume that they start from a situation in which no one participates. Will they win or lose if they resort to coercion? You can use the table below to help work out the problem. (4 points)

Lower threshold	Upper threshold	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
S		n								
S		n								
S		n								
S		n								
M		n								
M		n								
M		n								
M		n								
M		n								

With coercion, we have the following table.

Lower threshold	Upper threshold	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
S 0	11	n	p	p	p	p	p	p	p	p
S 1	4	n		p	p	p	p	p	p	p
S 1	4	n		p	p	p	p	p	p	p
S 2	4	n			p	p	p	p	p	p
M 5	3	n								
M 5	4	n								
M 6	6	n								
M 8	6	n								
M 9	9	n								

Note that they never get above four participants and hence they lose. Coercion succeeds in getting the sympathizers to all participate but prevents the moderates from joining in.

c. Now say that they give out benefits. Again, assume that they start from a situation in which no one participates. Will they win or lose if they give out benefits? You can use the table below to help work out the problem. (4 points)

Lower threshold	Upper threshold	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
S		n								
S		n								
S		n								
S		n								
M		n								
M		n								
M		n								
M		n								
M		n								

With benefits, we have the following table.

Lower threshold	Upper threshold	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
S -2	9	n	p	p	p	p	p	p	p	p
S -1	2	n	p		p		p		p	
S -1	2	n	p		p		p		p	
S 0	2	n	p		p		p		p	
M 3	1	n								
M 3	2	n								
M 4	4	n		p		p		p		p
M 6	4	n								
M 7	7	n								

Note that again they never get above four participants and hence they lose.