

Answers to

Final exam

PS 30

December 2008

Name:

UID:

TA and section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are eight parts in this exam. Each part is weighted equally (12 points for each part). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If the TA responsible for a given question is not in the room at the time, work on other parts of the exam and hold the question until that TA rotates to your exam location. When the end of the exam is announced, please stop working immediately. People who continue working after the end of the exam is announced will have their grades penalized by 25 percent. If you need to leave the room to use the bathroom during the exam, please write your name down on the bathroom log before you leave. A person cannot leave the room more than once during the exam (a person who leaves for a second time will be considered to have completed his or her exam). Please turn in your exam to one of the TAs. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Please turn off all cell phones and other electronic gadgets. Good luck!

1	
2	
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7	
8	
total	

Part 1. Examine the following game. Player 1 can choose among alternatives W, X, Y, and Z, and Player 2 can choose among alternatives A, B, and C.

		Player 2		
		A	B	C
Player 1	W	4,2	3,4 ⁺	11,3
	X	*6,6 ⁺	4,1	7,5
	Y	3,3	6,5 ⁺	*13,4
	Z	*6,7	*7,6	10,8 ⁺

a. Find all pure strategy Nash equilibria of this game (4 points).

pure strategy NE = (X, A)

b. By iteratively eliminating strongly or weakly dominated strategies, eliminate as many strategies as possible. Please show the order of elimination (4 points).

	A	B	C	
W	4,2	3,4	11,3	③
X	6,6	4,1	7,5	①
Y	3,3	6,5	13,4	
Z	6,7	7,6	10,8	
	②			

- ① Z weakly dominates X
- ② C strongly dominates A
- ③ Y strongly dominates W

c. Find all mixed strategy Nash equilibria of this game (4 points).

		$[q]$	$[1-q]$
	B		C
$[p]$	Y	6, 5	13, 4
$[1-p]$	Z	7, 6	10, 8

$$6q + 13(1-q) = 7q + 10(1-q)$$

$$13 - 7q = 10 - 3q$$

$$3 = 4q$$

$$q = \frac{3}{4}$$

$$5p + 6(1-p) = 4p + 8(1-p)$$

$$6 - p = 8 - 4p$$

$$3p = 2$$

$$p = \frac{2}{3}$$

MSNE:

(1 plays Y with probability $\frac{2}{3}$, Z with probability $\frac{1}{3}$, 2 plays B with prob $\frac{3}{4}$, C with prob $\frac{1}{4}$)

Part 2. Two candidates, Hilton and O'Hara, are competing to be the Democratic Party's presidential nominee. Two states, Brewery and Winery, are considering whom to vote for at the party convention. Brewery prefers Hilton and Winery prefers O'Hara.

Because the party is facing a general election soon, party unity is very important. The party is united if both states vote for the same candidate. The best outcome for a state is for the party to be united around the state's preferred candidate (payoff = 4). For example, the best outcome for Winery is for both states to vote for O'Hara.

The second best outcome is being united under the candidate whom they don't prefer (payoff = 3). The second worst case is when each state votes for its own preferred candidate and their votes are split (payoff = 2). The absolute worst case is when each state votes for its not-preferred candidate and their votes are split (payoff = 1).

a. Say that the states choose on the same day. Represent this as a strategic form game and find all pure strategy Nash equilibria and mixed strategy Nash equilibria. (2 points)

		Winery	
		[2] H	[1-2] O
Brewery	[p] H	* 4, 3 +	2, 2
	[1-p] O	1, 1	* 3, 4 +

pure strategy NE: (H, H) (0, 0)

$$4q + 2(1-q) = q + 3(1-q)$$

$$2 + 2q = 3 - 2q$$

$$4q = 1$$

$$q = \frac{1}{4}$$

$$3p + 1(1-p) = 2p + 4(1-p)$$

$$1 + 2p = 4 - 2p$$

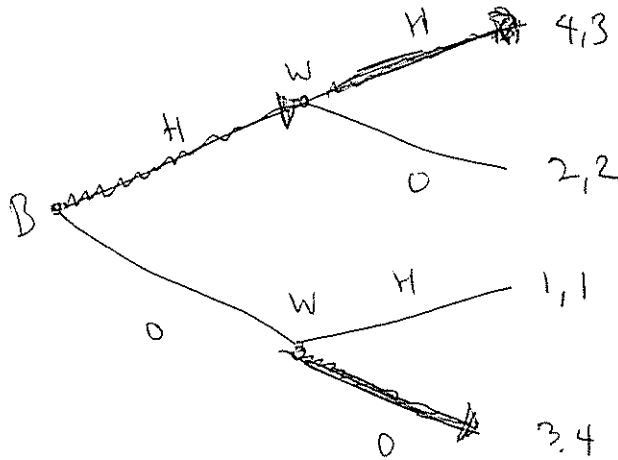
$$4p = 3$$

$$p = \frac{3}{4}$$

Mixed strategy NE: (Brewery plays H with prob $\frac{3}{4}$
O $\frac{1}{4}$,

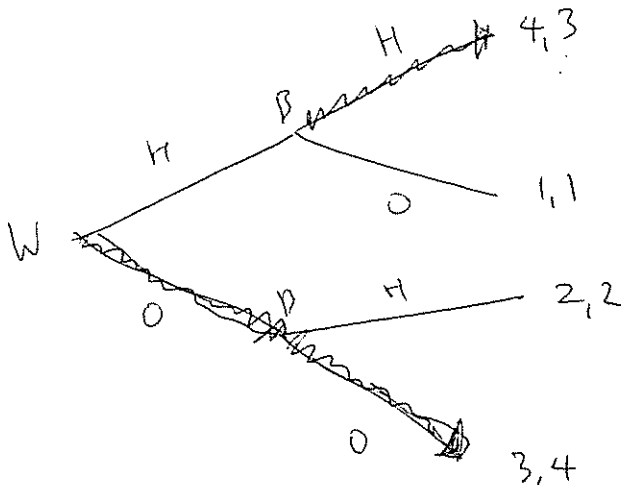
Winery plays H with prob $\frac{1}{4}$
O $\frac{3}{4}$)

b. Now say that Brewery's primary is in September and Winery's primary is in October. In other words, Brewery chooses first. Model this as an extensive form game and find all subgame perfect Nash equilibria. For clarity's sake, write payoffs as (Brewery, Winery). (2 points)



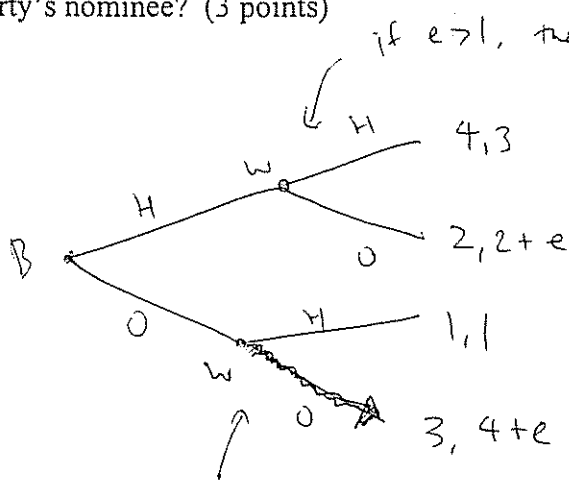
SPNE : (Brewery plays H, Winery plays H if Brewery played H)

c. Now say that Winery's primary is earlier and so Winery chooses first. Model this as an extensive form game and find all subgame perfect Nash equilibria. For clarity's sake, write payoffs as (Brewery, Winery). (2 points)



SPNE : (W plays O, B plays H if W played H)

d. Say again that Brewery chooses first. However, now Winery cares more about voicing its true opinion. In addition to its payoffs from before, Winery has an additional payoff e when it votes for O'Hara, no matter who is finally chosen as the party's nominee, where e is positive. Model this as an extensive form game. For clarity's sake, write payoffs as (Brewery, Winery). How large must e be for O'Hara to be chosen as the party's nominee? (3 points)



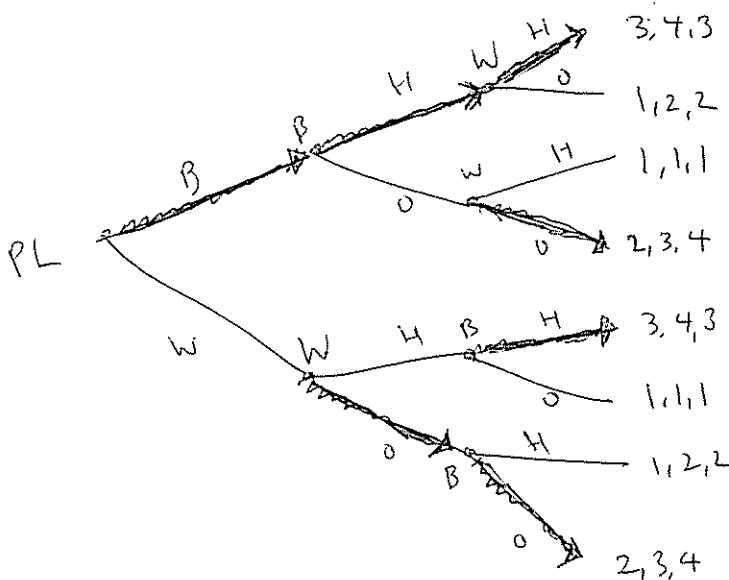
W always votes for O here because $4+e > 1$ since $e > 0$.

If $e > 1$, then W always plays O and thus B plays O also.

If $e < 1$, then W plays H if B plays H and thus B plays H.

So e has to be at least 1 for O to be chosen.

e. Now say that national party leaders decide which state will have its primary first. The party leaders prefer Hilton, who is better-known to the public. Thus, the party leaders' top preference (payoff 3) is being united under Hilton, their second preference (payoff 2) is being united under O'Hara, and their worst case (payoff 1) is having the party split. Represent this as an extensive form game and find all subgame perfect Nash equilibria. For clarity's sake, write payoffs as (Party leaders, Brewery, Winery). Who will be the party nominee? (3 points)



The party nominates Hilton.

SPNE

Part 3. Consider the following game, where X and Y are varying payoffs.

		Player 2	
		A	B
Player 1	A	3, 2	X, 3
	B	X, Y	6, 4

a. In the grid below, X ranges from 1.5 to 8.5 and Y ranges from 1.5 to 8.5. For each box in the grid, write "Yes" if the game has exactly two pure strategy Nash equilibria and one mixed strategy Nash equilibrium. Otherwise, write "No."

For example, the shaded box corresponds to $X = 1.5$ and $Y = 3.5$. If the game has exactly two pure strategy Nash equilibria and one mixed strategy Nash equilibrium when $X = 1.5$ and $Y = 3.5$, then you would write "Yes" in the shaded box. Otherwise, you would write "No."

Note that every box below should be filled in with either "Yes" or "No." Please justify your answer. (4 points)

no pure strategy NE (it "cycles")

Y = 8.5	No	No	No	No	No	Yes	Yes	Yes
Y = 7.5	No	No	No	No	No	Yes	Yes	Yes
Y = 6.5	No	No	No	No	No	Yes	Yes	Yes
Y = 5.5	No	No	No	No	No	Yes	Yes	Yes
Y = 4.5	No	No	No	No	No	Yes	Yes	Yes
Y = 3.5	No	No	No	No	No	No	No	No
Y = 2.5	No	No	No	No	No	No	No	No
Y = 1.5	No	No	No	No	No	No	No	No
	X = 1.5	X = 2.5	X = 3.5	X = 4.5	X = 5.5	X = 6.5	X = 7.5	X = 8.5

(B, A) and (A, B) are pure strategy NE.

If $Y < 4$, then B s. dominates A for person 2 and thus we can't have 2 pure strategy NE.

If $X > 3$ and $X < 6$, then B s. dominates A for person 1.

If $Y > 4$ and $X < 3$, we have

	A	B
A	*3, 2	X, 3+
B	X, Y+	*6, 4

which has no pure strategy NE.

If $Y > 4$ and $X > 6$, we have

	A	B
A	3, 2	*X, 3+
B	*X, Y+	6, 4

which has two pure strategy NE: (A, B) and (B, A).
It also has one mixed NE.

Here is the game again:

		Player 2	
		A	B
Player 1	A	3, 2	X, 3
	B	X, Y	6, 4

b. In the grid below, X ranges from 1.5 to 8.5 and Y ranges from 1.5 to 8.5. For each box in the grid, write "Yes" if the game has no pure strategy Nash equilibria and exactly one mixed strategy Nash equilibrium. Otherwise, write "No."

For example, the shaded box corresponds to $X = 1.5$ and $Y = 3.5$. If game has no pure strategy Nash equilibria and exactly one mixed strategy Nash equilibrium when $X = 1.5$ and $Y = 3.5$, then you would write "Yes" in the shaded box. Otherwise, you would write "No."

Note that every box below should be filled in with either "Yes" or "No." Please justify your answer. (4 points)

Y = 8.5	Yes	Yes	No	No	No	No	No	No
Y = 7.5	Yes	Yes	No	No	No	No	No	No
Y = 6.5	Yes	Yes	No	No	No	No	No	No
Y = 5.5	Yes	Yes	No	No	No	No	No	No
Y = 4.5	Yes	Yes	No	No	No	No	No	No
Y = 3.5	No	No	No	No	No	No	No	No
Y = 2.5	No	No	No	No	No	No	No	No
Y = 1.5	No	No	No	No	No	No	No	No
	X = 1.5	X = 2.5	X = 3.5	X = 4.5	X = 5.5	X = 6.5	X = 7.5	X = 8.5

Like we said before, if $Y > 4$ and $X < 3$, we have

	A	B
A	* 3, 2	X, 3
B	X, Y	* 6, 4

which has no pure strategy NE and one mixed strategy NE.

Here is the game again:

		Player 2	
		A	B
Player 1	A	3, 2	X, 3
	B	X, Y	6, 4

c. Find the values of X and Y such that the following is a mixed strategy Nash equilibrium: Player 1 plays A with probability 1/4 and B with probability 3/4, and Player 2 plays A with probability 4/11 and B with probability 7/11. (4 points)

$$3q + X(1-q) = Xq + 6(1-q)$$
~~$$3q + X - Xq = Xq + 6 - 6q$$~~

$$3q + X - Xq = Xq + 6 - 6q$$
~~$$(3 - 2X + 6)q = 6 - X$$~~

$$(3 - 2X + 6)q = 6 - X$$

$$q = \frac{6 - X}{3 - 2X + 6}$$

$$2p + Y(1-p) = 3p + 4(1-p)$$
~~$$2p + Y - Yp = 3p + 4 - 4p$$~~
~~$$Y(1-p) = 4 - 3p$$~~
~~$$Y = \frac{4 - 3p}{1-p}$$~~

$$Y - 4 = -2p + Yp + 3p - 4p$$

$$Y - 4 = Yp - 3p$$

$$p = \frac{Y - 4}{Y - 3}$$

We need $p = \frac{1}{4}$ and $q = \frac{4}{11}$.

$$\text{So } \frac{4}{11} = \frac{6 - X}{3 - 2X + 6}$$

$$12 - 8X + 24 = 66 - 11X$$

$$3X = 30$$

$$X = 10$$

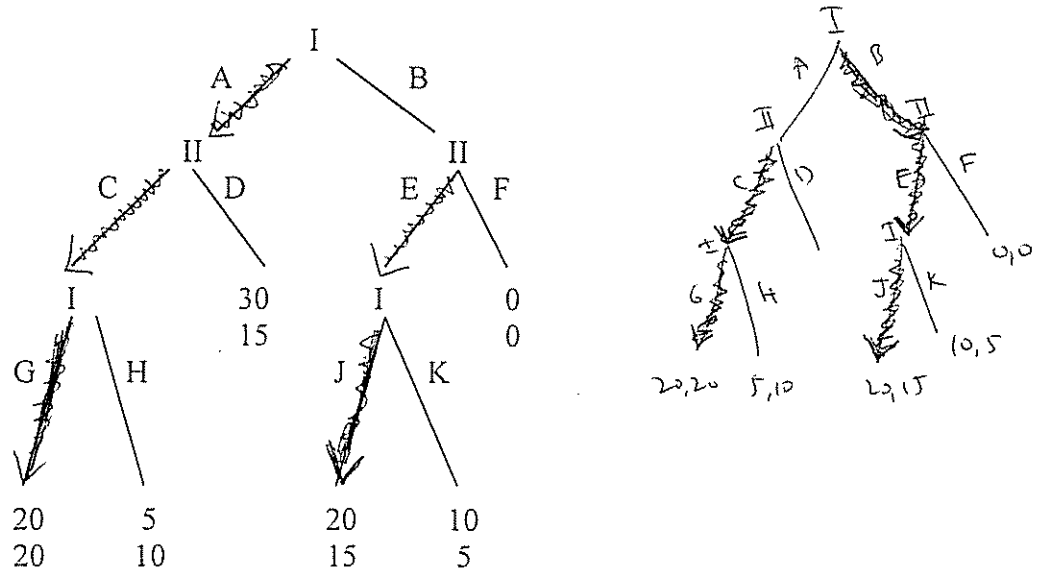
$$\frac{1}{4} = \frac{Y - 4}{Y - 3}$$

$$Y - 3 = 4Y - 16$$

$$13 = 3Y$$

$$Y = \frac{13}{3}$$

Part 4. Consider the following extensive form game. There are two players, Player I and Player II. For example, if Player I plays A and then Player II plays C, and then Player I plays H, then Player I gets a payoff of 5 and Player II gets a payoff of 10.



a. Find all pure strategy Nash equilibria of this game. Find all subgame perfect Nash equilibria of this game. (4 points)

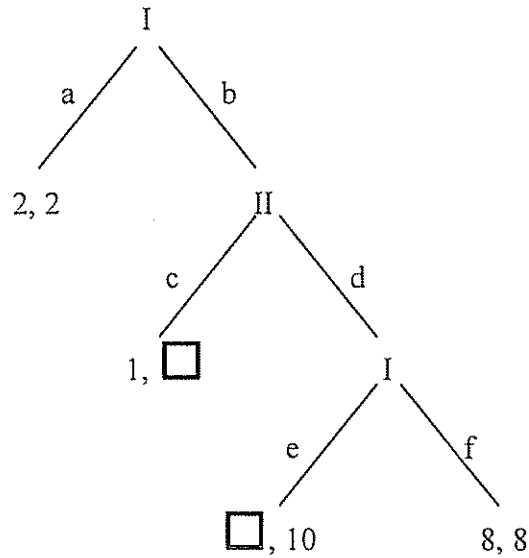
2

	CE	CF	DE	DF
AGJ	* 20, 20+	* 20, 20+	* 30, 15	* 30, 15
AGK	* 20, 20+	* 20, 20+	* 30, 15	* 30, 15
AHJ	5, 10	5, 10	* 30, 15+	* 30, 15+
AHK	5, 10	5, 10	* 30, 15+	* 30, 15+
BGJ	* 20, 15+	0, 0	20, 15+	0, 0
BGK	10, 5	0, 0	10, 5	0, 0
BHJ	* 20, 15+	0, 0	20, 15+	0, 0
BHK	10, 5	0, 0	10, 5	0, 0

PSNE: (AGJ, CE) (AGJ, CF) (AGK, CE) (AGK, CF)
 (AHJ, DE) (AHJ, DF) (AHK, DE) (AHK, DF)
 (BGJ, CE) (BHJ, CE)

SPNE: (AGJ, CE) and (BGJ, CE) ~~shown above~~
 (shown above)

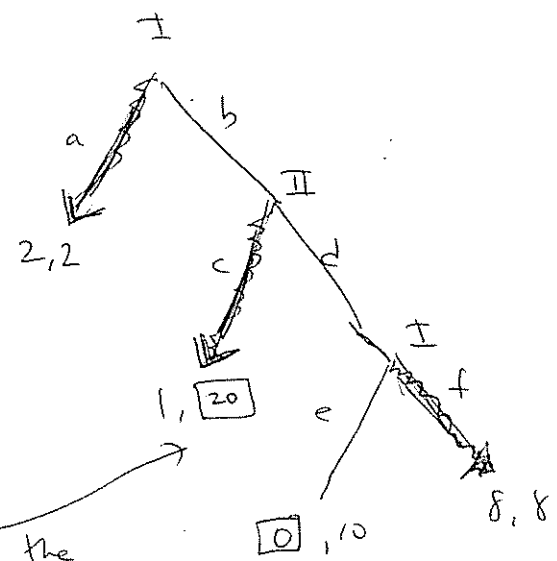
b. Consider the following game.



The payoffs in the boxes are left unspecified. Is it possible to fill in these boxes so that in the subgame perfect Nash equilibrium of the resulting game, both players end up getting payoff 2? (You can put a different number in each box—you don't have to put the same number in both boxes.) If so, please fill in the boxes and show the subgame perfect Nash equilibrium. If not, explain why not. (4 points)

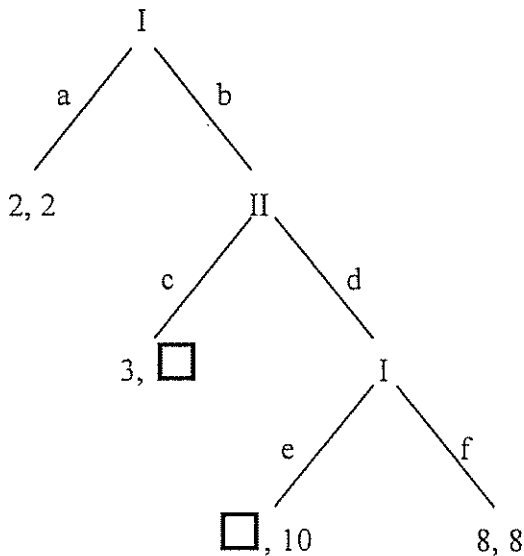
~~If I gets to choose between e and f, it will get at least 8.~~

For example



As long as the payoff here is big enough, then II will play c here. Thus I will play a at the beginning.

c. Now consider this game.



The payoffs in the boxes are left unspecified. Is it possible to fill in these boxes so that in the subgame perfect Nash equilibrium of the resulting game, both players end up getting payoff 2? (You can put a different number in each box—you don't have to put the same number in both boxes.) If so, please fill in the boxes and show the subgame perfect Nash equilibrium. If not, explain why not. (4 points)

It is not possible to fill in the boxes such that I plays a at the start.

If I gets to choose between e and f, he will get at least 8 regardless of what his payoff from e is.

~~Thus if by playing d, II gets at least 8.~~

Thus if II gets to choose between c and d, no matter what happens, I gets at least 3.

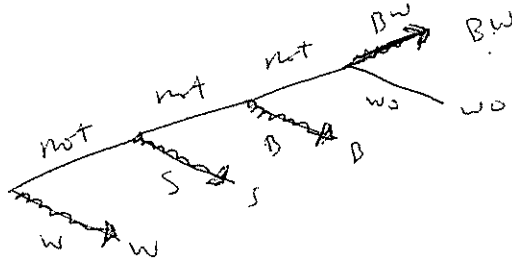
Thus if I plays B at the start, he gets at least 3 no matter what the payoffs in the boxes are.

So I plays a at the start.

Part 5. The Board of Supervisors is the legislative branch of the City of Johnson and consists of 5 members. The Board of Supervisors has decided to designate an official Johnson City Tree for the upcoming town Centennial Celebration. The Board makes decisions by majority rule. A marketing firm has conducted focus groups in Johnson and found that the population likes the following five trees: Buckeye, Willow, Sycamore, White Oak, and Black Walnut. The preferences of the Board of Supervisors are:

	Councilperson 1	Councilperson 2	Councilperson 3	Councilperson 4	Councilperson 5
Favorite Tree	White Oak	Sycamore	Willow	Willow	Black Walnut
	Buckeye	Buckeye	Black Walnut	Sycamore	Sycamore
	Willow	Willow	Buckeye	Buckeye	Buckeye
	Sycamore	Black Walnut	Sycamore	White Oak	Willow
Least Favorite Tree	Black Walnut	White Oak	White Oak	Black Walnut	White Oak

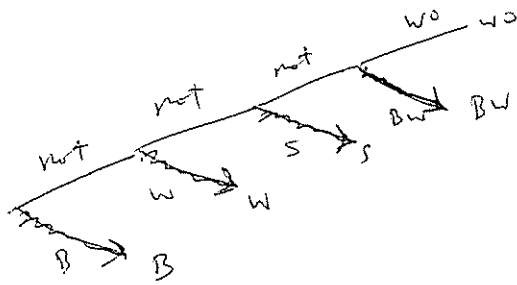
a. The secretary for the board creates the agenda for the meeting. The secretary is a member of the national Arbor Day foundation and is a huge fan of the Willow tree. She wants it to be voted on first and schedules the voting to take place as follows: First the Board votes for the Willow or not. If the Willow loses, then the Board votes for the Sycamore or not. If the Sycamore loses, then the Board votes for the Buckeye or not. If the Buckeye loses, then the Board votes between the White Oak and the Black Walnut. Which outcome does the Board choose? (3 points)



$BW > wo$
 $B > BW$
 $S > B$
 $W > S$

The Board chooses Willow.

b. Now the voting takes place as follows: First the Board votes for the Buckeye or not. If the Buckeye loses, then the Board votes for the Willow or not. If the Willow loses, then the Board votes for the Sycamore or not. If the Sycamore loses, then the Board votes between the White Oak and The Black Walnut. Which outcome does the Board choose? (3 points)



$BW > WO$
 $S > BW$
 $W > S$
 $B > W$

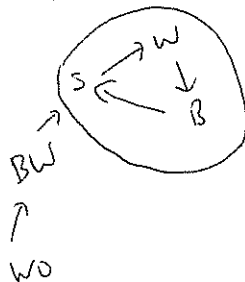
The Board chooses Buckeye.

c. Is there a voting agenda in which the Black Walnut wins? If so, write it out as a tree. If not, explain why not. (3 points)

$BW > WO$

BW beats WO but WO does not beat any other alternative. So there is ~~no~~ no agenda in which BW wins.

d. What is the top cycle in this example? (3 points)



top cycle is $\{W, S, B\}$

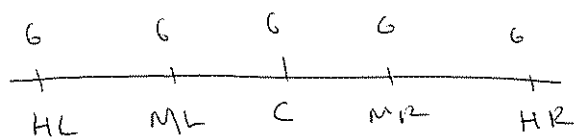
Part 6. There are 30 people in a society. 6 are hard left (HL), 6 are moderately left (ML), 6 are central (C), 6 are moderately right (MR), and 6 are hard right (HR).

There are three candidates in an election. As in the example discussed in class, each voter votes for the candidate who is closest to her own position. If two (or three) candidates are equally far away, then the voters' votes are split equally among the two (or three) candidates who are equally far away.

For example, say candidate 1 takes position C, candidate 2 takes position C, and candidate 3 takes position HL. Then the HL voters vote for candidate 3, the ML voters split equally between all three candidates, and the C, MR, and HR voters split equally between candidates 1 and 2. Candidate 1 thus gets 11 votes, candidate 2 gets 11 votes, and candidate 3 gets 8 votes.

a. Candidate 1 can take any position, HL, ML, C, MR, or HR. Similarly, candidate 2 can take any position, HL, ML, C, MR, or HR. However, Candidate 3 is ideologically committed to the left and chooses only among the positions HL or ML.

Model this as a strategic form game and find all pure strategy Nash equilibria (6 points).



2

	HL	ML	C	MR	HR
HL	10, 10, 10	3, 24, 3	5, 20, 5	6, 18, 6	8, 14, 8
ML	24, 3, 3	12, 12, 6	6, 18, 6	9, 15, 6	12, 12, 6
C	20, 5, 5	18, 6, 6	11, 11, 8	9, 12, 9	12, 9, 9
MR	18, 6, 6	15, 9, 6	12, 9, 9	9, 9, 12	12, 6, 12
HR	14, 8, 8	12, 12, 6	9, 12, 9	6, 12, 12	8, 8, 14

2

	HL	ML	C	MR	HR
HL	3, 3, 24	6, 12, 12	6, 18, 6	6, 15, 9	6, 12, 12
ML	12, 6, 12	10, 10, 10	6, 18, 6	8, 14, 8	9, 12, 9
C	18, 6, 6	18, 6, 6	9, 9, 12	6, 12, 12	9, 9, 12
MR	15, 6, 9	14, 8, 8	12, 6, 12	8, 8, 14	9, 6, 15
HR	12, 6, 12	12, 9, 9	9, 9, 12	6, 9, 15	6, 6, 18

3: HL

3: ML

pure strategy NE: (MR, MR, ML)

note that for 3, ML w. dominates HL

b. Now, Candidate 1 is committed to the left and chooses only among HL or ML. Candidate 2 is committed to the right and chooses only among MR and HR. Candidate 3 can take any position.

Model this as a strategic form game and find all pure strategy Nash equilibria (6 points).

2

	MR	HR
HL	6, 18, 6	8, 14, 8
ML	9, 15, 6	12, 12, 6

3: HL

	MR	HR
HL	6, 15, 9	6, 12, 12
ML	8, 14, 8	9, 12, 9

3: ML

	MR	HR
HL	9, 12, 9	9, 9, 12
ML	12, 12, 6	12, 9, 9

3: C

	MR	HR
HL	12, 9, 9	12, 6, 12
ML	14, 8, 8	15, 6, 9

3: MR

	MR	HR
HL	12, 9, 9	12, 6, 12
ML	12, 12, 6	15, 6, 9

3: MR

	MR	HR
HL	12, 12, 6	14, 8, 8
ML	15, 9, 6	18, 6, 6

3: HR

PS NE : (ML, MR, ML)
~~(HL, MR, MR)~~
 (ML, MR, MR)

Part 7. As the new First Lady, Michelle Obama is charged with the task of being down with pop cultures across the world. She invites 10 famous people to the inaugural ball. She needs to match the men and women together to create five heterosexual dance couples. The men invited are French footballer Zinedine Zidane, rapper Talib Kweli, Korean superstar Rain, R. Kelly, and Israeli spy/hairstylist Zohan. The women invited are Mariah Carey, 13-year-old Chinese gymnast He Kexin, Benin superstar Angelique Kidjo, Venus Williams, and Judi Dench. Their preferences are as follows:

	Zidane	Kweli	Rain	R. Kelly	Zohan
Best	Venus	Kidjo	Mariah	He	Mariah
	Kidjo	Venus	He	Mariah	Kidjo
	Dench	Mariah	Kidjo	Kidjo	Venus
	He	He	Venus	Venus	He
Worst	Mariah	Dench	Dench	Dench	Dench

	Mariah	He	Kidjo	Venus	Dench
Best	Kweli	Rain	Kweli	Zidane	Zidane
	Zohan	Zohan	Zidane	Kweli	Zohan
	Zidane	Kweli	Rain	Zohan	Kweli
	Rain	Zidane	Zohan	Rain	Rain
Worst	R. Kelly	R. Kelly	R. Kelly	R. Kelly	R. Kelly

a. Does there exist a stable match in which R. Kelly is paired with He Kexin and Zohan is paired with Mariah Carey? If so, write down the stable match. If not, specify which of the five men and which of the five women would form a new couple and thus successfully deviate from any match in which R. Kelly is paired with He Kexin and Zohan is paired with Mariah Carey. (4 points)

~~(TK=HK, Z=MC, ...)~~ stable?

If Zohan is paired with Mariah, then Rain must be paired with someone other than Mariah.

Thus He Kexin would like to dump R. Kelly and join with Rain. Rain will accept He Kexin because he prefers her over all women other than Mariah.

b. Find the stable matching that makes the men as happy as possible. (4 points)

- (Zidane Venus, Kweli Kidjo, Ravi Mariah, RK He, Zohan Mariah) Mariah chooses Zohan
- (Zidane Venus, Kweli Kidjo, Ravi He, RK He, Zohan Mariah) He chooses Ravi
- (Zidane Venus, Kweli Kidjo, Ravi He, RK Mariah, Zohan Mariah) Mariah chooses Zohan
- (Zidane Venus, Kweli Kidjo, Ravi He, RK Kidjo, Zohan Mariah) Kidjo chooses Kweli
- (Zidane Venus, Kweli Kidjo, Ravi He, RK Venus, Zohan Mariah) Venus chooses Zidane
- (Zidane Venus, Kweli Kidjo, Ravi He, RK Deuch, Zohan Mariah)

stable because all women get exactly one offer

c. Find the stable matching that makes the women as happy as possible. (4 points)

- (Mariah Kweli, He Ravi, Kidjo Kweli, Venus Zidane, Deuch Zidane) Zidane chooses Venus
- ~~(Mariah Kweli, He Ravi, Kidjo Kweli, Venus Zidane, Deuch Zohan) Zohan chooses~~
- (Mariah Zohan, He Ravi, Kidjo Kweli, Venus Zidane, Deuch Zohan) Zohan chooses Mariah
- (Mariah Zohan, He Ravi, Kidjo Kweli, Venus Zidane, Deuch Kweli) Kweli chooses Kidjo
- (Mariah Zohan, He Ravi, Kidjo Kweli, Venus Zidane, Deuch Ravi) Ravi chooses He
- (Mariah Zohan, He Ravi, Kidjo Kweli, Venus Zidane, Deuch RK)

stable because all men get no offer

Part 8. Seven students are deciding whether to go to their TA's weekly review session. A student wants to participate if the number of other students who participate is greater or equal to the student's threshold. Suppose that student A has threshold 1, students B and C have threshold 2, students D and E have threshold 4, and students F and G have threshold 6.

a. Find all pure strategy Nash equilibria. You should write up the equilibria like (p, n, n, p, p, n, n) for example (student A participates, student B does not, student C does not, students D and E participate, and students F and G do not). Please use the "graph paper" below if you need to. (4 points)

th

A	1	n	p			p	p												
B	2	n	p			p	p												
C	2	n	p			p	p												
D	4	n	p			n	p												
E	4	n	p			n	p												
F	6	n	p			n	n												
G	6	n	p			n	n												

NE:

- (n, n, n, n, n, n, n) no one participates
- (p, p, p, n, n, n, n) the threshold 1 + 2 people participate
- (p, p, p, p, p, n, n) everyone except the threshold 6 people participate
- (p, p, p, p, p, p, p) everyone participates

b. Now the TA switches the review session from 8pm to 5pm, which is more convenient for all students. Thus all students' thresholds decrease by 1.

However, some students prefer not having so many other students attend, since it makes the session more crowded. In addition to their "lower thresholds," the students now have "upper thresholds." Student A wants to participate if no one else comes, or if exactly one other person comes, but if two or more other students come, she does not want to. Students B and C want to participate if one or more other students participate, but do not if three or more other students come. Students D and E want to participate if three or more other students come, but do not if six or more other students come. Students F and G want to participate if five or more other students come.

Find all pure strategy Nash equilibria. You should write up the equilibria like (p, n, n, p, p, n, n) for example (student A participates, student B does not, student C does not, students D and E participate, and students F and G do not). If there are no pure strategy Nash equilibria, explain why. Please use the "graph paper" below if you need to. (4 points)

LT UT

A	0	1																		
B	1	2																		
C	1	2																		
D	3	5																		
E	3	5																		
F	5																			
G	5																			

If A, B, C, D, E all participate, then A will drop out. So this will never happen. Thus F+G will never participate.

A participates only alone or with one other person. If she goes alone, B+C will jump in. It's not possible for only one other person to participate - if B participates, so will C. So A never participates.

Thus the only issue is whether ~~B+C~~ B+C will participate and whether D+E will.

It is easy to check that (n, p, p, n, n, n, n)

~~and (n, p, p, p, p, n, n)~~

is ~~the~~ the only NE.

c. Now go back to the situation in part a. In other words, student A has threshold 1, students B and C have threshold 2, students D and E have threshold 4, and students F and G have threshold 6.

Now say that any pair of students can have lunch together. When two students have lunch together, the student with the lower threshold becomes more conservative, and her threshold becomes the threshold of her lunch partner. The student with the higher threshold does not change. For example, if student B and student G have lunch, then afterward student B has threshold 6 and student G has threshold 6.

Is it possible that after a series of lunches, the students will become so conservative that the only pure strategy Nash equilibrium is (n, n, n, n, n, n, n) , in which no one participates? If so, write down the series of lunches, detailing which pairs have lunch together. If not, explain why not. Please use the "graph paper" below if you need to. (4 points)

A	1																							
B	2																							
C	2																							
D	4																							
E	4																							
F	6																							
G	6																							

It is not possible. The worst that can happen after many ~~lunches~~ lunches is that everyone will have threshold 6. Even then, (P, P, P, P, P, P, P) is a NE.