

Final exam PS 30 December 2008

Name:

UID:

TA and section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are eight parts in this exam. Each part is weighted equally (12 points for each part). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If the TA responsible for a given question is not in the room at the time, work on other parts of the exam and hold the question until that TA rotates to your exam location. When the end of the exam is announced, please stop working immediately. People who continue working after the end of the exam is announced will have their grades penalized by 25 percent. If you need to leave the room to use the bathroom during the exam, please write your name down on the bathroom log before you leave. A person cannot leave the room more than once during the exam (a person who leaves for a second time will be considered to have completed his or her exam). Please turn in your exam to one of the TAs. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Please turn off all cell phones and other electronic gadgets. Good luck!

1	
2	
3	
4	
5	
6	
7	
8	
total	

Part 1. Examine the following game. Player 1 can choose among alternatives W, X, Y, and Z, and Player 2 can choose among alternatives A, B, and C.

		Player 2		
		A	B	C
Player 1	W	4,2	3,4	11,3
	X	6,6	4,1	7,5
	Y	3,3	6,5	13,4
	Z	6,7	7,6	10,8

a. Find all pure strategy Nash equilibria of this game (4 points).

b. By iteratively eliminating strongly or weakly dominated strategies, eliminate as many strategies as possible. Please show the order of elimination (4 points).

c. Find all mixed strategy Nash equilibria of this game (4 points).

Part 2. Two candidates, Hilton and O'Hara, are competing to be the Democratic Party's presidential nominee. Two states, Brewery and Winery, are considering whom to vote for at the party convention. Brewery prefers Hilton and Winery prefers O'Hara.

Because the party is facing a general election soon, party unity is very important. The party is united if both states vote for the same candidate. The best outcome for a state is for the party to be united around the state's preferred candidate (payoff = 4). For example, the best outcome for Winery is for both states to vote for O'Hara.

The second best outcome is being united under the candidate whom they don't prefer (payoff = 3). The second worst case is when each state votes for its own preferred candidate and their votes are split (payoff = 2). The absolute worst case is when each state votes for its not-preferred candidate and their votes are split (payoff = 1).

a. Say that the states choose on the same day. Represent this as a strategic form game and find all pure strategy Nash equilibria and mixed strategy Nash equilibria. (2 points)

b. Now say that Brewery's primary is in September and Winery's primary is in October. In other words, Brewery chooses first. Model this as an extensive form game and find all subgame perfect Nash equilibria. For clarity's sake, write payoffs as (Brewery, Winery). (2 points)

c. Now say that Winery's primary is earlier and so Winery chooses first. Model this as an extensive form game and find all subgame perfect Nash equilibria. For clarity's sake, write payoffs as (Brewery, Winery). (2 points)

d. Say again that Brewery chooses first. However, now Winery cares more about voicing its true opinion. In addition to its payoffs from before, Winery has an additional payoff e when it votes for O'Hara, no matter who is finally chosen as the party's nominee, where e is positive. Model this as an extensive form game. For clarity's sake, write payoffs as (Brewery, Winery). How large must e be for O'Hara to be chosen as the party's nominee? (3 points)

e. Now say that national party leaders decide which state will have its primary first. The party leaders prefer Hilton, who is better-known to the public. Thus, the party leaders' top preference (payoff 3) is being united under Hilton, their second preference (payoff 2) is being united under O'Hara, and their worst case (payoff 1) is having the party split. Represent this as an extensive form game and find all subgame perfect Nash equilibria. For clarity's sake, write payoffs as (Party leaders, Brewery, Winery). Who will be the party nominee? (3 points)

Part 3. Consider the following game, where X and Y are varying payoffs.

		Player 2	
		A	B
Player 1	A	3, 2	X, 3
	B	X, Y	6, 4

a. In the grid below, X ranges from 1.5 to 8.5 and Y ranges from 1.5 to 8.5. For each box in the grid, write “Yes” if the game has exactly two pure strategy Nash equilibria and one mixed strategy Nash equilibrium. Otherwise, write “No.”

For example, the shaded box corresponds to $X = 1.5$ and $Y = 3.5$. If the game has exactly two pure strategy Nash equilibria and one mixed strategy Nash equilibrium when $X = 1.5$ and $Y = 3.5$, then you would write “Yes” in the shaded box. Otherwise, you would write “No.”

Note that every box below should be filled in with either “Yes” or “No.” Please justify your answer. (4 points)

Y = 8.5								
Y = 7.5								
Y = 6.5								
Y = 5.5								
Y = 4.5								
Y = 3.5								
Y = 2.5								
Y = 1.5								
	X = 1.5	X = 2.5	X = 3.5	X = 4.5	X = 5.5	X = 6.5	X = 7.5	X = 8.5

Here is the game again:

		Player 2	
		A	B
Player 1	A	3, 2	X, 3
	B	X, Y	6, 4

b. In the grid below, X ranges from 1.5 to 8.5 and Y ranges from 1.5 to 8.5. For each box in the grid, write “Yes” if the game has no pure strategy Nash equilibria and exactly one mixed strategy Nash equilibrium. Otherwise, write “No.”

For example, the shaded box corresponds to $X = 1.5$ and $Y = 3.5$. If game has no pure strategy Nash equilibria and exactly one mixed strategy Nash equilibrium when $X = 1.5$ and $Y = 3.5$, then you would write “Yes” in the shaded box. Otherwise, you would write “No.”

Note that every box below should be filled in with either “Yes” or “No.” Please justify your answer. (4 points)

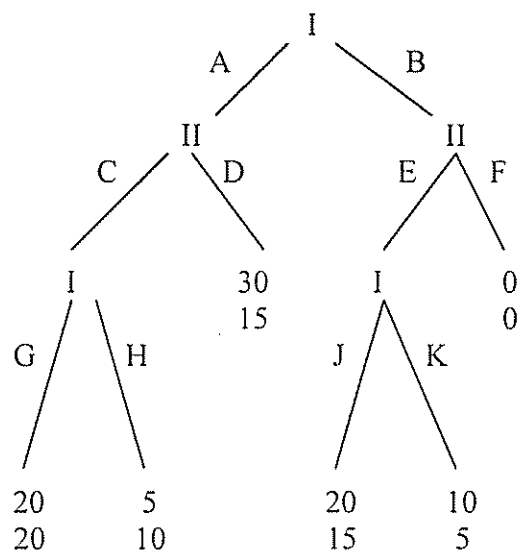
Y = 8.5									
Y = 7.5									
Y = 6.5									
Y = 5.5									
Y = 4.5									
Y = 3.5									
Y = 2.5									
Y = 1.5									
	X = 1.5	X = 2.5	X = 3.5	X = 4.5	X = 5.5	X = 6.5	X = 7.5	X = 8.5	

Here is the game again:

		Player 2	
		A	B
Player 1	A	3, 2	X, 3
	B	X, Y	6, 4

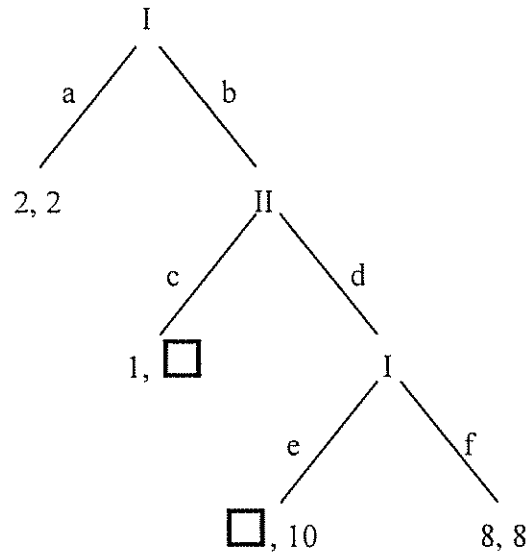
c. Find the values of X and Y such that the following is a mixed strategy Nash equilibrium: Player 1 plays A with probability $1/4$ and B with probability $3/4$, and Player 2 plays A with probability $4/11$ and B with probability $7/11$. (4 points)

Part 4. Consider the following extensive form game. There are two players, Player I and Player II. For example, if Player I plays A and then Player II plays C, and then Player I plays H, then Player I gets a payoff of 5 and Player II gets a payoff of 10.



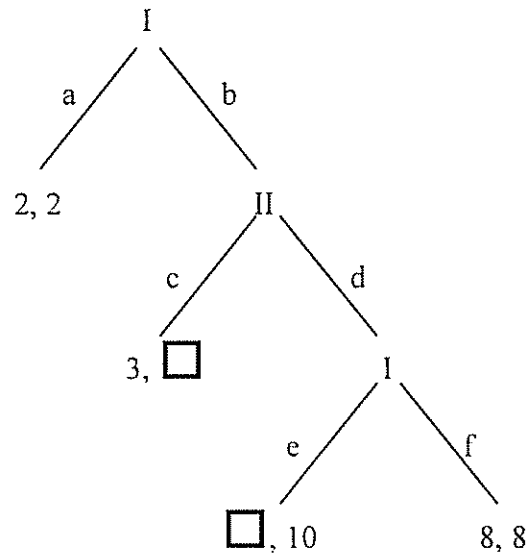
a. Find all pure strategy Nash equilibria of this game. Find all subgame perfect Nash equilibria of this game. (4 points)

b. Consider the following game.



The payoffs in the boxes are left unspecified. Is it possible to fill in these boxes so that in the subgame perfect Nash equilibrium of the resulting game, both players end up getting payoff 2? (You can put a different number in each box—you don't have to put the same number in both boxes.) If so, please fill in the boxes and show the subgame perfect Nash equilibrium. If not, explain why not. (4 points)

c. Now consider this game.



The payoffs in the boxes are left unspecified. Is it possible to fill in these boxes so that in the subgame perfect Nash equilibrium of the resulting game, both players end up getting payoff 2? (You can put a different number in each box—you don't have to put the same number in both boxes.) If so, please fill in the boxes and show the subgame perfect Nash equilibrium. If not, explain why not. (4 points)

Part 5. The Board of Supervisors is the legislative branch of the City of Johnson and consists of 5 members. The Board of Supervisors has decided to designate an official Johnson City Tree for the upcoming town Centennial Celebration. The Board makes decisions by majority rule. A marketing firm has conducted focus groups in Johnson and found that the population likes the following five trees: Buckeye, Willow, Sycamore, White Oak, and Black Walnut. The preferences of the Board of Supervisors are:

	Councilperson 1	Councilperson 2	Councilperson 3	Councilperson 4	Councilperson 5
Favorite Tree	White Oak	Sycamore	Willow	Willow	Black Walnut
	Buckeye	Buckeye	Black Walnut	Sycamore	Sycamore
	Willow	Willow	Buckeye	Buckeye	Buckeye
	Sycamore	Black Walnut	Sycamore	White Oak	Willow
Least Favorite Tree	Black Walnut	White Oak	White Oak	Black Walnut	White Oak

a. The secretary for the board creates the agenda for the meeting. The secretary is a member of the national Arbor Day foundation and is a huge fan of the Willow tree. She wants it to be voted on first and schedules the voting to take place as follows: First the Board votes for the Willow or not. If the Willow loses, then the Board votes for the Sycamore or not. If the Sycamore loses, then the Board votes for the Buckeye or not. If the Buckeye loses, then the Board votes between the White Oak and the Black Walnut. Which outcome does the Board choose? (3 points)

b. Now the voting takes place as follows: First the Board votes for the Buckeye or not. If the Buckeye loses, then the Board votes for the Willow or not. If the Willow loses, then the Board votes for the Sycamore or not. If the Sycamore loses, then the Board votes between the White Oak and The Black Walnut. Which outcome does the Board choose? (3 points)

c. Is there a voting agenda in which the Black Walnut wins? If so, write it out as a tree. If not, explain why not. (3 points)

d. What is the top cycle in this example? (3 points)

Part 6. There are 30 people in a society. 6 are hard left (HL), 6 are moderately left (ML), 6 are central (C), 6 are moderately right (MR), and 6 are hard right (HR).

There are three candidates in an election. As in the example discussed in class, each voter votes for the candidate who is closest to her own position. If two (or three) candidates are equally far away, then the voters' votes are split equally among the two (or three) candidates who are equally far away.

For example, say candidate 1 takes position C, candidate 2 takes position C, and candidate 3 takes position HL. Then the HL voters vote for candidate 3, the ML voters split equally between all three candidates, and the C, MR, and HR voters split equally between candidates 1 and 2. Candidate 1 thus gets 11 votes, candidate 2 gets 11 votes, and candidate 3 gets 8 votes.

a. Candidate 1 can take any position, HL, ML, C, MR, or HR. Similarly, candidate 2 can take any position, HL, ML, C, MR, or HR. However, Candidate 3 is ideologically committed to the left and chooses only among the positions HL or ML.

Model this as a strategic form game and find all pure strategy Nash equilibria (6 points).

b. Now, Candidate 1 is committed to the left and chooses only among HL or ML. Candidate 2 is committed to the right and chooses only among MR and HR. Candidate 3 can take any position.

Model this as a strategic form game and find all pure strategy Nash equilibria (6 points).

Part 7. As the new First Lady, Michelle Obama is charged with the task of being down with pop cultures across the world. She invites 10 famous people to the inaugural ball. She needs to match the men and women together to create five heterosexual dance couples. The men invited are French footballer Zinedine Zidane, rapper Talib Kweli, Korean superstar Rain, R. Kelly, and Israeli spy/hairstylist Zohan. The women invited are Mariah Carey, 13-year-old Chinese gymnast He Kexin, Benin superstar Angelique Kidjo, Venus Williams, and Judi Dench. Their preferences are as follows:

	Zidane	Kweli	Rain	R. Kelly	Zohan
Best	Venus	Kidjo	Mariah	He	Mariah
	Kidjo	Venus	He	Mariah	Kidjo
	Dench	Mariah	Kidjo	Kidjo	Venus
	He	He	Venus	Venus	He
Worst	Mariah	Dench	Dench	Dench	Dench

	Mariah	He	Kidjo	Venus	Dench
Best	Kweli	Rain	Kweli	Zidane	Zidane
	Zohan	Zohan	Zidane	Kweli	Zohan
	Zidane	Kweli	Rain	Zohan	Kweli
	Rain	Zidane	Zohan	Rain	Rain
Worst	R. Kelly	R. Kelly	R. Kelly	R. Kelly	R. Kelly

a. Does there exist a stable match in which R. Kelly is paired with He Kexin and Zohan is paired with Mariah Carey? If so, write down the stable match. If not, specify which of the five men and which of the five women would form a new couple and thus successfully deviate from any match in which R. Kelly is paired with He Kexin and Zohan is paired with Mariah Carey. (4 points)

b. Find the stable matching that makes the men as happy as possible. (4 points)

c. Find the stable matching that makes the women as happy as possible. (4 points)

Part 8. Seven students are deciding whether to go to their TA's weekly review session. A student wants to participate if the number of other students who participate is greater or equal to the student's threshold. Suppose that student A has threshold 1, students B and C have threshold 2, students D and E have threshold 4, and students F and G have threshold 6.

a. Find all pure strategy Nash equilibria. You should write up the equilibria like (p, n, n, p, p, n, n) for example (student A participates, student B does not, student C does not, students D and E participate, and students F and G do not). Please use the "graph paper" below if you need to. (4 points)

