

Answers to

Final exam

PS 30

December 2007

Name:

TA:

Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are eight parts in this exam. Each part is weighted equally (12 points for each part). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If the TA responsible for a given question is not in the room at the time, work on other parts of the exam and hold the question until that TA rotates to your exam location. When the end of the exam is announced, please stop working immediately. People who continue working after the end of the exam is announced will have their grades penalized by 25 percent. If you need to leave the room to use the bathroom during the exam, please write your name down on the bathroom log before you leave. A person cannot leave the room more than once during the exam (a person who leaves for a second time will be considered to have completed his or her exam). Please turn in your exam to one of the TAs. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Please turn off all cell phones and other electronic gadgets. Good luck!

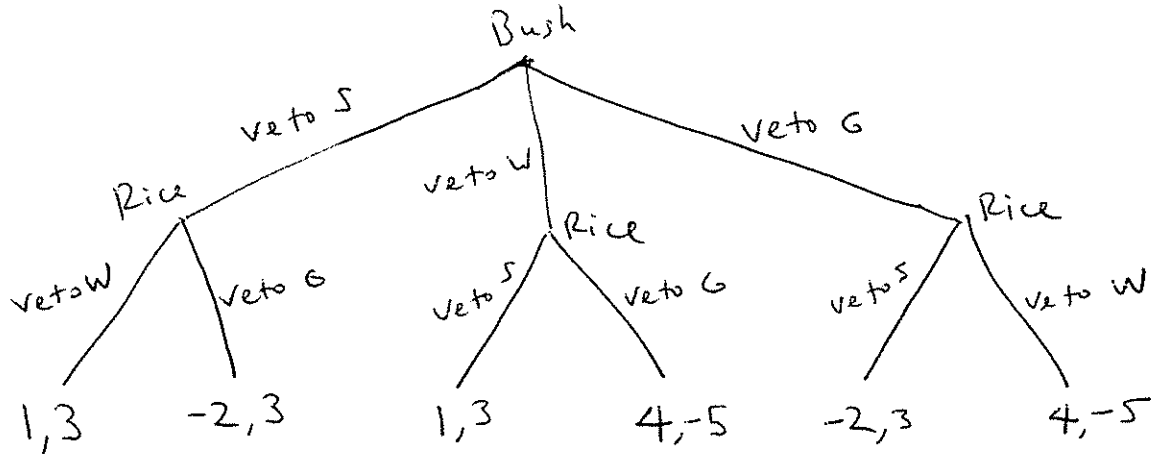
1	
2	
3	
4	
5	
6	
7	
8	
total	

Part 1. President Bush and Secretary of State Rice choose a policy on Iraq.

There are three policies: stay in Iraq forever (S), withdraw troops gradually (W), or get out of Iraq immediately (G). Bush and Rice choose the policy in the following manner. First, Bush vetoes a policy and thus leaves two remaining policies. Then, Rice vetoes one of the two remaining policies. The policy left after these two vetoes is the policy which is chosen.

Bush likes S the best, G second best, and likes W the least. Rice is indifferent between G and W, and likes either better than S.

a. Model this situation as an extensive form game. For Bush, assign a payoff of 4 to the best outcome, 1 to the second best and -2 to the worst. For Rice, assign 3 to the best and -5 to the worst. (2 points)



b. Convert the extensive form game into a strategic form game. (4 points)

if Bush vetoes S
if Bush vetoes W
if Bush vetoes G

Rice

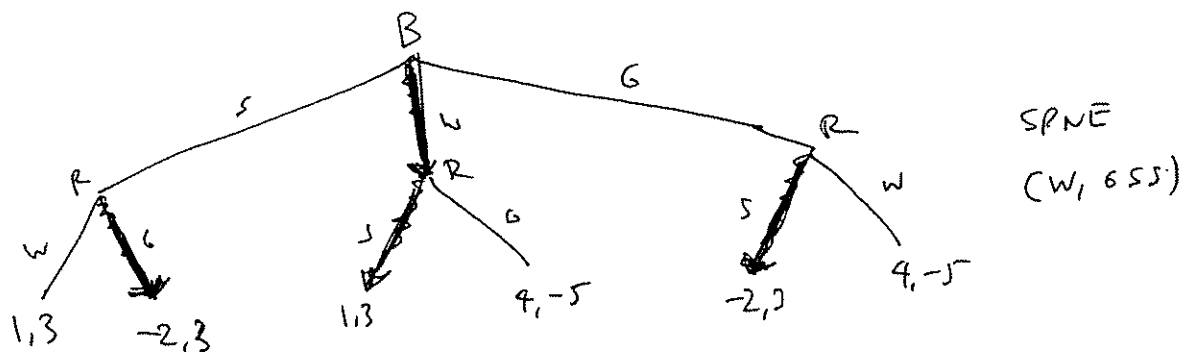
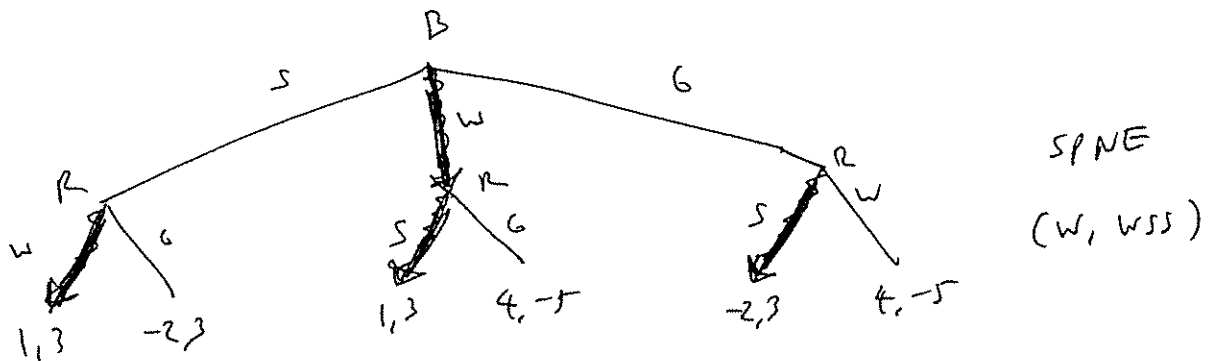
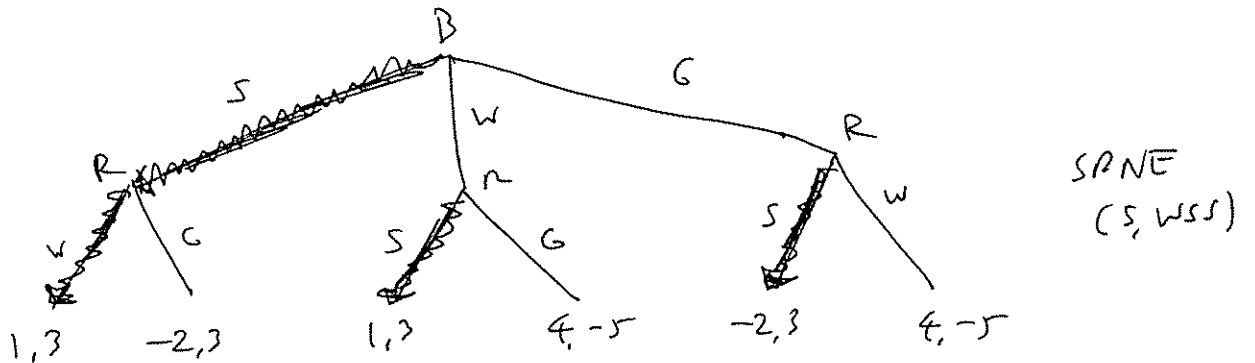
		WS S	WS W	WS G	WG W	GS S	GS W	GS G	GW W
Bush	S	*1, 3 ⁺	1, 3 ⁺	1, 3 ⁺	1, 3 ⁺	-2, 3 ⁺	-2, 3 ⁺	-2, 3 ⁺	-2, 3 ⁺
	W	*1, 3 ⁺	1, 3 ⁺	*4, -5	*4, -5	*1, 3 ⁺	1, 3 ⁺	*4, -5	*4, -5
	G	-2, 3 ⁺	*4, -5	-2, 3 ⁺	*4, -5	-2, 3 ⁺	*4, -5	-2, 3 ⁺	*4, -5

c. Find all pure strategy Nash equilibria of the game. (2 points)

From the * and + in the above game, we see that

the NE are (S, WSS)
 (W, WSS)
 (W, GSS)

d. Draw all subgame perfect Nash equilibria of the game. (Draw them using arrows in a game tree.) (4 points)



Part 2. There are three men A, B, and C and three women X, Y, and Z. Each person is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman Z best, then Y second-best, then X worst. Man B prefers woman Y best, then Z, then X worst. Man C prefers woman X best, then Y, then Z worst. Woman X prefers man A best, then B, then C worst. Woman Y prefers man A best, then C, then B worst. Finally, woman Z prefers man C best, then B, then A worst.

a. Write down all possible matchings and determine which of them are stable and which are not stable. (4 points)

(AX, BY, CZ) not stable - A and Y want to get together
 (AX, BZ, CY) not stable - A and Y want to get together
 (AY, BX, CZ) stable
 (AY, BZ, CX) stable
 (AZ, BX, CY) not stable - B and Z want to get together
 (AZ, BY, CX) stable

A	B	C
Z	Y	X
Y	Z	Y
X	X	Z

X	Y	Z
A	A	C
B	C	B
C	B	A

b. Among the set of stable matchings, which matching is most preferred by the women?
 Among the set of stable matchings, which matching is most preferred by the men? (4 points)

(AY, BX, CZ) most preferred by women
 (AZ, BY, CX) " " men.

c. Now say that there are four men, A, B, C, and D, and four women W, X, Y, and Z.
 Man A's preferences (from best to worst) are W, Y, Z, X. Man B's preferences are X, Z, W, Y.
 Man C's preferences are Y, W, Z, X. Man D's preferences are Y, W, X, Z.
 Woman W's preferences are B, A, D, C. Woman X's preferences are B, D, C, A.
 Woman Y's preferences are A, B, C, D. Woman Z's preferences are A, D, C, B. Find the set of all stable matchings. (4 points)

Men-first algorithm:

(AW, BX, CY, D~~Y~~) D rejected
 (AW, BX, C~~Y~~, DW) D rejected
 (AW, BX, C~~Y~~, DX) D rejected
 (AW, BX, CY, DZ) stable match. best for men, worst for women

A	B	C	D	W	X	Y	Z
W	X	Y	Y	B	B	A	A
Y	Z	W	W	A	D	B	D
Z	W	Z	X	D	C	C	C
X	Y	X	Z	C	A	D	B

Women-first algorithm:

(WB, XD, YA, ZA) W rejected, Z rejected
 (WA, XB, YA, ZD) Y rejected
 (WA, XB, YB, ZD) Y rejected
 (WA, XB, YC, ZD) stable match, best for women, worst for men

Since the stable match best for men and the stable match worst for men are the same, there is only one stable match: (AW, BX, CY, DZ).

Another way to solve this problem is to first realize that since B and X are each other's first choices, they must be paired. Given this, A and W must be paired since W is A's first choice and the only person W likes.

better than A is B, who is certainly matched with X. Then the only action is between C and D and Y and Z. C and D both like Y better than Z, and Y prefers C, so C and Y must match. Z and D are stuck with each other.

Another way to do this problem is to look at all 24 possible matchings. This is very time-consuming.

Part 3. The North Korean Refugee problem is getting serious attention from international society. Refugees from North Korea frequently cross borders into China, Russia, and even Southeast Asian countries. Since North Korean refugees have not obtained "refugee" status as stipulated in international law, they live under the threat of unwanted repatriation back to North Korea. Hence, for humanitarian purposes, the United Nations tries to pass "North Korean Refugee Resolution No. 1" to set up refugee camps in neighboring countries in order to protect them from local threats and also enable them to freely choose their future destinations.

The UN Security Council, which is composed of 5 Permanent members and 10 temporary members, votes on this issue. To pass the resolution, 9 or more members of the Security Council must vote "yes". However, a "no" vote, or "veto," by any of the five permanent members prevents adoption of a resolution, even if it has received the required number of "yes" votes. Permanent members of the Security Council are the U.S, Great Britain, France, Russia and China.

a. Model this as a threshold model, where the thresholds of each of the 15 Security Council member countries are shown below. Assume that the initial state is that no one supports the resolution. Will the resolution eventually be adopted in the Security Council? You can use the following table as a worksheet if you like. (4 points)

	Threshold	Initial	t=1	t=2	t=3	t=4	t=5	t=6
Belgium	1	N		Y	Y	Y	Y	Y
Burkina Faso	9	N					Y	Y
China	14	N						
Costa Rica	9	N					Y	Y
Croatia	5	N				Y	Y	Y
France	1	N		Y	Y	Y	Y	Y
Great Britain	1	N		Y	Y	Y	Y	Y
Indonesia	3	N			Y	Y	Y	Y
Italy	3	N			Y	Y	Y	Y
Libya	5	N				Y	Y	Y
Panama	9	N					Y	Y
Russia	14	N						
South Africa	5	N				Y	Y	Y
U.S	0	N	Y	Y	Y	Y	Y	Y
Vietnam	3	N			Y	Y	Y	Y

Not written for clarity

total

1 4 7 10 13 13
threshold 0 starts
threshold 1 jump in
threshold 3 jump in
threshold 5 jump in
threshold 9 jump in
no more

The "snowball" eventually includes 13 countries, but China + Russia don't jump in. The resolution is not adopted.

b. Now say that the thresholds of China and Russia change: China's threshold is x and Russia's threshold is $x-1$. Everyone else's thresholds stay the same. Again assume that the initial state is that no one supports the resolution.

	Threshold	Initial	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	
Belgium	1	N		Y	Y	Y	Y		
Burkina Faso	9	N					Y		
China	x	N							
Costa Rica	9	N					Y		
Croatia	5	N				Y	Y		
France	1	N		Y	Y	Y	Y		
Great Britain	1	N		Y	Y	Y	Y		
Indonesia	3	N			Y	Y	Y		
Italy	3	N			Y	Y	Y		
Libya	5	N				Y	Y		
Panama	9	N					Y		
Russia	$x-1$	N							
South Africa	5	N				Y	Y		
U.S	0	N	Y	Y	Y	Y	Y		
Vietnam	3	N			Y	Y	Y		

Circle the values of x below for which the Security Council adopts the resolution. (4 points)

- $x=1$
- $x=2$
- $x=3$
- $x=4$
- $x=5$
- $x=6$
- $x=7$
- $x=8$
- $x=9$
- $x=10$
- $x=11$
- $x=12$
- $x=13$
- $x=14$
- $x=15$

Regardless of what x is, all ¹³ countries except China + Russia will join by period $t=5$. If $x=14$, then Russia will join in at $t=6$ and China will join at $t=7$. If $x=15$, China will never join. If $x=13$ or lower, Russia will join at $t=6$ or earlier, and so will China.

c. Now say that the thresholds of Croatia and France also change: their thresholds are both x . Like before, China's threshold is x and Russia's threshold is $x-1$. Everyone else's thresholds stay the same. Again assume that the initial state is that no one supports the resolution.

	Threshold	Initial	$t=1$	$t=2$	$t=3$	$t=4$			
Belgium	1	N		Y	Y	Y			
Burkina Faso	9	N							
China	x	N							
Costa Rica	9	N							
Croatia	x	N							
France	x	N							
Great Britain	1	N		Y	Y	Y			
Indonesia	3	N			Y	Y			
Italy	3	N			Y	Y			
Libya	5	N				Y			
Panama	9	N							
Russia	$x-1$	N							
South Africa	5	N				Y			
U.S	0	N	Y	Y	Y	Y			
Vietnam	3	N			Y	Y			

Circle the values of x below for which the Security Council adopts the resolution. (4 points)

- $x=1$
- $x=2$
- $x=3$
- $x=4$
- $x=5$
- $x=6$
- $x=7$
- $x=8$
- $x=9$
- $x=10$
- $x=11$
- $x=12$
- $x=13$
- $x=14$
- $x=15$

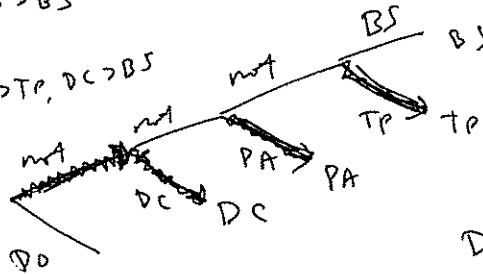
Regardless of x , the ~~five~~^{eight} countries above will join in by $t=4$.
 If $x=9$, then Russia will join at $t=5$ and everyone else will join at $t=6$.
 If $x=8$ or lower, then Russia and the others will join at the same time or earlier.
 If $x=10$, then Russia and the others will never join in.
 or higher

Part 4. It's 2052 and Borat becomes the president of Khazeetan. His cabinet, consisting of five members, is voting to see whom to give the "Greatest Person in the World" medal to. The candidates are Borat's producer (PO), Pomel-a Anderson (PA), Dave Chappelle (DC), Tony Parker (TP), and Britney Spears (BS). The cabinet makes decisions by majority rule. The cabinet's preferences are as follows:

	1	2	3	4	5
Best	PO	PA	DC	DC	PO
	DC	DC	PO	PA	PA
	PA	PO	PA	PO	TP
	BS	TP	TP	BS	DC
Worst	TP	BS	BS	TP	BS

a. Say that the agenda is as follows. First the cabinet votes on PO or not. If PO loses, then the cabinet votes on DC or not. If DC loses, then the cabinet votes on PA or not. If PA loses, then the cabinet votes between BS and TP. Note that the order of voting corresponds to Person 1's preferences (starting from Person 1's most preferred to his least preferred). Which outcome does the cabinet choose? Does Person 1 get his most favored candidate approved? (2 points)

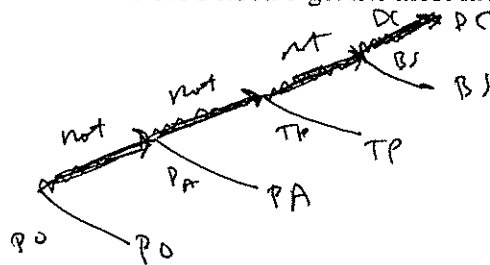
$PO > PA, PO > TP, PO > BS$
 $PA > TP, PA > BS$
 $DC > PO, DC > PA, DC > TP, DC > BS$
 $TP > BS$



DC is chosen

Person 1 doesn't get PO approved.

b. Now say that the agenda is as follows. First the cabinet votes on PO or not. If PO loses, then the cabinet votes on PA or not. If PA loses, then the cabinet votes on TP or not. If TP loses, then the cabinet votes between DC and BS. Note that the order of voting corresponds to Person 5's preferences (starting from Person 5's most preferred to his least preferred). Which outcome does the cabinet choose? Does Person 5 get her most favored candidate approved? (2 points)



DC is chosen

Person 5 doesn't get PO approved.

c. Is there any agenda in which the cabinet chooses PO? If so, write down the agenda. If not, explain why not. (2 points)

PO is never chosen because DC beats it
and no other candidate beats DC.

d. Is there any agenda in which the cabinet chooses BS? If so, write down the agenda. If not, explain why not. (2 points)

BS is never chosen because DC beats it
and no other candidate beats DC.

e. What is the top cycle? (2 points)

DC is the Condorcet winner (beating all others)
so the top cycle is $\{DC\}$.

f. Now say that DC is disqualified because he went to Africa and cannot be reached. Now the cabinet chooses among four possible candidates: PO, PA, TP, and BS. Their preferences over these four are the same as before (the only thing which has changed is that DC is no longer a candidate). Now what is the top cycle? (2 points)

1	2	3	4	5
PO	PA	PO	PA	PO
PA	PO	PA	PO	PA
BS	TP	TP	BS	TP
TP	BS	BS	TP	BS

PO beats all others.

PO is the Condorcet winner so
the top cycle is $\{PO\}$.

Part 5. Say that there are three islands, A, B, and C. Island A has \$4 billion in treasure, island B has \$6 billion in treasure, and island C has \$8 billion in treasure.

There are two competing pirate captains, Jolly Roger and Pegleg Petunia. Each has command over two ships. Each pirate captain simultaneously decides where to send his or her ships. A captain can decide to send both ships to one island, or one ship each to different islands. If a captain sends a ship to an island and it arrives uncontested (no ship arrives from the other side), then the captain gets all the treasure from that island. If both captains send ships to an island, the one that sends more ships to that island gets all that island's treasure. If both captains send the same number of ships to an island, then it is a draw (on that island) and they split that island's treasure equally. If no ship appears from either side on an island, no one gets that island's treasure.

For example, if Jolly Roger sends one ship to island A and one ship to island B, and Pegleg Petunia sends one ship to island B and one ship to island C, then Jolly Roger gets \$7 billion (all of island A's treasure and half of island B's) and Pegleg Petunia gets \$11 billion (half of island B's treasure and all of island C's).

As another example, if Jolly Roger sends both ships to island B, and Pegleg Petunia sends one ship to island B and one ship to island C, then Jolly Roger gets \$6 billion (all of island B's treasure) and Pegleg Petunia gets \$8 billion (all of island C's treasure). Island A's treasure goes untouched.

a. Model this as a strategic form game. (3 points)

A B C
4 6 8

		2A	2B	2C	AB	AC	BC
(both ships at A)	2A	2, 2	4, 6	4, 8	4, 6	4, 8	4, 14 ⁺
(B)	6, 4	3, 3	6, 8	6, 4	6, 12 ⁺	6, 8
(C)	8, 4	8, 6	4, 4	8, 10 ⁺	8, 4	*8, 6
(one at A, one at B)	AB	5, 4	4, 6	*10, 8	5, 5	8, 10	7, 11 ⁺
(A	C)	8, 4	*12, 6	4, 8	10, 8	6, 6	*8, 11 ⁺
(B	C)	*14, 4	8, 6	6, 8	*11, 7	*10, 8 ⁺	7, 7

b. Are there any strongly dominated strategies? Are there any weakly dominated strategies? If so, write them down. (2 points)

2A is strongly dominated by BC.
 .. weakly .. 2C, AB, AC.
 2B is weakly dominated by BC.

c. Find all pure strategy Nash equilibria of this game. If there are no pure strategy Nash equilibria, explain why there are none. (2 points)

(AC, BC)
 (BC, AC) are the two NE (look at the *, + in the game matrix).

d. Now say that there is a fourth island, island D, which has a treasure of \$40 billion. (Island A still has \$4 billion in treasure, island B has \$6 billion, and island C has \$8 billion.) Find all pure strategy Nash equilibria. (Try to do this without writing down the entire game.) Explain your work. (3 points)

A	B	C	D
4	6	8	40

There is no NE in which no ships are sent to D because if

By sending both ships to D, you get at least 20. So there is no NE in which one side sends no ship to D (because if you send no ship to D, the most you can get is 18).

So both sides send at least one ship to D.

If the other side sends one ship to D, then your best response is to send both ships to D and get 40. If the other side sends two ships to D, your best response is to send both ships to D and get 20, which is greater than the most you could have got from A, B, C.

So the only NE is $(2D, 2D)$. Both sides send both ships to D.

e. Is it true that putting both ships on island D weakly dominates all other strategies? (2 points)

No. If the other side ~~plays~~ plays ~~AC~~ AC, for example, playing BD and getting 46 is better than playing 2D and getting 40.

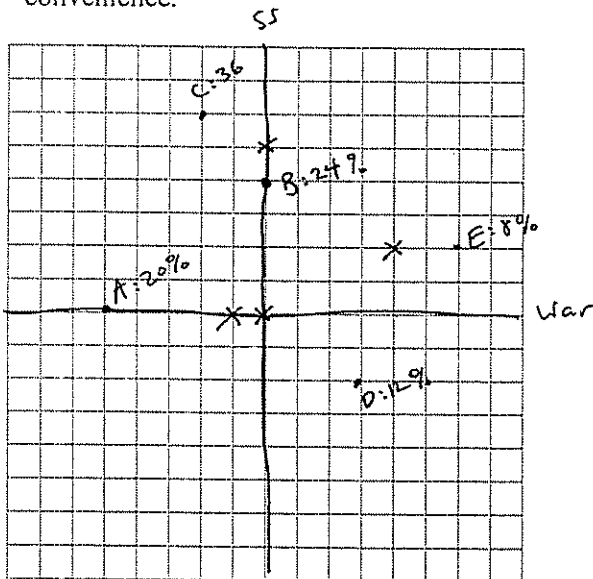
If the ~~other~~ other side doesn't touch D, you don't gain by putting two ships on D.

You can also solve this problem by setting up a 10×10 game, but this takes a long time.

Part 6. Say that a country is holding an election with two candidates running. In this election, the critical issues are whether to allocate more or less money to the war and more or less money to social security. This country has five groups of voters. Group A makes up 20% of the population and prefers to reduce spending on the war by \$5 billion and leave social security funding unchanged. Group B makes up 24% of the population and prefers to leave war spending unchanged and spend \$4 billion more on social security. Group C is 36% of the population and prefers to spend \$2 billion less on war and \$6 billion more on social security. Group D is 12% of the population and prefers to spend \$3 billion more on the war and \$2 billion less on social security. Group E is 8% of the population and prefers to spend \$6 billion more on the war and \$2 billion more on social security.

This is a two-dimensional Downsian model, where each group has an ideal point. For example, Group A's ideal point is (-5, 0) and Group C's ideal point is (-2, 6).

a. There are two candidates, Candidate 1 and Candidate 2, and for simplicity assume that the positions that candidates can take are limited to the following: (0, 0), (0, 5), (4, 2), or (-1, 0). Again, position (0, 5) means that the candidate leaves war funding unchanged and increases social security funding by \$5 billion. Model this as a strategic form game and make a prediction by finding all pure strategy Nash equilibria (4 points). You can use the "graph paper" below for convenience.



(0,0) vs. (0,5)	40%	60%
AED		BC

(0,0) vs. (4,2)	92%	8%
ABCD		E

(0,0) vs. (-1,0)	44%	56%
BDE		AC

(4,2) vs. (0,5)	20%	80%
DE		ABC

(0,5) vs. (-1,0)	BCE	AD
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(4,2) vs. (-1,0)	DE	ABC
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D is $\sqrt{13}$ away from (0,0)
D is $\sqrt{7}$ away from (4,2)

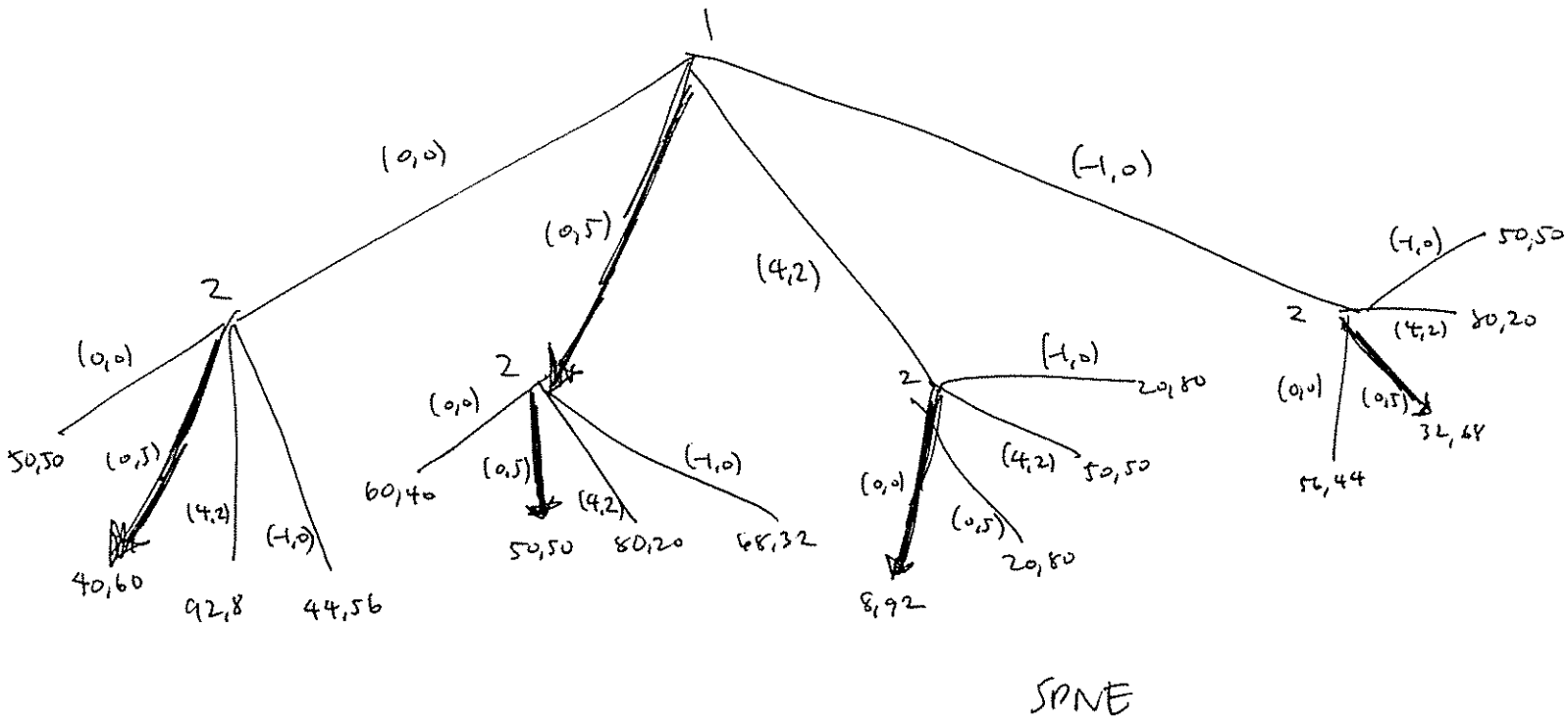
D is $\sqrt{20}$ away from (-1,0)
D is $\sqrt{58}$ away from (0,5)
E is $\sqrt{53}$ away from (-1,0)
E is $\sqrt{45}$ away from (0,5)

B is $\sqrt{20}$ away from (4,2)
B is $\sqrt{17}$ away from (-1,0)
C is $\sqrt{37}$ away from (-1,0)
C is $\sqrt{52}$ away from (-1,0)
D is $\sqrt{17}$ away from (4,2)
D is $\sqrt{20}$ away from (-1,0)

	(0,0)	(0,5)	(4,2)	(-1,0)
(0,0)	50,50	40,60	92,8	44,56
(0,5)	60,40	50,50	80,20	68,32
(4,2)	8,92	20,80	50,50	20,80
(-1,0)	56,44	32,68	80,20	50,50

NE: ((0,5), (0,5))
Both will locate at (0,5).

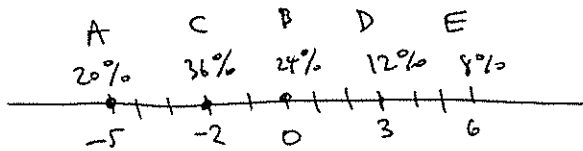
b. Now say that Candidate 1 announces her issue position first. As in part a. above, both candidates choose their policy positions among $(0, 0)$, $(0, 5)$, $(4, 2)$ or $(-1, 0)$. After Candidate 1 chooses his position, Candidate 2 chooses her position, knowing what Candidate 1's position is. Represent this game as an extensive form game and find all subgame perfect Nash equilibria. (4 points)



c. Now say that the two candidates again choose their positions simultaneously. However, because of constitutional protection of social security funding, social security funding is "off the table" and candidates are forced to take the position that social security funding must remain unchanged. Candidates now take positions only on changing war funding, and can change it by any amount. In other words, candidates choose their positions from $(-6, 0)$, $(-5, 0)$, $(-4, 0)$, $(-3, 0)$, $(-2, 0)$, $(-1, 0)$, $(0, 0)$, $(1, 0)$, $(2, 0)$, $(3, 0)$, $(4, 0)$, $(5, 0)$, $(6, 0)$. What positions will the two candidates take? (2 points)

Too difficult ~~to do~~ and time-consuming to write out the same.

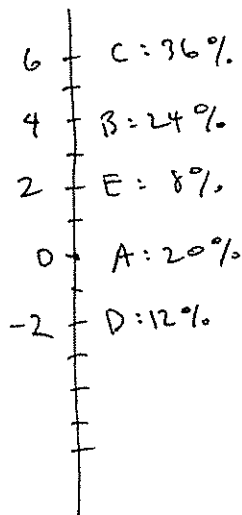
Instead, just realize that it is a one-dimensional problem and thus both candidates will pick the median voter position.



Median voter is in group C.

Both candidates will take position $(-2, 0)$.

d. Again, the two candidates choose their positions simultaneously. Now assume that the country is in an arms race with another nation and thus war spending is "off the table." Candidates are forced to have the same position on war spending: war spending must increase by \$8 billion. Candidates now take positions only on social security funding, and can change it by any amount. In other words, candidates choose their positions from $(8, -6)$, $(8, -5)$, $(8, -4)$, $(8, -3)$, $(8, -2)$, $(8, -1)$, $(8, 0)$, $(8, 1)$, $(8, 2)$, $(8, 3)$, $(8, 4)$, $(8, 5)$, $(8, 6)$. What positions will the two candidates take? (2 points)



Median voter is in group B.

Both candidates will take position $(8, 4)$.

Part 7. There are 100 legislators in the parliament of the country of Ecopia. They are considering four different bills for controlling air pollution: A, B, C, or D. There are four political parties which the members of parliament belong to. Party W members make up 30 percent of the parliament, Party X members make up 28 percent, Party Y members make up 24 percent, and Party Z members make up 18 percent.

Members of Party W prefer A to D to C to B (in other words, Party W members like A best, D second-best, C third-best, and B worst). Members of Party X prefer B to C to A to D. Members of Party Y prefer C to D to B to A. Members of Party Z prefer D to A to C to B.

Three voting rules are available: runoff, Borda count, and approval voting (where each legislator votes for her top two choices).

a. Which bill will be the winner if the runoff procedure is used? Explain why by showing how many votes each bill receives under runoff. (2 points)

30 W	28 X	24 Y	18 Z
A	B	C	D
D	C	D	A
C	A	B	C
B	D	A	B

First vote:

A	30%	} top two
B	28%	
C	24%	
D	18%	

Runoff between A + B:

A: 48% (parties W + Z)

B: 52% (parties X + Y)

B wins the runoff
and is the winner.

b. Which bill will be the winner if the Borda count is used? Explain why by showing how many points each bill receives under the Borda count. (2 points)

Borda count:

$$A: 3 \cdot 30 + 1 \cdot 28 + 0 \cdot 24 + 2 \cdot 18 = 154$$

$$B: 0 \cdot 30 + 3 \cdot 28 + 1 \cdot 24 + 0 \cdot 18 = 108$$

$$C: 1 \cdot 30 + 2 \cdot 28 + 3 \cdot 24 + 1 \cdot 18 = 176 \leftarrow$$

$$D: 2 \cdot 30 + 0 \cdot 28 + 2 \cdot 24 + 3 \cdot 18 = 162$$

C is the Borda count winner.

c. Which bill will be the winner if approval voting is used? Explain why by showing how many points each bill receives under approval voting. (2 points)

Approval (top two):

$$A: 30 + 18 = 48$$

$$B: 28$$

$$C: 28 + 24 = 52$$

$$D: 30 + 24 + 18 = 72 \leftarrow$$

D is the winner under approval voting (top two).

d. Say that Party W controls which voting rule the parliament uses. Which voting rule will the parliament use? Explain why this voting rule would be the best for Party W among the three rules. (3 points)

Runoff: B wins
 Borda: C wins
 Approval: D wins

among these, Party W prefers D the best. Hence Party W chooses approval voting.

e. Now say that some members of Party Y defect to Party Z. So now Party W members make up 30 percent of the parliament, Party X members make up 28 percent, Party Y members make up $24 - x$ percent, and Party Z members make up $18 + x$ percent. Party W still controls the voting system. For x very close to 0, Party W will of course still choose the voting rule it chose in part d. above. But as x increases, Party W might decide to choose a different voting rule. How large does x have to be to make Party W choose a different voting rule than in part d. above? (3 points)

	W	X	Y	Z
30				
28				
$24-x$				
$18+x$				
A	B	C	D	
D	C	D	A	
C	A	B	C	
B	D	A	B	

approval (top two):

A: $48 + 2x$
 B: 28
 C: $52 - 2x$
 D: 72

unless $x > 24$,
 D will still win under approval voting.
 But $x \leq 24$ and so D will win regardless of x .

Borda count:

$$A: 3 \cdot 30 + 1 \cdot 28 + 0 \cdot (24 - x) + 2 \cdot (18 + x) = 154 + 2x$$

$$B: 0 \cdot 30 + 2 \cdot 28 + 1 \cdot (24 - x) + 0 \cdot (18 + x) = 108 - x$$

$$C: 1 \cdot 30 + 2 \cdot 27 + 3(24 - x) + 1 \cdot (18 + x) = 176 - 2x$$

$$D: 2 \cdot 30 + 0 \cdot 28 + 2 \cdot (24 - x) + 3 \cdot (18 + x) = 162 + 2x$$

C still wins as long as $176 - 2x > 162 + 2x$ and $176 - 2x > 154 + 2x$

$$14 > 3x$$

$$\frac{14}{3} > x$$

$$\text{and } 22 > 4x$$

$$\frac{11}{2} = \frac{22}{4} > x$$

So as long as $x < \frac{14}{3} \approx 4.67$, C still wins.

Runoff:

A: 30
 B: 28
 C: $24 - x$
 D: $18 + x$

top two (unless $x > 10$)

Runoff:
 A: $48 + 2x$
 B: $52 - 2x$

If $x > 2$, then A wins, which is W's first choice. So if $x > 2$, party W chooses plurality.

Part 8. Consider the following four person game. Each person can play either A or B. For example, if player 1 plays B, player 2 plays B, player 3 plays A, and player 4 plays B, then the payoffs are 5, -5, 5, -5 (player 1 gets 5, player 2 gets -5, player 3 gets 5, and player 4 gets -5). Find all (pure strategy and mixed strategy) Nash equilibria of this game (12 points).

		P2	
		A	B
P1	A	*15, 5, 10, 10	*20, 5, 0, 10
	B	10, 5, 0, 20	0, 10, 0, 20

3A (r)

		P2	
		A	B
P1	A	5, 10, 0, 10	*10, 5, 0, 20
	B	*10, 5, 0, 5	0, 5, 10, 10

3B (1-r)

4A

		P2	
		A	B
P1	A	20, -10, 0, 0	10, 10, 0, 5
	B	0, 20, -5, 5	5, -5, 5, -5

3A

		P2	
		A	B
P1	A	10, 5, -5, 5	20, 0, 0, 0
	B	15, 0, 0, 0	0, 0, 0, 0

3B

4B

First note that 4A strictly dominates 4B. So ignore 4B. It is now a 3-person game.

From the *, +, ^ above, the pure strategy NE are

(A, A, A), (A, B, A) and (B, A, B).

Mixed NE:

If you randomize, you must be indifferent among the strategies you randomize over.

So 1's exp. utility from a = ~~15q~~ 15qr + 20(1-q)r + 5q(1-r) + 10(1-q)(1-r) } must be equal.
 1's " b = 10qr + 0(1-q)r + ~~10~~ 10q(1-r) + 0(1-q)(1-r)

$$\sum 5qr + 20(1-q)r - 5q(1-r) + 10(1-q)(1-r) = 0$$

$$\underline{5qr} + 20r - \underline{20qr} - 5q + \underline{5qr} + 10 - 10q - 10r + \underline{10qr} = 0$$

$$20r - 5q + 10 - 10q - 10r = 0$$

$$10r - 15q + 10 = 0$$

$$2r - 3q + 2 = 0$$

$$2 = 3q - 2r$$

