

Final exam PS 30 December 2007

Name:

TA:

Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. There are eight parts in this exam. Each part is weighted equally (12 points for each part). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If the TA responsible for a given question is not in the room at the time, work on other parts of the exam and hold the question until that TA rotates to your exam location. When the end of the exam is announced, please stop working immediately. People who continue working after the end of the exam is announced will have their grades penalized by 25 percent. If you need to leave the room to use the bathroom during the exam, please write your name down on the bathroom log before you leave. A person cannot leave the room more than once during the exam (a person who leaves for a second time will be considered to have completed his or her exam). Please turn in your exam to one of the TAs. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Please turn off all cell phones and other electronic gadgets. Good luck!

1	
2	
3	
4	
5	
6	
7	
8	
total	

Part 1. President Bush and Secretary of State Rice choose a policy on Iraq. There are three policies: stay in Iraq forever (S), withdraw troops gradually (W), or get out of Iraq immediately (G). Bush and Rice choose the policy in the following manner. First, Bush vetoes a policy and thus leaves two remaining policies. Then, Rice vetoes one of the two remaining policies. The policy left after these two vetoes is the policy which is chosen.

Bush likes S the best, G second best, and likes W the least. Rice is indifferent between G and W, and likes either better than S.

a. Model this situation as an extensive form game. For Bush, assign a payoff of 4 to the best outcome, 1 to the second best and -2 to the worst. For Rice, assign 3 to the best and -5 to the worst. (2 points)

b. Convert the extensive form game into a strategic form game. (4 points)

c. Find all pure strategy Nash equilibria of the game. (2 points)

d. Draw all subgame perfect Nash equilibria of the game. (Draw them using arrows in a game tree.) (4 points)

Part 2. There are three men A, B, and C and three women X, Y, and Z. Each person is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman Z best, then Y second-best, then X worst. Man B prefers woman Y best, then Z, then X worst. Man C prefers woman X best, then Y, then Z worst. Woman X prefers man A best, then B, then C worst. Woman Y prefers man A best, then C, then B worst. Finally, woman Z prefers man C best, then B, then A worst.

- a. Write down all possible matchings and determine which of them are stable and which are not stable. (4 points)

b. Among the set of stable matchings, which matching is most preferred by the women? Among the set of stable matchings, which matching is most preferred by the men? (4 points)

c. Now say that there are four men, A, B, C, and D, and four women W, X, Y, and Z. Man A's preferences (from best to worst) are W, Y, Z, X. Man B's preferences are X, Z, W, Y. Man C's preferences are Y, W, Z, X. Man D's preferences are Y, W, X, Z. Woman W's preferences are B, A, D, C. Woman X's preferences are B, D, C, A. Woman Y's preferences are A, B, C, D. Woman Z's preferences are A, D, C, B. Find the set of all stable matchings. (4 points)

Part 3. The North Korean Refugee problem is getting serious attention from international society. Refugees from North Korea frequently cross borders into China, Russia, and even Southeast Asian countries. Since North Korean refugees have not obtained "refugee" status as stipulated in international law, they live under the threat of unwanted repatriation back to North Korea. Hence, for humanitarian purposes, the United Nations tries to pass "North Korean Refugee Resolution No. 1" to set up refugee camps in neighboring countries in order to protect them from local threats and also enable them to freely choose their future destinations.

The UN Security Council, which is composed of 5 Permanent members and 10 temporary members, votes on this issue. To pass the resolution, 9 or more members of the Security Council must vote "yes". However, a "no" vote, or "veto," by any of the five permanent members prevents adoption of a resolution, even if it has received the required number of "yes" votes. Permanent members of the Security Council are the U.S, Great Britain, France, Russia and China.

a. Model this as a threshold model, where the thresholds of each of the 15 Security Council member countries are shown below. Assume that the initial state is that no one supports the resolution. Will the resolution eventually be adopted in the Security Council? You can use the following table as a worksheet if you like. (4 points)

	Threshold	Initial							
Belgium	1	N							
Burkina Faso	9	N							
China	14	N							
Costa Rica	9	N							
Croatia	5	N							
France	1	N							
Great Britain	1	N							
Indonesia	3	N							
Italy	3	N							
Libya	5	N							
Panama	9	N							
Russia	14	N							
South Africa	5	N							
U.S	0	N							
Vietnam	3	N							

b. Now say that the thresholds of China and Russia change: China's threshold is x and Russia's threshold is $x-1$. Everyone else's thresholds stay the same. Again assume that the initial state is that no one supports the resolution.

	Threshold	Initial							
Belgium	1	N							
Burkina Faso	9	N							
China	x	N							
Costa Rica	9	N							
Croatia	5	N							
France	1	N							
Great Britain	1	N							
Indonesia	3	N							
Italy	3	N							
Libya	5	N							
Panama	9	N							
Russia	$x-1$	N							
South Africa	5	N							
U.S.	0	N							
Vietnam	3	N							

Circle the values of x below for which the Security Council adopts the resolution. (4 points)

- $x=1$
- $x=2$
- $x=3$
- $x=4$
- $x=5$
- $x=6$
- $x=7$
- $x=8$
- $x=9$
- $x=10$
- $x=11$
- $x=12$
- $x=13$
- $x=14$
- $x=15$

c. Now say that the thresholds of Croatia and France also change: their thresholds are both x . Like before, China's threshold is x and Russia's threshold is $x-1$. Everyone else's thresholds stay the same. Again assume that the initial state is that no one supports the resolution.

	Threshold	Initial							
Belgium	1	N							
Burkina Faso	9	N							
China	x	N							
Costa Rica	9	N							
Croatia	x	N							
France	x	N							
Great Britain	1	N							
Indonesia	3	N							
Italy	3	N							
Libya	5	N							
Panama	9	N							
Russia	$x-1$	N							
South Africa	5	N							
U.S	0	N							
Vietnam	3	N							

Circle the values of x below for which the Security Council adopts the resolution. (4 points)

- $x=1$
- $x=2$
- $x=3$
- $x=4$
- $x=5$
- $x=6$
- $x=7$
- $x=8$
- $x=9$
- $x=10$
- $x=11$
- $x=12$
- $x=13$
- $x=14$
- $x=15$

Part 4. It's 2052 and Borat becomes the president of Khazeetan. His cabinet, consisting of five members, is voting to see whom to give the "Greatest Person in the World" medal to. The candidates are Borat's producer (PO), Pomel-a Anderson (PA), Dave Chappelle (DC), Tony Parker (TP), and Britney Spears (BS). The cabinet makes decisions by majority rule. The cabinet's preferences are as follows:

	1	2	3	4	5
Best	PO	PA	DC	DC	PO
	DC	DC	PO	PA	PA
	PA	PO	PA	PO	TP
	BS	TP	TP	BS	DC
Worst	TP	BS	BS	TP	BS

a. Say that the agenda is as follows. First the cabinet votes on PO or not. If PO loses, then the cabinet votes on DC or not. If DC loses, then the cabinet votes on PA or not. If PA loses, then the cabinet votes between BS and TP. Note that the order of voting corresponds to Person 1's preferences (starting from Person 1's most preferred to his least preferred). Which outcome does the cabinet choose? Does Person 1 get his most favored candidate approved? (2 points)

b. Now say that the agenda is as follows. First the cabinet votes on PO or not. If PO loses, then the cabinet votes on PA or not. If PA loses, then the cabinet votes on TP or not. If TP loses, then the cabinet votes between DC and BS. Note that the order of voting corresponds to Person 5's preferences (starting from Person 5's most preferred to his least preferred). Which outcome does the cabinet choose? Does Person 5 get her most favored candidate approved? (2 points)

c. Is there any agenda in which the cabinet chooses PO? If so, write down the agenda. If not, explain why not. (2 points)

d. Is there any agenda in which the cabinet chooses BS? If so, write down the agenda. If not, explain why not. (2 points)

e. What is the top cycle? (2 points)

f. Now say that DC is disqualified because he went to Africa and cannot be reached. Now the cabinet chooses among four possible candidates: PO, PA, TP, and BS. Their preferences over these four are the same as before (the only thing which has changed is that DC is no longer a candidate). Now what is the top cycle? (2 points)

Part 5. Say that there are three islands, A, B, and C. Island A has \$4 billion in treasure, island B has \$6 billion in treasure, and island C has \$8 billion in treasure.

There are two competing pirate captains, Jolly Roger and Pegleg Petunia. Each has command over two ships. Each pirate captain simultaneously decides where to send his or her ships. A captain can decide to send both ships to one island, or one ship each to different islands. If a captain sends a ship to an island and it arrives uncontested (no ship arrives from the other side), then the captain gets all the treasure from that island. If both captains send ships to an island, the one that sends more ships to that island gets all that island's treasure. If both captains send the same number of ships to an island, then it is a draw (on that island) and they split that island's treasure equally. If no ship appears from either side on an island, no one gets that island's treasure.

For example, if Jolly Roger sends one ship to island A and one ship to island B, and Pegleg Petunia sends one ship to island B and one ship to island C, then Jolly Roger gets \$7 billion (all of island A's treasure and half of island B's) and Pegleg Petunia gets \$11 billion (half of island B's treasure and all of island C's).

As another example, if Jolly Roger sends both ships to island B, and Pegleg Petunia sends one ship to island B and one ship to island C, then Jolly Roger gets \$6 billion (all of island B's treasure) and Pegleg Petunia gets \$8 billion (all of island C's treasure). Island A's treasure goes untouched.

a. Model this as a strategic form game. (3 points)

b. Are there any strongly dominated strategies? Are there any weakly dominated strategies? If so, write them down. (2 points)

c. Find all pure strategy Nash equilibria of this game. If there are no pure strategy Nash equilibria, explain why there are none. (2 points)

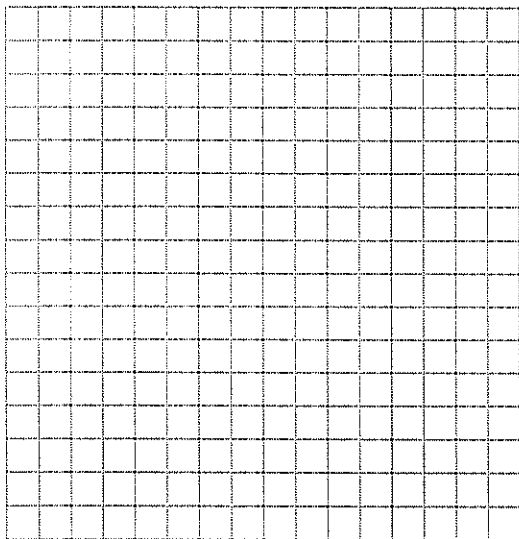
d. Now say that there is a fourth island, island D, which has a treasure of \$40 billion. (Island A still has \$4 billion in treasure, island B has \$6 billion, and island C has \$8 billion.) Find all pure strategy Nash equilibria. (Try to do this without writing down the entire game.) Explain your work. (3 points)

e. Is it true that putting both ships on island D weakly dominates all other strategies? (2 points)

Part 6. Say that a country is holding an election with two candidates running. In this election, the critical issues are whether to allocate more or less money to the war and more or less money to social security. This country has five groups of voters. Group A makes up 20% of the population and prefers to reduce spending on the war by \$5 billion and leave social security funding unchanged. Group B makes up 24% of the population and prefers to leave war spending unchanged and spend \$4 billion more on social security. Group C is 36% of the population and prefers to spend \$2 billion less on war and \$6 billion more on social security. Group D is 12% of the population and prefers to spend \$3 billion more on the war and \$2 billion less on social security. Group E is 8% of the population and prefers to spend \$6 billion more on the war and \$2 billion more on social security.

This is a two-dimensional Downsian model, where each group has an ideal point. For example, Group A's ideal point is $(-5, 0)$ and Group C's ideal point is $(-2, 6)$.

a. There are two candidates, Candidate 1 and Candidate 2, and for simplicity assume that the positions that candidates can take are limited to the following: $(0, 0)$, $(0, 5)$, $(4, 2)$, or $(-1, 0)$. Again, position $(0, 5)$ means that the candidate leaves war funding unchanged and increases social security funding by \$5 billion. Model this as a strategic form game and make a prediction by finding all pure strategy Nash equilibria (4 points). You can use the "graph paper" below for convenience.



b. Now say that Candidate 1 announces her issue position first. As in part a. above, both candidates choose their policy positions among $(0, 0)$, $(0, 5)$, $(4, 2)$ or $(-1, 0)$. After Candidate 1 chooses his position, Candidate 2 chooses her position, knowing what Candidate 1's position is. Represent this game as an extensive form game and find all subgame perfect Nash equilibria. (4 points)

c. Now say that the two candidates again choose their positions simultaneously. However, because of constitutional protection of social security funding, social security funding is "off the table" and candidates are forced to take the position that social security funding must remain unchanged. Candidates now take positions only on changing war funding, and can change it by any amount. In other words, candidates choose their positions from $(-6, 0)$, $(-5, 0)$, $(-4, 0)$, $(-3, 0)$, $(-2, 0)$, $(-1, 0)$, $(0, 0)$, $(1, 0)$, $(2, 0)$, $(3, 0)$, $(4, 0)$, $(5, 0)$, $(6, 0)$. What positions will the two candidates take? (2 points)

d. Again, the two candidates choose their positions simultaneously. Now assume that the country is in an arms race with another nation and thus war spending is "off the table." Candidates are forced to have the same position on war spending: war spending must increase by \$8 billion. Candidates now take positions only on social security funding, and can change it by any amount. In other words, candidates choose their positions from $(8, -6)$, $(8, -5)$, $(8, -4)$, $(8, -3)$, $(8, -2)$, $(8, -1)$, $(8, 0)$, $(8, 1)$, $(8, 2)$, $(8, 3)$, $(8, 4)$, $(8, 5)$, $(8, 6)$. What positions will the two candidates take? (2 points)

Part 7. There are 100 legislators in the parliament of the country of Ecopia. They are considering four different bills for controlling air pollution: A, B, C, or D. There are four political parties which the members of parliament belong to. Party W members make up 30 percent of the parliament, Party X members make up 28 percent, Party Y members make up 24 percent, and Party Z members make up 18 percent.

Members of Party W prefer A to D to C to B (in other words, Party W members like A best, D second-best, C third-best, and B worst). Members of Party X prefer B to C to A to D. Members of Party Y prefer C to D to B to A. Members of Party Z prefer D to A to C to B.

Three voting rules are available: runoff, Borda count, and approval voting (where each legislator votes for her top two choices).

a. Which bill will be the winner if the runoff procedure is used? Explain why by showing how many votes each bill receives under runoff. (2 points)

b. Which bill will be the winner if the Borda count is used? Explain why by showing how many points each bill receives under the Borda count. (2 points)

c. Which bill will be the winner if approval voting is used? Explain why by showing how many points each bill receives under approval voting. (2 points)

d. Say that Party W controls which voting rule the parliament uses. Which voting rule will the parliament use? Explain why this voting rule would be the best for Party W among the three rules. (3 points)

e. Now say that some members of Party Y defect to Party Z. So now Party W members make up 30 percent of the parliament, Party X members make up 28 percent, Party Y members make up $24 - x$ percent, and Party Z members make up $18 + x$ percent. Party W still controls the voting system. For x very close to 0, Party W will of course still choose the voting rule it chose in part d. above. But as x increases, Party W might decide to choose a different voting rule. How large does x have to be to make Party W choose a different voting rule than in part d. above? (3 points)

Part 8. Consider the following four person game. Each person can play either A or B. For example, if player 1 plays B, player 2 plays B, player 3 plays A, and player 4 plays B, then the payoffs are 5, -5, 5, -5 (player 1 gets 5, player 2 gets -5, player 3 gets 5, and player 4 gets -5). Find all (pure strategy and mixed strategy) Nash equilibria of this game (12 points).

		P2	
		A	B
P1	A	15, 5, 10, 10	20, 5, 0, 10
	B	10, 5, 0, 20	0, 10, 0, 20

3A

		P2	
		A	B
P1	A	5, 10, 0, 10	10, 5, 0, 20
	B	10, 5, 0, 5	0, 5, 10, 10

3B

4A

		P2	
		A	B
P1	A	20, -10, 0, 0	10, 10, 0, 5
	B	0, 20, -5, 5	5, -5, 5, -5

3A

		P2	
		A	B
P1	A	10, 5, -5, 5	20, 0, 0, 0
	B	15, 0, 0, 0	0, 0, 0, 0

3B

4B

(continue your answers to Part 8 on this page if necessary)