2006 FINAL EXAM ANSWERS

Answers to Part 1:

a. Say Player X is Player 3. List all strategies for each player (P1, P2, P3).

Self-evident. =)

b. Represent the above game as a strategic form game.

		P2			
		AC	AD	BC	BD
P1	L	4,4,4	4,4,4	3,3,3	3,3,3
	M	6,6,5	6,6,5	6,6,5	6,6,5
	R	8,8,8	0,0,0	8,8,8	0,0,0

e

		P2			
		AC	AD	BC	BD
P1	L	4,4,4	4,4,4	1,1,1	1,1,1
	M	6,6,5	6,6,5	6,6,5	6,6,5
	R	8,8,8	0,0,0	8,8,8	0,0,0

f

		P2			
		AC	AD	BC	BD
P1	L	4,4,4	4,4,4	2,2,2	2,2,2
	M	6,6,5	6,6,5	6,6,5	6,6,5
	R	8,8,8	0,0,0	8,8,8	0,0,0

g

c. Say Player X is Player 1. List all strategies for each player (P1, P2).

So, now P1 has 6 strategies because depending on what P2 chooses, P1 can choose between c, f, and z. So, the strategies are:

Le
Lf
Lg
Me
Mf
Mg
Re
Rf
Rg

P2 same as last time.

AC
AD
BC
BD

d. Represent the above game as a strategic form game.

	P2				
		AC	AD	BC	BD
P1	Le	4,4,4	4,4,4	3,3,3	3,3,3
	Lf	4,4,4	4,4,4	1,1,1	1,1,1
	Lg	4,4,4	4,4,4	2,2,2	2,2,2
	Me	6,6,5	6,6,5	6,6,5	6,6,5
	Mf	6,6,5	6,6,5	6,6,5	6,6,5
	Mg	6,6,5	6,6,5	6,6,5	6,6,5
	Re	8,8,8	0,0,0	8,8,8	0,0,0
	Rf	8,8,8	0,0,0	8,8,8	0,0,0
	Rg	8,8,8	0,0,0	8,8,8	0,0,0

Answer for Part 2:

a. (4 points) Find all pure and mixed strategy Nash Equilibrium of the following game:

		P2	
		L	R
P1	T	3, 5	-2, 3
	D	-1, -2	4, 2

PSNE are (T,L) and (R,D). MSNE exists where P1 plays T with prob 2/3 and P2 plays L with prob 3/5.

$5p + (-2)(1-p) = 3p + 2(1-p)$
$7p-2=p+2$
$6p=4$
$p=2/3$

$3q + (-2)(1-q) = -q + 4(1-q)$
$5q-2 = -5q +4$
$10q = 6$
$q=3/5$

b. (4 points) Now consider this modified version of the game where the payoffs for D, R are *x, y*. For what values of *x* and *y* will there be a mixed strategy Nash Equilibrium where Player 1 plays D with probability 2/3 and Player 2 plays R with probability 2/3?

		P2	
		L	R
P1	T	3, 5	-2, 3
	D	-1, -2	*x*, *y*

Say P1 plays T with prob p and D with prob (1-p). P2 plays L with prob q and R with prob (1-q).

$EU_T = EU_D$
$5p + (-2)(1-p) = 3p + y(1-p)$
$p= (y+2) / (y+4)$

$EU_L = EU_R$
$3q + (-2)(1-q) = -q + x(1-q)$
$q= (x+2) / (x+6)$

We want p=1/3 and q=1/3. It is easy to see that the only values of x and y that provide these values are:

$x = 0$, $y = -1$

c. (4 points) In the game directly above, is there a value of *x* and *y* where there is no mixed strategy Nash Equilibrium? If yes, indicate all possible values of *x* and *y* where there will be no mixed strategy Nash Equilibrium.

There is no MSNE if $x \leq -2$ and $y \leq -2$

Answer for Part 3:

Consider the following 3-player strategic form game

P3 X

P1 \ P2	a	b	c	d
A	2,5,2	3,2,5	2,1,2	2,0,1
B	3,2,1	2,5,3	2,1,1	2,0,2
C	1,2,2	1,2,1	1,1,2	2,0,1
D	0,2,1	0,2,2	0,1,1	2,0,2

P3 Y

P1 \ P2	a	b	c	d
A	2,5,2	3,2,4	2,1,2	2,0,1
B	3,2,1	2,5,2	2,1,1	2,0,2
C	4,2,2	1,2,1	1,1,2	2,0,1
D	0,2,1	0,2,2	0,1,1	2,0,2

1. Find all PSNEs. Write the strategy profile like (P1's, P2's, P3's). (3 points).

(C, a, Y)

b. Find all MSNE.

① D is elimated
② c and d are eliminated
③ Y is eliminated
④ C is eliminated
☞ ① and ② are interchangeable.

P1 \ P2	a	b
A	2,5,2	3,2,5
B	3,2,1	2,5,3

P3X

P1 will choose A(or B) with probability ½
P2 will choose a (or b) with probability ½
P3 will choose X.

Answer to Part 4:

Consider 100 residents attending a town meeting in Westwood. They must choose among three proposals for dealing with town garbage. Proposal 1 asks the town to provide garbage collection; Proposal 2 calls for the town to hire a private garbage collector; Proposal 3 calls for residents to be responsible for their own garbage. There are 3 groups of residents as below. For example, residents in Group 1 like Proposal 1 best and Proposal 3 worst.

Group 1 (40 residents)	Group 2 (26 residents)	Group 3 (35 residents)
Proposal 1	Proposal 2	Proposal 3
Proposal 2	Proposal 3	Proposal 2
Proposal 3	Proposal 1	Proposal 1

a. Is there a Condorcet winner? If so, which proposal is the Condorcet winner.

We know 1 cannot be the winner because it's in the bottom for 61 residents total. Proposal 2 is the Condorcet winner because 2 groups (40+26 residents) prefer that to Proposal 3.

b. Suppose there are three possible voting procedures; plurality rule, Borda count and a Runoff system. What are the results from each of these procedures? In order for Group 3 to get its way, which voting method(s) are most advantageous for Group 3?

Plurality rule => tied because each gets 1 vote
Borda count => Proposal 2 wins

Let's put points: top row gets 2, middle row gets 1, bottom row gets 0. So now the points that each Proposal gets is:

Proposal 1 = 2 x 40 residents = 80
Proposal 2 = [26 residents x (2)] + [35+40residents x (1)] = 101
Proposal 3 = 35 residents x (2) + 26 residents x (1) = 96

Runoff => Proposal 3 wins

Proposal 1 gets 40 votes, while Proposal 3 gets 35 votes, so 1 and 3 moves on to the runoff round. Now, 35 + 26 residents prefer 3 to 1, so 3 wins.

Runoff procedure helps group 3 because proposal 3 wins here.

Answer for Part 5:

A town is deciding on whether to allocate more or less money to highways and to buses in the coming year. The town has four groups of voters. Aristocrats make up 30% of the population and prefer to reduce spending on buses by $2 million and reduce spending on highways by $6 million. Burghers make up 40% and prefer to increase spending on buses by $6 million and leave funding for highways unchanged. Cosmopolitans make up 20% and want to spend $6 million less on buses and $6 million more on highways. Urbanites are 10% of the population and want no change in bus spending but $1 million more for highways. New spending proposals are made by candidates in increments of $1 million (for example, a candidate can propose to spend $2 million more on buses but cannot propose $2.5 million more). Voters will vote for the candidate whose proposal is closest to their preferences. In the event that there is more than one proposal that is equally close to a voting group's preference, the group's vote will be split equally amongst the proposals. Whichever candidate receives the most votes wins.

a. Say that there are two candidates. Candidate 1 proposes to cut spending on buses by $3 million and increase spending on highways by $1 million. Candidate 2 proposes to increase spending on buses by $3 million and cut spending on highways by $3 million. Which candidate's proposal will win? Show your work by specifying what percentage of the vote will be received by each candidate.

Place all preferences and proposals in a 2-dimensional space with coordinates (bus, highway).

Aristocrats (-2,-6)
Burghers (6,0)
Cosmopolitans (-6,6)
Urbanites (0,1)

Candidate 1 proposes (-3,1)
Candidate 2 proposes (3,-3)

Candidate 2 wins. Candidate 1 gets the votes of C and U (30%). Candidate 2 gets the vote of A and B (70%).

b. Consider the losing candidate from part a) above. They decide that they will change their policy proposal in order to attract more votes than the competing proposal. However, they want to keep their new proposal as close to their original proposal from part a) as possible. Assuming that the other candidate will not change their proposal, what policy proposal will the losing candidate now make and what proportion of the vote will they receive?

Candidate 1 is the loser. Instead of (-3,1), if they propose (-3,-1), they will capture the votes of the Aristocrats, keep the votes of the Cosmopolitans and Urbanites, and win with

60% of the vote. Their new policy proposal is ideological distance \$2 million away from their original proposal.

c. Consider now a different situation where there are three candidates. As in part a), Candidate 1 proposes to cut spending on buses by \$3 million and increase spending on highways by \$1 million. Candidate 2 proposes to increase spending on buses by \$3 million and cut spending on highways by \$3 million. Candidate 3 does not want spending to change on highways but is willing to propose anything for bus spending to ensure victory. List all possible bus spending proposals that Candidate 3 can make to ensure victory.

Any increase in highway spending equal to or greater than \$2 million and equal to or less than \$10 million will ensure that Candidate 3 captures at least the vote of the Burghers which will be enough to win given that the remaining vote is split between Candidates 1 and 2. Any other proposal on highway spending will not ensure victory for Candidate 3.

Answer to Part 6:

Here "n" = non-participation and "p" = participation.
Each player has a lower threshold, which is the minimum number of players other than him or herself who must be participating in order for him or her to start participating.
Each player also has an upper threshold, which is the maximum number of other players he or she can tolerate participating at the same time as him or herself.

Consider the following situation:

Player	Lower Threshold	Upper Threshold	Initial Condition	t = 1	t = 2	t = 3	t = 4	t = 5
A	0	6						
B	1	6						
C	2	6						
D	x	x						
E	x	x						
F	5	6						

a. With an initial condition of all non-participation (n,n,n,n,n,n), find all the value(s) of x that will lead to player F participating at some point, even if F is no longer participating when an equilibrium is reached. Demonstrate the participation of player F for each of your values of x in the tables provided below.

Well, we know that x can't be 6 because the lower threshold is 6, and x will again never participate. So x has a possibility to be between 0 to 5, inclusive.
Now, we know that because the lower threshold of A, B, and C are 0, 1, and 2, the three players at some point will participate, so you can guarantee that at some point, at the very least, (Y,Y,Y,N,N,N) will occur.
x cannot be equal to 4 because D and E will never rebel then. X can't equal to 5 either because D and E will never rebel (due to lower threshold constraint).
So, now if x = 3, then we know that D and E will participate at least at one point. Let's see what happens then.

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	N	Y	Y	Y	Y	Y	Y
B	1	6	N	N	Y	Y	Y	Y	Y
C	2	6	N	N	N	Y	Y	Y	Y
D	3	3	N	N	N	N	Y	N	N
E	3	3	N	N	N	N	Y	N	N
F	5	6	N	N	N	N	N	Y	N

What if x = 2?

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	N	Y	Y	Y	Y	Y	Y
B	1	6	N	N	Y	Y	Y	Y	Y
C	2	6	N	N	N	Y	Y	Y	Y
D	2	2	N	N	N	Y	N	N	N
E	2	2	N	N	N	Y	N	N	N
F	5	6	N	N	N	N	Y	N	N

Let's see what happens when x = 1:

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	N	Y	Y	Y	Y	Y	Y
B	1	6	N	N	Y	Y	Y	Y	Y
C	2	6	N	N	N	Y	Y	Y	Y
D	1	1	N	N	Y	N	N	N	N
E	1	1	N	N	Y	N	N	N	N
F	5	6	N	N	N	N	N	N	N

F will never participate, so 1 can't be it.

Let's see what happens when x = 0:

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	N	Y	Y	Y	Y	Y	Y
B	1	6	N	Y	Y	Y	Y	Y	Y
C	2	6	N	N	Y	Y	Y	Y	Y
D	0	0	N	Y	N	N	N	N	N
E	0	0	N	Y	N	N	N	N	N
F	5	6	N	N	N	N	N	N	N

So the two values of x are: 1 and 2.

b. With an initial condition of all participation (p,p,p,p,p,p), find all value(s) of x that will allow player F to continue participating indefinitely. Demonstrate how this occurs for each of your x values answers in the tables provided below.

Since the initial condition has changed, we know that as long as the upper threshold of x is 5 and above, the player will continue to participate. But x can't exceed 6 because then the lower threshold will be 6! So, let's try 5 first:

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	P	Y	Y	Y	Y	Y	Y
B	1	6	P	Y	Y	Y	Y	Y	Y
C	2	6	P	Y	Y	Y	Y	Y	Y
D	5	5	P	Y	Y	Y	Y	Y	Y
E	5	5	P	Y	Y	Y	Y	Y	Y
F	5	6	P	Y	Y	Y	Y	Y	Y

X = 5 works.

What is x =4? Let's see:

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	P	Y	Y	Y	Y	Y	Y
B	1	6	P	Y	Y	Y	Y	Y	Y
C	2	6	P	Y	Y	Y	Y	Y	Y
D	4	4	P	Y	N	Y	N	Y	N
E	4	4	P	Y	N	Y	N	Y	N
F	5	6	P	Y	Y	N	Y	N	Y

Nope. If X is anything lower than 4, F will cease to participate at one point, then decides to participate again at the next period. This is not the same as indefinitely…

If x = 3:

Player	Lower threshold	Upper Threshold	Initial Condition	T=1	T=2	T=3	T=4	T=5	T=6
A	0	6	P	Y	Y	Y	Y	Y	Y
B	1	6	P	Y	Y	Y	Y	Y	Y
C	2	6	P	Y	Y	Y	Y	Y	Y
D	3	3	P	Y	N	N	N	N	N
E	3	3	P	Y	N	N	N	N	N
F	5	6	P	Y	Y	N	N	N	N

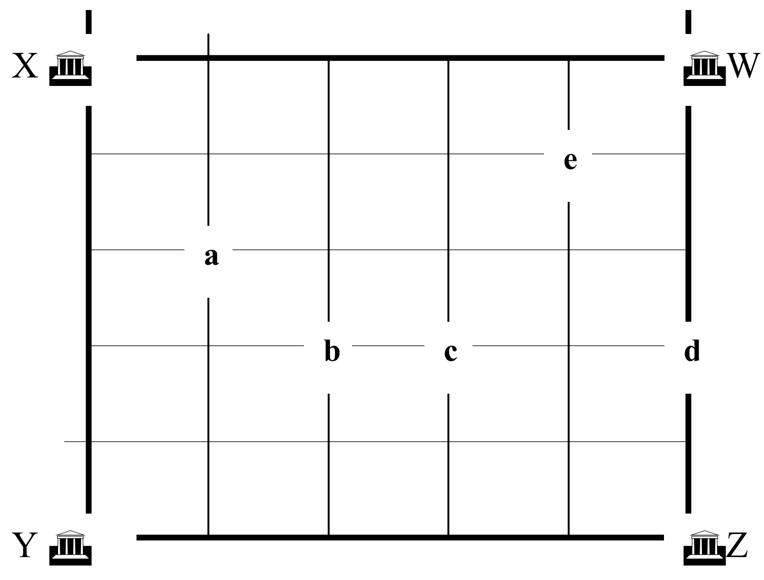
So, x=5 is the only value.

Answer to Part 7:

(Background) Say there is a small county called Kyrandia, which consists of four towns, W, X, Y, and Z. The population in these towns is distributed such that 30 people reside in Town W, 35 people reside in Town X, 13 people reside in Town Y, and 22 people reside in Town Z.

The County is run by "the Board," to which each town can send one deputy. By convention, the chairperson of "the Board" is the deputy from the largest town. Importantly, the chairperson holds agenda control power.

(Issue) Wal Mart wants to open a new branch in Kyrandia and its plan is under "the Board's" consideration. The members of the Board know that people in Kyrandia want to have Wal Mart closer to their town but that only 5 locations (a, b, c, d, e) are available. A map of possible locations is provided below. For simplicity's sake, assume people in a town hold identical preferences in terms of location.



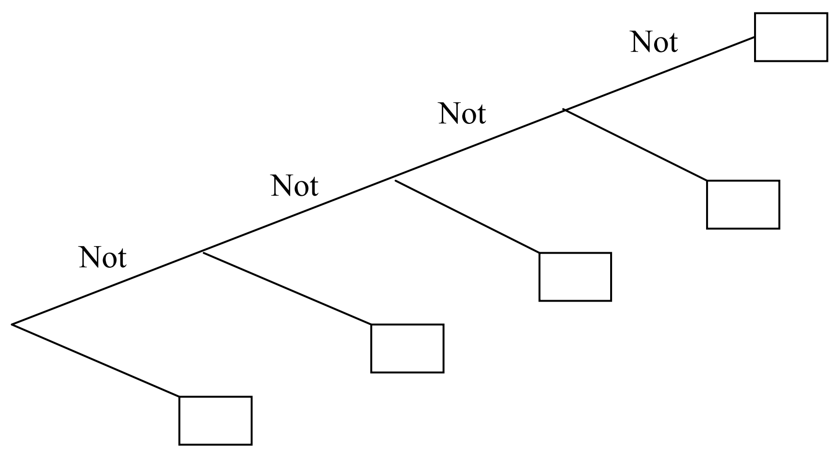
1. Since the location of Wal Mart is the most salient issue for everyone in Kyrandia, "the Board" decided to determine its location based on a voting procedure in which people vote on candidate locations for Wal Mart sequentially. For example, people vote on "a" or not. If "a" loses, then they vote on "b" or not, and so on and so forth (recall the agenda tree from the slides presented in lecture). Though not a critical issue, members of "the Board" are assumed to vote. If you were the chairperson whose preference were identical to people's in his/her hometown, which location is the best outcome you can bring to yoru hometown people.? Explain why? (6 points)

W (30)	X (35)	Y (13)	Z (22)
e	a	B	d
d	b	A	c
c	e	C	b
b	c	D	e
a	d	E	a

b > a, b > e
c > a, c > b
d > a, d > b, d > c
e > a, e > c, e > d

b is the best location

2. Draw the agenda tree in which you arrive at the answer from the above question. (3points)

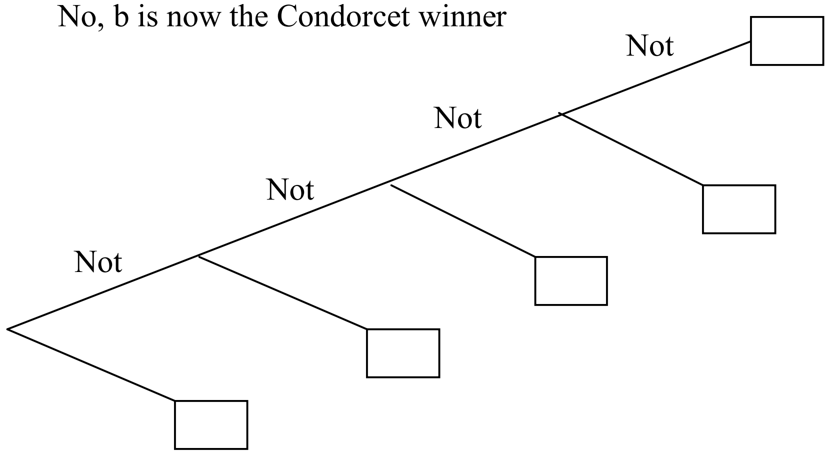


b – e – d- c – a (c and a interchangeable)

3. People moved their town (to get easier access to Wal Mart). The population is distributed such that 15 people live in Town W, 40 people live in Town X, 25 people live in Town Y and 20 people live in Town Z. In addition, reforms were made to "the Board" due to complaints that the larger town had too much influence on decisions. As a result of these reforms, the deputy from the smallest town holds the chair. Assume that you are the chairperson who wants to relocate Wal Mart. Can you control the voting agenda to move

Wal Mart to a location that your townspeople would prefer? If you can, draw an agenda tree demonstrating voting agenda. If you cannot, explain why? (3points)

a > c, a > d, a > e
b > a, b > c, b > d, b > e
c > d
e > c, e > d No, b is now the Condorcet winner



Answer to Part 8

You are a professional match maker. The most desperate customers are four 40-year-old single men, Alex, Brian, Chuck, and David and four 40-year-old single women, Evie, Frida, Geanette, and Heidi. You organized a group blind date, and found Alex prefers Heidi best, Geanette next, Frida next, and Evie least. Brian prefers Frida best, Geanette next, Heidi next, and Evie least. Chuck's preference ordering (from best to worst) is Frida, Geanette, Heidi, Evie. David's ordering is Geanette, Frida, Heidi, Evie. Evie's ordering is Alex, Chuck, Brian, David. Frida's ordering is Alex, David, Chuck, Brian. Geanette's ordering is Alex, David, Brian, Chuck. Heidi's ordering is Chuck, Brian, David, Alex.

a. How many possible matching are there?
   4! = 24

b. Among the possible sets of stable matchings, which matching is most preferred by the men?

   Find the match for men-ask. The match is the most preferred one (dating prefers the askers)

   The preferences for the men and women are listed as follows

A	**B**	**C**	**D**
H	F	F	G
G	G	G	F
F	H	H	H
E	E	E	E

E	**F**	**G**	**H**
A	A	A	C
C	D	D	B
B	C	B	D
D	B	C	A

Let's use the men-ask algorithm. All men ask their most preferred, so:
(A-H, B-F, C-F, D-G)
Frida likes C more than B, so she rejects B's invitation. Now, B asks his next favorite, G:
(A-H, B-G, C-F, D-G)
Granette likes D better than B, so she rejects B. Now, B asks his next favorite, G. BUT since he knows that G will reject B, he now asks H:
(A-H, B-H, C-F, D-G)
H likes B more than A, so A is rejected. A now asks G:
(A-G, B-H, C-F, D-G)
G likes A better and so rejects D. D now asks F:
(A-G, B-H, C-F, D-F)
F likes D better than C, so rejects C. C now asks G. Looking at G's preferences, she likes C the least and will surely reject her. Now C asks H:
(A-G, B-H, C-H, D-F)
H likes C better than B, so rejects B. B now asks E because he already asked F, G, and H:
(A-G, B-E, C-H, D-F)
THIS IS STABLE SINCE EVERYONE GOT AN INVITATION.

c. Among the possible sets of stable matchings, which matching is most preferred by the women?
   Let's use the women-ask algorithm. All women ask their most preferred men, so:
      (E-A, F-A, G-A, H-A)
   A likes H best, so rejects everyone else. G now asks D, F asks D and E asks C:
      (E-C, F-D, G-D, H-A)
   D likes G more than F. So F asks C:
      (E-C, F-C, G-D, H-A)
   C likes E the least and so rejects her. E now ask B:
      (E-B, F-C, G-D, H-A)
   THIS IS STABLE SINCE EVERYONE GOT AN INVITAITON.

d. Recommend men-ask or women-ask. Why?

   This depends on what you think is fair. For women-ask, all men except for B got their first choice, but E got worst. For men-ask, not everyone got first choice and B still got worst.