Part 1.

(a) There are two taco sellers in a town, Jose and Alex. They each set their taco prices at either \$5, \$10, or \$12. Jose has 3,000 loyal customers who will buy Jose's tacos regardless of price. Alex has 4,000 loyal customers who will buy Alex's tacos regardless of price. There are also 3,000 floating customers in a town. If Jose and Alex have different prices, these floating customers will buy from whoever offers the lower price. However, they will NOT buy any taco if Jose and Alex have same price because these floating customers will lose their interest in having tacos.

(a) Represent this as a strategic form game. Payoffs are each seller's revenues. (2 points)

Alex

Jose	\$5	\$10	\$12
\$5	3K, 4K	6K, 4K	6K, 4K
\$10	3K, 7K	3K, 4K	6K, 4K
\$12	3K, 7K	3K, 7K	3K, 4K

$\Rightarrow$

Alex

Jose	\$5	\$10	\$12
\$5	15K, 20K	30K, 40K	30K, $\boxed{48K}$
\$10	30K, 35K	30K, 40K	60K, 48K
\$12	36K, 35K	36K, 70K	36K, 48K

(b) Find all pure strategy and mixed strategy Nash equilibria. (3 points)

PSNE : (Jose \$10, Alex \$12)

(Jose \$12, Alex \$10)

MSNE

Alex

Jose	~~\$5~~	\$10 (q)	\$12 (1−q)
~~\$5~~	~~15, 20~~	~~30, 40~~	~~30, 48~~
\$10 (p)	30, 35	30, 40	60, 48
\$12 (1−p)	36, 35	36, 70	36, 48

① ②

$EP_J(\$10) = 30q + 60(1-q) = -30q + 60$

$EP_J(\$12) = 36q + 36(1-q) = 36$

$\Rightarrow 30q = 24. \quad \boxed{q = 4/5}$

$EP_A(\$10) = 40p + 70(1-p) = -30p + 70$

$EP_A(\$12) = 48$

$\Rightarrow 30p = 22 \quad \boxed{p = 11/15}$

(Part 1 continued)

Now, 1,000 customers who used to be loyal to Alex have changed their minds and become loyal to Jose. Therefore, Jose now has 4,000 loyal customers and Alex has 3,000 loyal customers. There are still 3,000 floating customers.

(c) Represent this as a strategic form game. Payoffs are each seller's revenues. (2 points)

Alex

Jose	\$5	\$10	\$12
\$5	4K, 3K	7K, 3K	7K, 3K
\$10	4K, 6K	4K, 3K	7K, 3K
\$12	4K, 6K	4K, 6K	4K, 3K

⇒

Alex

Jose	\$5	\$10	\$12
\$5	20K, 15K	35K, 30K	35K, 36K
\$10	40K, 30K	40K, 30K	70K, 36K
\$12	48K, 30K	48K, 60K	48K, 36K

(d) Find all pure strategy and mixed strategy Nash equilibria. (3 points).

PSNE : (Jose \$10, Alex \$12)
(Jose \$12, Alex \$10)

MSNE

Alex

Jose	\$5	\$10 (q)	\$12 (1-q)
~~\$5~~	~~20. 15~~	~~35. 30~~	~~35. 36~~
\$10 (p)	40. 30	40. 30	70. 36
\$12 (1-p)	48. 30	48. 60	48. 36

① ②

$EPJ(\$10) = 40q + 70(1-q) = -30q + 70$

$EPJ(\$12) = 48$

$\Rightarrow 30q = 22$ $\boxed{q = 11/15}$

$EPA(\$10) = 30p + 60(1-p) = -30p + 60$

$EPA(\$12) = 36$

$\Rightarrow 30p = 24$ $\boxed{p = 4/5}$

(e) Jose has more loyal customers in the second story. How did this change affect Jose's behavior? Specifically, how did it change the probability that Jose sets the highest price? (2 points) $1-p$: $4/15 \rightsquigarrow 1/5$ (decreased)

Part 2.

(a) Here are five people's (1, 2, 3, 4, 5) preference orders over four candidates (a, b, c, d), where the most-preferred is listed first and the least-preferred is listed last (for example, person 1 likes a best and d worst).

1	2	3	4	5
a	a	a	c	c
b	c	b	b	b
c	d	c	d	d
d	b	d	a	a

(a) Who is the plurality winner? (2 points)

a

(b) Who is the Borda count winner? (2 points)

$a = 3(3) + 2(0) = 9$

$b = 4(2) + 1(0) = 8$

$c = 2(3) + 1(2) + 2(1) = 10$

$d = 3(1) + 2(0) = 3$

↳ c is the Borda count winner

(c) Who is the approval voting winner, when people vote for their top two choices? (2 points)

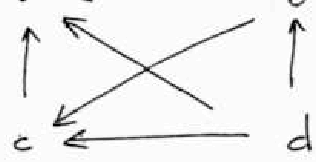
a = 3 counts

b = 4 counts

c = 3 counts

↳ b is the approval voting winner

(d) Is there a Condorcet winner? If so, who? (2 points)



a is the Condorcet winner

shortcut: just by looking at the char

the majority of ppl prefers a

(Part 2 continued)

(e) There are 15 people in a town. They are going to elect a representative. There are 7 candidates (a, b, c, d, e, f, g). The 15 people will vote for candidates who are on the top of their preference orders. If a candidate gets a majority (8 votes), the candidate will win. However, if no candidate wins a majority, there will be a second round. In the second round, the candidate(s) who got the least votes will be eliminated. Then, the 15 people will vote again, given that some candidate or candidates were eliminated already. (For example, if candidate 'b' was eliminated after the first round, the person 1 will vote for candidate 'g' in the second round.) If there is a candidate who gets a majority, the candidate is a winner. However, if there is no winner even in the second round, they will have a third round, and so forth. Who will be the winner in the end AND how many votes will that candidate get? (4 points)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
~~b~~	e	e	~~f~~	~~d~~	~~c~~	e	~~a~~	g	g	~~c~~	~~d~~	e	g	g
g	b	d	~~a~~	~~c~~	e	a	e	f	c	~~d~~	~~b~~	d	c	f
c	a	f	~~d~~	~~b~~	b	b	g	a	f	~~b~~	~~a~~	c	d	a
a	g	a	~~b~~	~~a~~	d	c	b	c	a	e	~~c~~	b	e	e
f	d	g	~~c~~	g	g	d	f	e	e	a	e	f	f	b
e	c	b	e	e	a	f	c	d	b	g	f	g	a	d
d	f	c	g	f	f	g	d	b	d	f	g	a	b	c

- No majority 1st round → 2nd round w/ b, f, a eliminated (only 1 vote

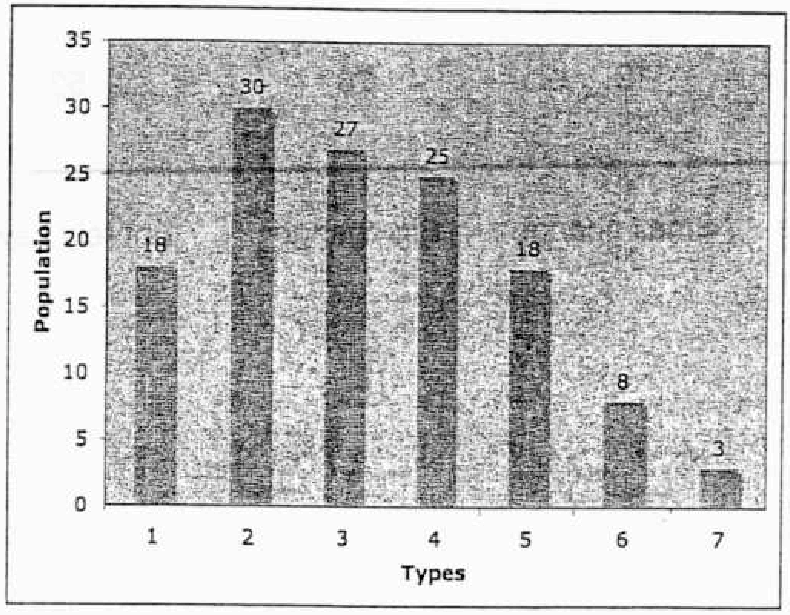
- 2nd round: c receives: 2
  e receives: 5 (voter 8 votes for e b/c a eliminated)
  d receives: 3 (voter 4 votes for d b/c f and a eliminated
  g receives: 5 (voter 1 votes for g b/c b eliminated)
  ↳ No majority → 3rd round w/ c eliminated

- 3rd round: e receives: 6 (voter 6 votes for e b/c c eliminated)
  d receives: 4 (voter 11 votes for d b/c c eliminated)
  g receives: 5
  ↳ No majority → 4th round w/ d eliminated
  see chart for ~~votes~~ votes

- 4th round: e receives: 9 ⇒ $\boxed{\text{e wins w/ 9 votes}}$
  g receives: 6

Part 3.

(a) Two candidates are running for President in "Polistan" in the newly independent country's first elections. There are 7 "types" of people in the population, as shown in the figure below. For example, there are 18 people of type 1 and 8 people of type 6. Each candidate chooses a position from 1 to 7, and each voter votes for the candidate whose position is closest to their own, as in the Downsian model. Each candidate tries to maximize his/her total number of votes. Where will each candidate locate? (4 points)



129

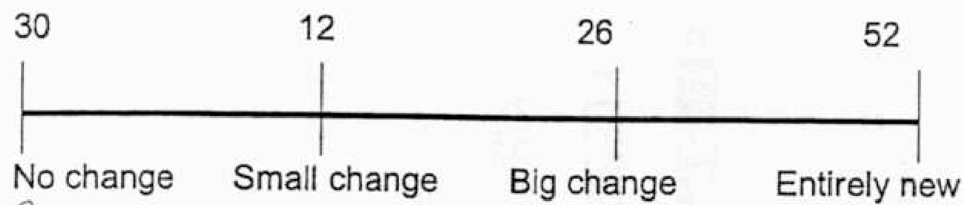
II

I	1	2	3	4	5	6	7
1	64.5 64.5	18. 111	33. 91	48 81	61.5 67.5	75. 54	87.5, 41.5
2	111 18	64.5 64.5	48 81	61.5 67.5	75 54	87.5. 41.5	100. 29
3	91. 33	81. 48	64.5 64.5	75. 54	87.5 41.5	100. 29	109 20
4	81 48	67.5. 61.5	54 75	64.5 64.5	100 29	109. 20	118 11
5	67.5 61.5	54 75	41.5 87.5	29 100	64.5 64.5	118. 11	122, 7
6	54 75	41.5 87.5	29 100	20. 109	11 118	64.5 64.5	126 3
7	41.5 87.5	29. 100	20. 109	11. 118	7. 122	3. 126	64.5 64.5

P

(b) Say Candidate 1 narrowly wins the election and becomes President. He now wants to revise the country's constitution entirely in order to improve property rights and other laws that will be conducive to economic growth. However, not everyone in Parliament agrees with him: 30 members want no change at all, 12 want only a small change, 26 want a big change, and 52 agree with the President's position of an entirely new constitution. A two-thirds majority is required for any change to be made. What will the President propose? (4 points)

$\underline{120} \to 2/3 = 8$



SQ

Propose "Entirely new" → 78 : 42 → No

Propose "Big change" → 84 : 36 → Yes

(c) The new President is not happy with the change in the constitution that was approved by the Parliament. He has therefore decided to take the issue to the public to vote for it in a referendum. The referendum will be held in 1 week. During this time the opposition will campaign against the President. A survey (shown below) shows the results of a survey in which citizens were asked their preferences. For example, 20% of the population most prefers a small change in the constitution. Using the Downsian model, which position should the two parties take (they decide simultaneously) to maximize the total number of votes they receive? (4 points)

Preference	% Respondents
No Change	24
Small Change	20
Big Change	26
New Constitution	30

No Sml Big New

24 20 26 30

Op

Pr	No	Small	Big	New
No	50 50	24 76	34.66	44 56
Small	76.24	50.50	44 56	57 43
Big	66 34	56 44	50 50	70 30
New	56 44	43.57	30 70	50 50

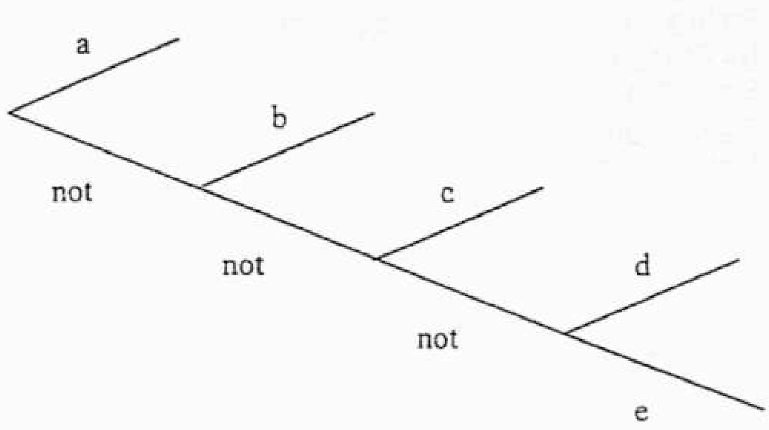
PSNG = (Pr Big, Op Big)

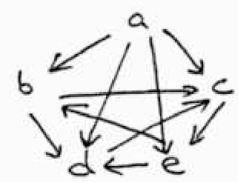
P

Part 4.

Say that there are three people deciding by majority rule over five candidates a, b, c, d, and e. Person 1's preferences (from best to worst) are c, d, b, a, e. Person 2's preferences (from best to worst) are b, d, e, a, c. Person 3's preferences (from best to worst) are e, c, d, b, a. Consider voting agendas in which people vote on candidates sequentially.

We can write an agenda as a tree. For example, in the following tree, people first vote on a or not. If a loses, then they vote on b or not. If b loses, then they vote on c or not. If c loses, then they vote on d versus e.



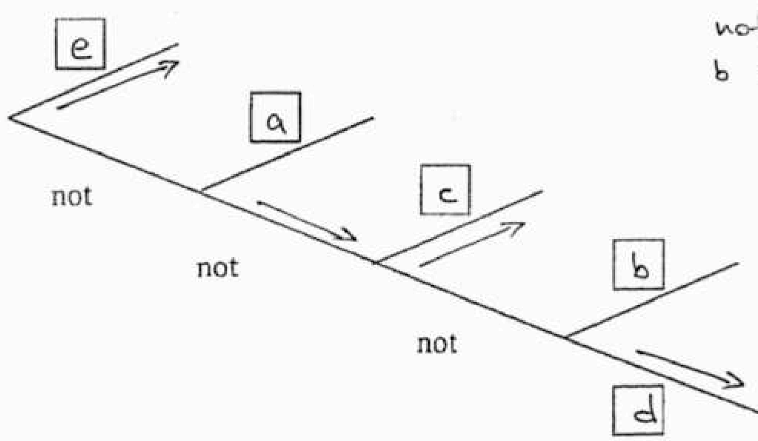


c & d [redacted]
most votes

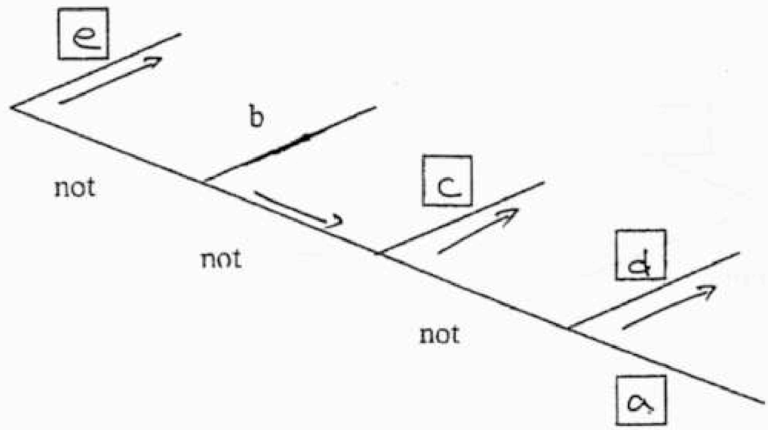
1	2	3
c	b	e
d	d	c
b	e	d
a	a	b
e	c	a

(a) Is there an agenda in which they decide on e? If so, fill in the blanks on the agenda below. If not, explain why not. (3 points)

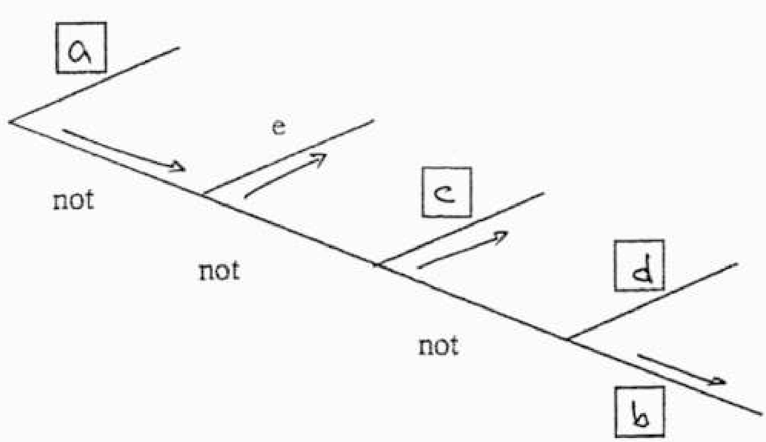
For e to be chosen, must arrange letters ~~[illegible]~~ s.t. e will not be directly compared to b or d.



(b) Assume that b is voted on in the second round (see the agenda below). Can you fill in the blanks in the agenda below so that e is chosen? If so, fill in the blanks. If not, explain why not. (3 points)

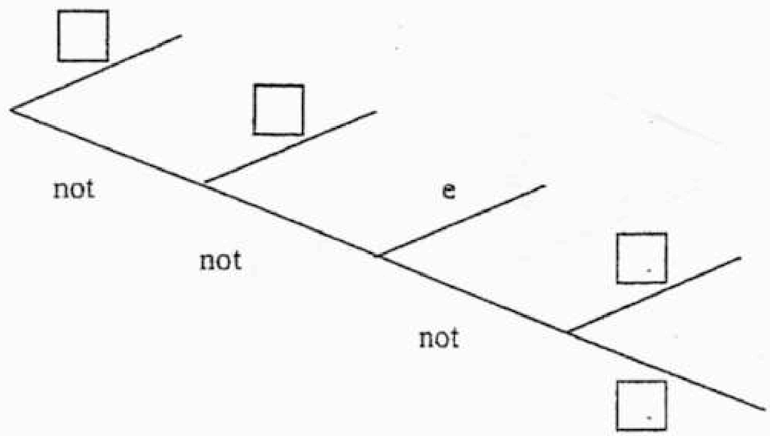


(c) Assume that e is voted on in the second round (see the agenda below). Can you fill in the blanks in the agenda below so that e is chosen? If so, fill in the blanks. If not, explain why not. (3 points)



P

(d) Assume that e is voted on in the third round (see the agenda below). Can you fill in the blanks in the agenda below so that e is chosen? If so, fill in the blanks. If not, explain why not. (3 points)



Not possible. Since both b & d beats e, we need both b & d to be eliminated s.t. e would only face c or a. Since here only b or d can be eliminated in the bottom (last round), e would eventually face b or d, & e would lose.

Part 5.

In a simplified version of "Battleship," say that there are six spaces, numbered 1, 2, 3, 4, 5, 6. Person 1 chooses to fire a missle at one of these six spaces. Person 2 has a ship which is three spaces long, and chooses where to put the ship on the board: she can either put it on spaces 1, 2, and 3, on spaces 2, 3, and 4, on spaces 3, 4, and 5, or on spaces 4, 5, and 6. The people make their choices simultaneously.

(a) Say that person 1 gets a payoff of 1 if her missile hits person 2's ship and a payoff of 0 if her missile misses. Say person 2 gets a payoff of 0 if the missile hits and a payoff of 1 if the missile misses. Model this as a strategic form game and make a prediction. (4 points)

II

I	123	234	345	456	
1	1, 0	0, 1	0, 1	0, 1	by 3
2	1, 0	1, 0	0, 1	0, 1	by 3
3	1, 0	1, 0	1, 0	0, 1	
4	0, 1	1, 0	1, 0	1, 0	
5	0, 1	0, 1	1, 0	1, 0	by 4
6	0, 1	0, 1	0, 1	1, 0	by 4

by 123   by 456

II

⇒

I	123	456
3	1, 0	0, 1
4	0, 1	1, 0

MSNE : $p = 1/2$ $q = 1/2$

P

(b) Now say that there are 7 spaces. Thus person 1 has 7 possible strategies and person 2 has 5 possible strategies. Model this as a strategic form game and make a prediction. (4 points)

II

I	(123)	234	345	456	567	
~~1~~	~~1.0~~	~~0.1~~	~~0.1~~	~~0.1~~	~~0.1~~	b/3
~~2~~	~~1.0~~	~~1.0~~	~~0.1~~	~~0.1~~	~~0.1~~	b/3
3	1.0	1.0	1.0	0.1	0.1	
4	0.1	1.0	1.0	1.0	0.1	
5	0.1	0.1	1.0	1.0	1.0	
~~6~~	~~0.1~~	~~0.1~~	~~0.1~~	~~1.0~~	~~1.0~~	b/5
~~7~~	~~0.1~~	~~0.1~~	~~0.1~~	~~0.1~~	~~1.0~~	b/5

b/567

b/123 b/123

II

⇒

I	(123)	567	
3	1.0	0.1	
~~4~~	~~0.1~~	~~0.1~~	b/3
5	0.1	1.0	

II

⇒

I	(123)	567
3	1.0	0.1
5	0.1	1.0

MSNE: $p = 1/2$, $q = 1/2$

(c) Assume again that there are 6 spaces. Now say that if person 1's missile hits the center of person 2's ship, then person 1 gets a payoff of 2. If person 1's missile hits the "edges" of person 2's ship, then person 1 gets a payoff of 1. If person 1's missile misses, she gets a payoff of 0. Person 2 still gets 0 if her ship is hit (anywhere) and 1 if the missile misses.

For example, if person 1 fires the missile at 4 and person 2's ship is on 3, 4, 5, then person 1 gets payoff 2 and person 2 gets payoff 0. If person 1 fires the missile at 3 and person 2's ship is on 3, 4, 5, then person 1 gets payoff 1 and person 2 gets payoff 0. Model this as a strategic form game and make a prediction. (4 points)

II

I	123	234	345	456	
~~1~~	~~1.0~~	~~0.1~~	~~0.1~~	~~0.1~~	by 3
2	2.0	1.0	0.1	0.1	
3	1.0	2.0	1.0	0.1	
4	0.1	1.0	2.0	1.0	
5	0.1	0.1	1.0	2.0	
~~6~~	~~0.1~~	~~0.1~~	~~0.1~~	~~1.0~~	by 5

b, 123 by 456

⇒

II

I	123	456	
2	2.0	0.1	
~~3~~	~~1.0~~	~~0.1~~	b, 2
~~4~~	~~0.1~~	~~1.0~~	by 5
5	0.1	2.0	

⇒

II

I	123	456
2	2.0	0.1
5	0.1	2.0

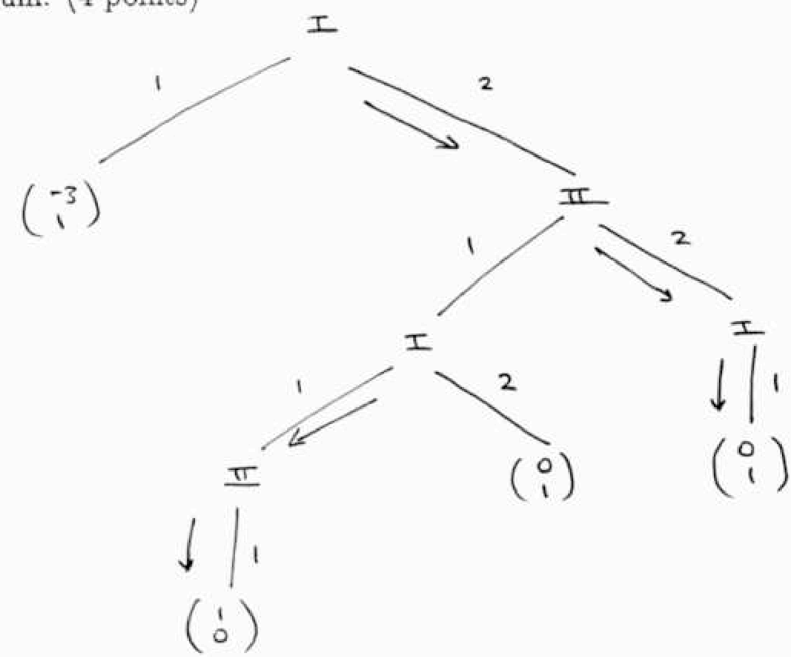
$EP_I(2) = 2q$
$EP_I(5) = 2-2q$ ⇒ $q = 1/2$

$EP_{II}(123) = 1-p$
$EP_{II}(456) = p$ ⇒ $p = 1/2$

P

Part 6. Say that person 1 and person 2 are playing a drinking game which goes like this. There are m beers in the refrigerator. Person 1 goes first by drinking either 1 or 2 beers. Then person 2 can drink either 1 or 2 beers. Then person 1 can drink either 1 or 2 beers, and so forth. In other words, when it is a person's turn to drink, she can drink either 1 or 2 beers. Whoever drinks the last beer loses, and the other person wins. Winning the game yields a payoff of 1 and losing yields a payoff of 0. However, there is an additional feature to the game: there is a "magic number" x (which is greater than 0 and less than m). If after your turn, there are exactly x beers left, then you lose the game and also have to go out and buy more beer; this has a payoff of -3 for the loser and a payoff of 1 for the winner.

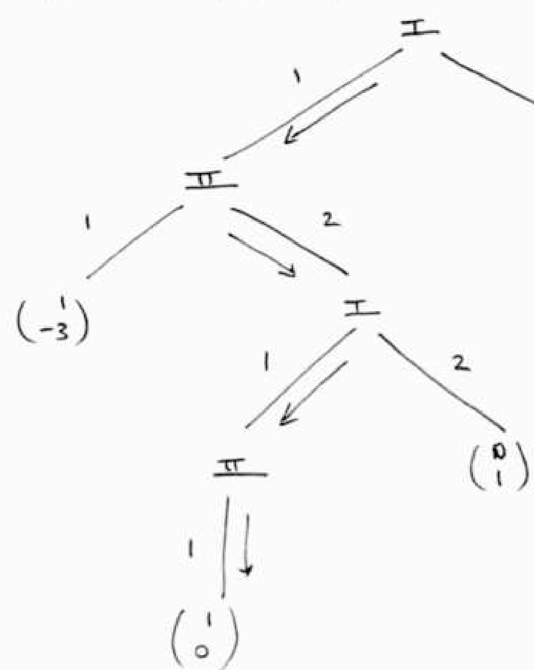
(a) Say that $m = 5$ and $x = 4$. Model this as an extensive form game and find a subgame perfect Nash equilibrium. (4 points)



SPNE:

$\{2,1,1 \,;\, 2,1\}$

(b) Say that $m = 5$ and $x = 3$. Model this as an extensive form game and find a subgame perfect Nash equilibrium. (4 points)



SPNE:

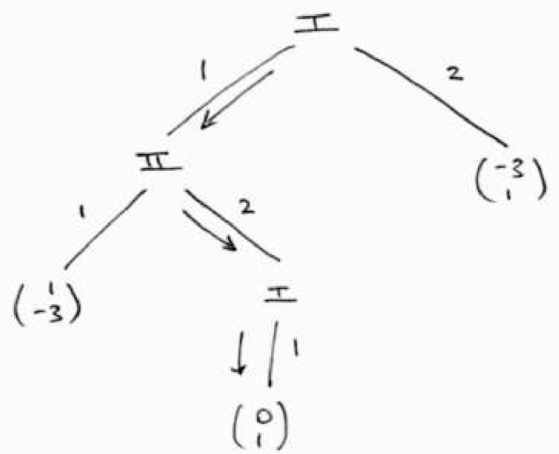
$\{1,1 \,;\, 2,1\}$

(c) Now let m and x be any number. Find a subgame perfect Nash equilibrium. For what values of m and x can person 1 guarantee a win? For what values of m and x can person 2 guarantee a win? (4 points)

So far we did the following:

m : odd	m : odd	m : even	m : ev
x : even	~~x~~ : odd	x : even	x : od
II wins	I wins		?

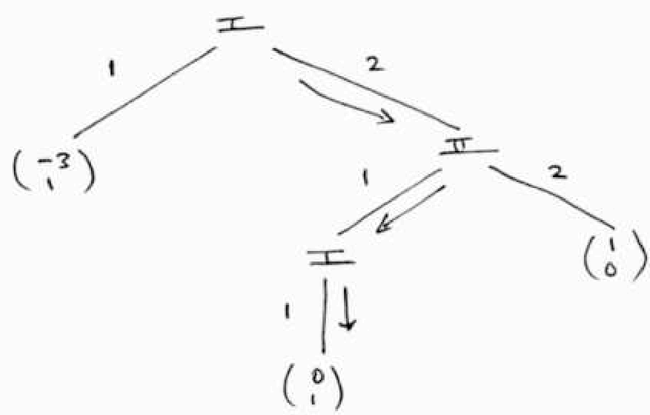
Let m & x be even: m = 4, x = 2



SPNE: {1, 1 ; 2}

$\boxed{\text{II wins}}$

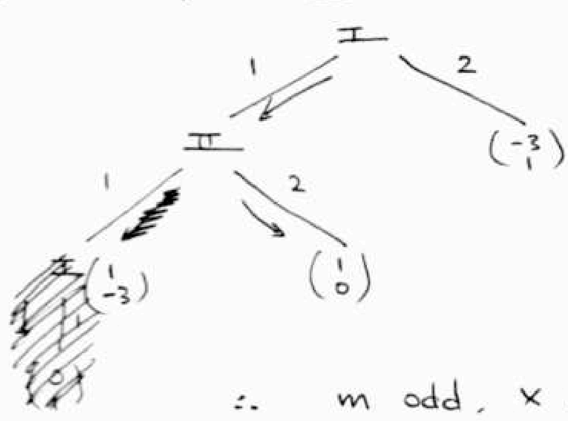
Let ~~m & x be~~ m = 4, x = 3:



II wins

So, candidate is m odd, x odd I wins. Double check:

Let m = 3, x = 1:



$\boxed{\text{I wins}}$

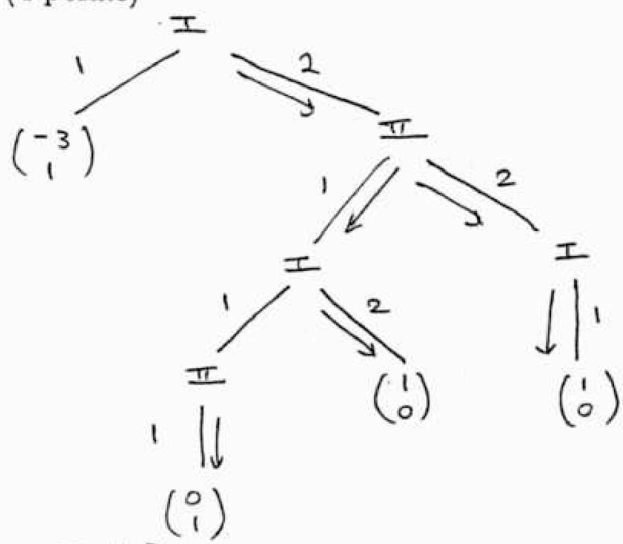
this repre[...] the last s[...] whenever [...] x odd. Al[...] it doesn't [...] look exactly [...] this, the [...] will be the [...]

∴ m odd, x odd I wins. II wins otherwi[...]

Example 2 for Q6: let's assume whoever drinks the last beer wins

Part 6. Say that person 1 and person 2 are playing a drinking game which goes like this. There are m beers in the refrigerator. Person 1 goes first by drinking either 1 or 2 beers. Then person 2 can drink either 1 or 2 beers. Then person 1 can drink either 1 or 2 beers, and so forth. In other words, when it is ~~a~~ person's turn to drink, she can drink either 1 or 2 beers. Whoever drinks the last beer ~~loses,~~ wins and the other person wins. Winning the game yields a payoff of 1 and losing yields a payoff of 0. However, there is an additional feature to the game: there is a "magic number" x (which is greater than 0 and less than m). If after your turn, there are exactly x beers left, then you lose the game and also have to go out and buy more beer; this has a payoff of -3 for the loser and a payoff of 1 for the winner.

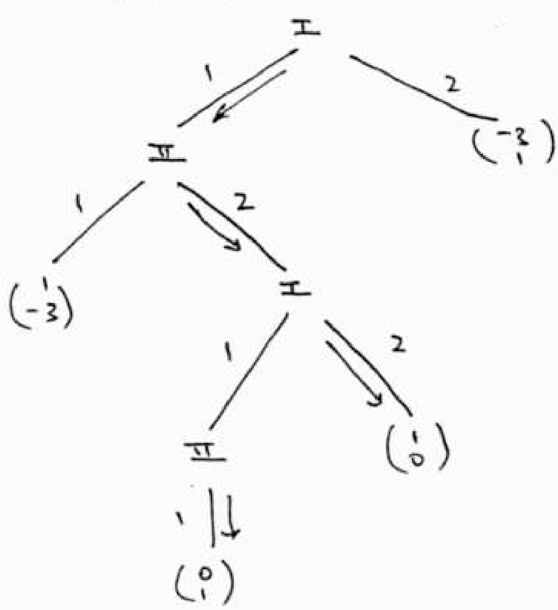
(a) Say that $m = 5$ and $x = 4$. Model this as an extensive form game and find a subgame perfect Nash equilibrium. (4 points)



SPNE 1 : {2, 2.1 ; 2, 1}

2 : {2.2.1 ; 1, 1}

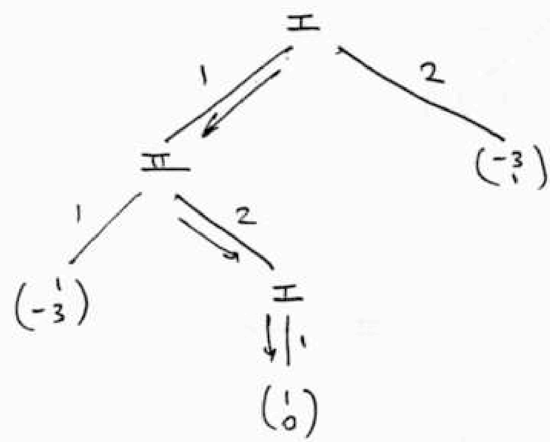
(b) Say that $m = 5$ and $x = 3$. Model this as an extensive form game and find a subgame perfect Nash equilibrium. (4 points)



SPNE:
{1.2 ; 2.1}

(c) Now let m and x be any number. Find a subgame perfect Nash equilibrium. For what values of m and x can person 1 guarantee a win? For what values of m and x can person 2 guarantee a win? (4 points)

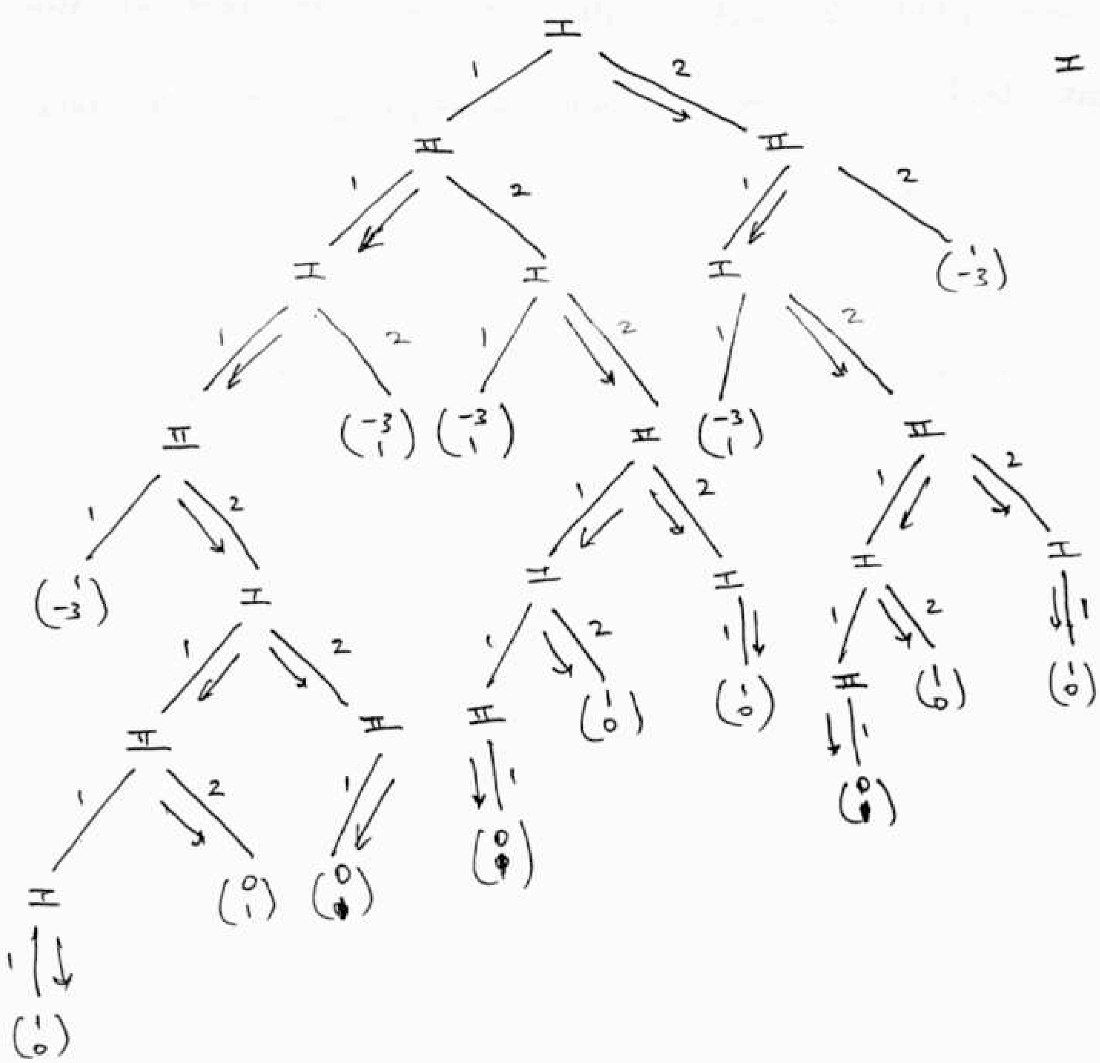
Let m be 4 & x be 2:



I wins

Let m be 8 & x be 4:

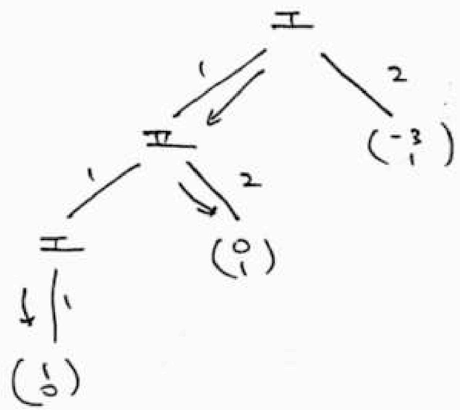
I wins



From HW 9 Q4, we know

$m = 6, \quad x = 4$ II wins.

Let $m = 3, \quad x = 2$:



II wins.

By backwards induction, we know if x is even number, any multip of 3 as m can yield 2 victory b/c if we are left w/ the above as the ~~last~~ last "subgame", which multiple of 3 will leave us II can guarantee a win.

$\therefore$ $m =$ multiple of 3, $x =$ even number can guarantee II a win. I wins otherwise

Part 7.

Letter-writing Campaign.

Genocide has been taking place in the southern province of Sudan called Darfur. Horrified by this, 7 UCLA students are wondering whether engage in a weekly letter-writing campaign in order to get the US government to act.
Each of these students has a propensity to act that is determined by the number of other people that they see acting: this is a collective action problem.
If someone sees that too few people are writing letters on a given week, she/he will think that the campaign will not be fruitful and will spend her/his energy on something else the following week (for example, fundraising to buy a full page in the daily bruin).
If someone sees that a lot of people are involved in the campaign on a given week, they will feel that their contribution does not matter so much because so many other people are writing letters already and she/he will engage in something else the following week (for example, joining a letter-writing campaign to convince their congressperson to support universal healthcare).

To sum up, someone will only be active with the Darfur Action Committee on a particular week if enough other people (no less than their lower threshold) but not too many other people (no more than their upper threshold) wrote letters the previous week. We denote an active participant with the letter **'P'** and a non-active student with the letter **'N'**. The name of the 7 students are Alice, Bob, Charlie, Diane, Emily, Frances and George (which we can shorten to A, B, C, D, E, F, and G)

The lower and upper thresholds for the 7 students are listed in the table below (feel free to use the rest of the table as draft paper).

Student	Lower Thresh.	Upper Thresh.
A	0	6
B	1	6
C	3	5
D	.3	5
E	4	6
F	4	5
G	5	6

p	p		n	p		p	p	
p	p		n	n		p	p	
p	n		n	n		n	n	
p	n		n	n		n	n	
p	p		n	n		n	n	
p	n		n	n		n	n	
p	p		n	n		n	n	

(a) Is the situation where everyone participates a Nash Equilibrium? What about when no-one participates? Circle the right answers below. (2 points)

(p,p,p,p,p,p,p) is a Nash Equilibrium yes no
(n,n,n,n,n,n,n) is a Nash Equilibrium yes no

(b) Write down a Nash Equilibrium which is neither (p,p,p,p,p,p,p) or (n,n,n,n,n,n,n). Fill in the table below. (2 points)

Student	A	B	C	D	E	F	G
Action	P	P	N	N	N	N	N

(c) Assume that on week 1 everyone but G is participating. What happens on week 2? What about on week 5? On week 6? Fill the corresponding columns in the table bellow. (6 points)

Student	Lower Thresh.	Upper Thresh.	Week 1	2	3	4	5	6
A	0	6	P	P	P	P	P	P
B	1	6	P	P	P	P	P	P
C	3	5	P	P	N	P	P	N
D	3	5	P	P	N	P	P	N
E	4	6	P	P	P	N	P	P
F	4	5	P	P	N	P	P	N
G	5	6	N	P	P	N	P	P

2. 5 8 11 14 17 20.

(d) Describe the situation on week 23 by filling the table below. (2 points)

Student	Week 23
A	P
B	P
C	P
D	P
E	P
F	P
G	P

Part 8.
Help me find my match !

After visiting UCLA's Career Center, you decide that you want to become a matchmaker.
You are applying to the *School of Matchmaking & Relationship Sciences* that describes itself as "an academy for would-be Cupids" but you have to prove that you are worthy of admission before they will let you in.
You are provided with the profiles of 10 heterosexual singles that are looking for their soul-mate. There are 5 men (A through E) and 5 women (V through Z). After careful study of their profiles, you have determined that their preference ordering is as described in the table below.

Man	Likes best	<-------------	--------------	------------->	Likes worst
A	V	W	X	Y	Z
B	V	X	W	Z	Y
C	V	W	Z	Y	X
D	Y	Z	V	W	X
E	Y	Z	V	W	X
Woman	Likes best	<-------------	--------------	------------->	Likes worst
V	A	B	C	D	E
W	C	D	A	B	E
X	D	B	C	A	E
Y	C	A	B	E	D
Z	C	B	D	A	E

(a) How many possible matchings are there? (4 points)

Write you answer here → 120

5!

↳ A takes on 5 possible, B takes on 4, C on 3, D on 2, E 1

(b) Assuming that the men are paying for access to your services, provide a stable matching that would make them as happy as possible. (4 points)

Man	Match
A	V
B	X
C	W
D	Z
E	Y

Men-ask algorithm:

A: V ✓

B: V rejected → X

C: V rejected → W

D: Y rejected → Z

E: Y ✓

Each gets ~~most preferred~~ 1 invite, stable

(c) Assuming that the women are paying for access to your services, provide a stable matching that would make them as happy as possible. (4 points)

Woman	Match
V	A
W	C
X	D
Y	E
Z	B

V: A

W: C ✓

X: D ✓

Y: C reject → A reject → B reject → E

Z: C reject → B ✓

Each gets 1 invite, stable