Final exam PS 30 June 2002

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. Everything you write should be your own work. Those involved in academic dishonesty will be summarily given a grade of F for the course; I will also write a letter recommending expulsion. Partial credit will be given: math mistakes will not jeopardize your grade. The problems all have the same weight. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

Make sure that your complete name is on each page. Number each page like this: "page 1 of 5," "page 2 of 5," and so forth (if you have 5 total pages for example). Write only on the front of each page. Staple your pages together when you turn them in and write your name down on the log. Good luck and have a good summer!

1. In a simplified version of "Battleship," say that there are four spaces, numbered $1, 2, 3, 4$. Person 1 chooses to fire a missle at one of these four spaces. Person 2 has a ship which is two spaces long, and chooses where to put the ship on the board: she can either put it on spaces 1 and 2, on spaces 2 and 3, or on spaces 3 and 4. The people make their choices simultaneously.

a. Model this as a strategic form game and find all mixed-strategy and pure-strategy Nash equilibria.

b. Now say that there are 5 spaces, numbered $1, 2, 3, 4, 5$. Model this as a strategic form game and find all mixed-strategy and pure-strategy Nash equilibria.

2. Say that person 1 and person 2 are playing a drinking game which goes like this. There are m beers in the refrigerator. Person 1 goes first by drinking either 1 or 2 beers. Then person 2 can drink either 1 or 2 beers. Then person 1 can drink either 1 or 2 beers, and so forth. In other words, when it is a person's turn to drink, she can drink either 1 or 2 beers. Whoever drinks the last beer wins the game. Winning the game yields a payoff of 1 and losing yields a payoff of 0. However, there is an additional feature to the game: there is a "magic number" x (which is greater than 0 and less than m). If after your turn, there are exactly x beers left, then you lose the game and also have to go out and buy more beer; this has a payoff of -3 for the loser and a payoff of 1 for the winner.

a. Say that $m = 6$ and $x = 4$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

b. Say that $m = 6$ and $x = 3$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

c. Now let m and x be any number. Find a subgame perfect Nash equilibrium. For what values of m and x can person 1 guarantee a win? For what values of m and x can person 2 guarantee a win?

3. Person 1 and person 2 are meeting at a club sometime tonight. The club is near person 2's apartment, and person 2 chooses to show up at either 8pm or 9pm. Person 1, however, works far away and has to drive the 405 freeway after work to get to the club. Person 1 chooses to leave work at either 6pm or 7pm. The traffic situation on the 405 is random: sometimes traffic is bad and it takes 2 hours for person 1 to get there, and sometimes traffic is good and it only takes 1 hour. The probability that traffic is good is 1/2 and the probability that traffic is bad is 1/2. The traffic situation does not depend on when person 1 leaves work. So if person 1 leaves work at 6pm, and traffic is bad, he gets to the club at 8pm, for example. If person 2 leaves work at 7pm and traffic is good, he gets to the club at 8pm, and so forth.

Person 1 and 2 are friends but they both hate waiting at the club alone. If one person shows up before the other person does, the person who gets there earlier and has to wait gets payoff -10 and the other person feels bad about it and gets payoff 0. If both person 1 and person 2 show up at the club at the same time, they both get payoff 4. Person 1's decision when to leave work and person 2's decision when to arrive at the club are made simultaneously.

a. Say that person 1 knows whether the 405 traffic situation is good or bad but person 2 does not. Model this as an incomplete information game and find all pure-strategy Nash equilbria.

b. Now say that person 2 knows whether the 405 traffic situation is good or bad but person 1 does not. Model this as an incomplete information game and find all pure-strategy Nash equilbria.

c. Now say that person 1 and person 2 both know whether the 405 traffic situation is good or bad. Model this as an incomplete information game and find all pure-strategy Nash equilbria.

d. Finally, say that neither person 1 nor person 2 knows whether the 405 traffic situation is good or bad. Model this as an incomplete information game and find all pure-strategy Nash equilbria.

4. In the Borda count with three alternatives, a person's most-preferred alternative gets 2 points, his second-best gets 1 point, and his worst gets 0 points. In the Borda count with four alternatives, a person's most-preferred alternative gets 3 points, his second-best gets 2 points, and so forth.

a. Say that there are three people who are deciding over three alternatives, a, b, c. Is it possible for the Condorcet winner to have a lower Borda count than some other alternative? (In other words, is it possible for the Condorcet winner to not be a Borda count winner?) If so, write down the preference orderings which make this possible. If not, explain in detail why not.

b. Now say that these three people decide among four alternatives, a, b, c, d. Is it possible for the Condorcet winner to have a lower Borda count than some other alternative? (In other words, is it possible for the Condorcet winner to not be a Borda count winner?) If so, write down the preference orderings which make this possible. If not, explain in detail why not.

5. Say that there are three men A, B, and C and three women X, Y, and Z. Each of these six people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman X best, woman Y next, and woman Z least. Man B prefers woman Y best, woman Z next, and woman X least. Man C prefers woman Y best, woman X next, and woman Z least. Woman X prefers man B best, man A next, and man C least. Woman Y prefers man A best, man B next, and man C least. Woman Z prefers man A best, man C next, and man B least.

a. Say that man A is matched with woman X, man B is matched with woman Y, and man C is matched with woman Z. Is this matching stable?

b. Write down all possible matchings and determine which of them are stable and which are not stable.

c. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?

6. There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability p and B with probability $1 - p$. Find p.