Homework answers IEE1149 Summer 2015

1. Read "You Yawn, We All Yawn—And Empathy May Explain Why" by Alison McCook (available at the "Media Clips" link on the course website). Is the explanation here of why a person yawns a rational choice explanation? Why or why not?
I would say that this is not a rational choice explanation because it does not assume that a person makes a choice of whether to yawn or not.

2. Read "Which Price is Right?" by Charles Fishman (available at the "Media Clips" link). The article mentions a behavior which seems to be a violation of our simple model of individual choice as we defined it in class. Point out the behavior and explain mathematically why a rational choice model (at least the most obvious one) cannot capture this behavior. What do you think explains the behavior?
The article says that if there are two versions of a product, one which sells for \$14.95 and a more glitzy one which sells for \$18.95, then a person typically chooses the \$14.95 one. However, if a \$34.95 version is introduced, then a person is more likely to buy the \$18.95 version.
The most obvious rational choice model of this assigns a payoff to each of the three alternatives. Since a typical person buys the \$14.95 version over the \$18.95 version, the payoff from buying the \$14.95 version (i.e. considering the price and how good the product is) must be higher than the payoff from the \$18.95 version. But when the \$34.95 version is introduced, the typical person buys the \$18.95 version, which means that the payoff from the \$18.95 version must be higher than the payoff from the \$14.95 version (and the payoff from the \$34.95 version). This is a contradiction. Hence a rational choice model cannot explain this behavior.
The standard interpretation of this kind of effect is "framing": when there are only two versions available, a person thinks she is wasting money buying the one which is more expensive but with the same functionality. However, if an even more expensive version is available, then a person thinks that he is "reasonable" if he buys the middle-priced product. For example, it seems that many stores (like stereo and video stores) have very expensive stuff on display which almost no one buys, and perhaps the reason is that these items make the less expensive (but still fairly pricey) stuff seem reasonable.
The interpretation of this which is more rational-choice-like in spirit says that there is always some uncertainty about the quality of a product, and that the relative price of a given product and its position in a product line provides some information about it. For example, a person might think that if a \$34.95 version of the product is available, quality of the product must be an important issue, and hence one shouldn't buy the absolute lowest quality product. Another interpretation is that perhaps a person always uses the rule "Buy the middle of the line product" not because it always is the best choice, but because it saves time and headaches, and that this strategy averaged over one's entire life is the best one.

3. Read "Does Blanket 'Don't Go to Graduate School!' Advice Ignore Race and Reality?" by Tressie McMillan Cottom (available at the "Media Clips" link). Can you make McMillan Cottom's argument using our simple model of individual choice? Or is McMillan Cottom making a different kind of argument?
I think that Cottom's argument can be understood using our simple model of individual choice. It's like the example discussed in class of a person who won't protest if the cost of protesting is too high, but will protest even when costs are high if she just can't stand staying at home and doing nothing.
Say that if you could go to graduate school for free, you would get a utility of 20. But graduate school costs 8. If you come from a middle class family and can thus can count on a lifestyle with utility 15 with just a bachelor's degree, then paying to go to graduate school doesn't make sense, because the net utility of going to graduate school would be $20-8=12$, which is less than 15. But if you come from a working family and expect a lifestyle of utility 10 with just a bachelor's degree, then paying to go to graduate school does make sense, because 12 is greater than 10. In other words, for the person from a working family, graduate school moves you farther ($20-10=10$) than for a person from a middle-class family ($20-15=5$).

4. Say that you and a friend are meeting for lunch. Both you and your friend can either be late or on time. If both of you are on time, you each get a utility of 3. If one is on time and the other is late, the prompt one gets a utility of 1 (since she has to wait around doing nothing) and the tardy one gets a utility of 4 (since she doesn't have to wait). However, if both are late, you don't find each other and you each get a utility of 0.

a. Model this as a strategic form game.
The game looks like this.

	Friend is late	Friend is on time
You are late	0,0	4,1
You are on time	1,4	3,3

This game is the same as the chicken game discussed in class, where late is "straight" and on time is "swerve."

5. Say you and a friend each privately choose a whole number between 0 and 5 (that is: 0, 1, 2, 3, 4, or 5). If you both choose the same number, I will give you both that number times \$100. If your number is exactly one less than your opponent's, however, you will get your opponent's number times \$100 plus a bonus \$100 and your opponent will get nothing. In any other case, both of you get nothing. So for example, if you both choose the number 5, I will give you both \$500. If you choose 4 and your opponent chooses 5, you will get \$600 and your opponent nothing. If you choose 3 and your opponent chooses 5, you both get nothing.

a. Model this as a strategic form game.
This game looks like this:

	2 chooses 0	2 chooses 1	2 chooses 2	2 chooses 3	2 chooses 4	2 chooses 5
1 chooses 0	0, 0	200, 0	0, 0	0, 0	0, 0	0, 0
1 chooses 1	0, 200	100, 100	300, 0	0, 0	0, 0	0, 0
1 chooses 2	0, 0	0, 300	200, 200	400, 0	0, 0	0, 0
1 chooses 3	0, 0	0, 0	0, 400	300, 300	500, 0	0, 0
1 chooses 4	0, 0	0, 0	0, 0	0, 500	400, 400	600, 0
1 chooses 5	0, 0	0, 0	0, 0	0, 0	0, 600	500, 500

Undercut your opponent

b. Read the article "Hollywood's Death Spiral" by Edward Jay Epstein on the web site. Can you use this game to think about the situation described in the article?
The game corresponds roughly to the situation described in the article in which each studio wants to release its DVDs slightly sooner than others. Getting your DVD on store shelves sooner is a competitive advantage. However, the sooner your DVD comes out, the less likely people will come to the theatrical release, since they know that the DVD will be out soon. As studios release their DVDs earlier and earlier, the industry as a whole loses profits.

6. Ann and Bob are each trying to win a prize in a school raffle (lottery). Each can buy either 0, 1, 2, or 3 raffle tickets. Ann and Bob are the only two people in the raffle, and each ticket has an equal chance of winning. So for example, if Ann buys 2 tickets and Bob buys 3 tickets, then Ann has a 2/5 chance of winning and Bob has a 3/5 chance of winning (if no one buys any tickets, the raffle is cancelled). The prize is worth \$60, and both Ann and Bob care about their "expected payoffs": for example, if Ann has a 2/5 chance of winning, her expected payoff is \$24. Model the following situations with strategic form games.

a. Say that raffle tickets are free. What does the game look like?
We simply do some expected value calculations to derive the following:

	2 buys 0 tickets	2 buys 1 ticket	2 buys 2 tickets	2 buys 3 tickets
1 buys 0 tickets	0, 0	0, 60	0, 60	0, 60
1 buys 1 ticket	60, 0	30, 30	20, 40	15, 45
1 buys 2 tickets	60, 0	40, 20	30, 30	24, 36
1 buys 3 tickets	60, 0	45, 15	36, 24	30, 30

Tickets are free

b. Now say that raffle tickets cost \$6 each. What does the game look like?

	2 buys 0 tickets	2 buys 1 ticket	2 buys 2 tickets	2 buys 3 tickets
1 buys 0 tickets	0, 0	0, 54	0, 48	0, 42
1 buys 1 ticket	54, 0	24, 24	14, 28	9, 27
1 buys 2 tickets	48, 0	28, 14	18, 18	12, 18
1 buys 3 tickets	42, 0	27, 9	18, 12	12, 12

Tickets cost \$6 each

c. Now say that raffle tickets cost \$10 each. What does the game look like?

Now the game looks like this:

	2 buys 0 tickets	2 buys 1 ticket	2 buys 2 tickets	2 buys 3 tickets
1 buys 0 tickets	0, 0	0, 50	0, 40	0, 30
1 buys 1 ticket	50, 0	20, 20	10, 20	5, 15
1 buys 2 tickets	40, 0	20, 10	10, 10	4, 6
1 buys 3 tickets	30, 0	15, 5	6, 4	0, 0

Tickets cost $10 each

7. Say that Spy 1 is trying to listen in on Spy 2. There are three rooms, A, B, and C, arranged in a line like this: A—B—C. In other words, A is on the left, B is in the middle, and C is on the right. Each spy must decide independently and simultaneously which room to enter. Their payoffs are determined as follows. If they both choose the same room, then they will see each other, a bloody gun battle will ensue, and both get payoff -10. If they are in adjacent rooms (for example, if Spy 1 is in room A and Spy 2 is in room B) then Spy 1 can set up her eavesdropping equipment and can intercept Spy 2's communications; hence Spy 1 gets a payoff of 5 and Spy 2 gets a payoff of -5. If they are not in adjacent rooms and they are not in the same room (for example, if Spy 1 is in room A and Spy 2 is in room C) then the distance between them is too great for the eavesdropping equipment to work; Spy 1 gets no secrets and Spy 2 gets to keep hers, and so both get a payoff of 0.

a. Model this as a strategic form game.
The game looks like this:

	$2A$	$2B$	$2C$
$1A$	$-10, -10$	$5, -5$	$0, 0$
$1B$	$5, -5$	$-10, -10$	$5, -5$
$1C$	$0, 0$	$5, -5$	$-10, -10$

8. Say that persons 1, 2, and 3 each decide whether to go to restaurant A or restaurant B. Person 1 wants the dinner group to be as large as possible. For person 1, the worst thing is if she goes to a restaurant alone, the best thing is if all three go to the same place, and going with one person (it doesn't matter which) is OK, neither best or worst. Person 2 is the exact opposite; she wants the dinner group to be as small as possible. All person 3 cares about is going to the same place as person 1, since he likes person 1.

a. Model this as a strategic form game.
Each person can go to A or go to B. The game looks like this.

3 goes to A	2 goes to A	2 goes to B
1 goes to A	10, 0, 10	5, 10, 10
1 goes to B	0, 5, 0	5, 5, 0

3 goes to B	2 goes to A	2 goes to B
1 goes to A	5, 5, 0	0, 5, 0
1 goes to B	5, 10, 10	10, 0, 10

b. Now say that person 3 loses interest in person 1 and becomes grouchy like person 2. Model this as a strategic form game.
Now the game looks like this.

	2 goes to A	2 goes to B
1 goes to A	10, 0, 0	5, 10, 5
1 goes to B	0, 5, 5	5, 5, 10

3 goes to A

	2 goes to A	2 goes to B
1 goes to A	5, 5, 10	0, 5, 5
1 goes to B	5, 10, 5	10, 0, 0

3 goes to B

9. Say that there are two people, a security guard and a thief. The security guard can either be vigilant or relax. The thief can either steal or do nothing. If the guard is vigilant, then the thief would rather do nothing than steal. If the guard is relaxed, however, the thief would rather steal than do nothing. If the thief steals, the guard would rather be vigilant than be relaxed. If the thief does nothing, however, the guard would rather be relaxed than vigilant.

a. Model this as a strategic form game. Feel free to choose payoffs which make sense to you. My version of this game looks like this:

	thief steals	thief does nothing
guard is vigilant	5,-10	-2,0
guard is relaxed	-10,10	0,0

10. Read "Running Mates: The Clark-Lieberman Iowa Jailbreak" by William Saletan (at the "Media Clips" link). Model the situation as a strategic form game.
Wesley Clark and Joe Lieberman can each decide whether to stay in the Iowa caucuses or quit. The article argues that the worst thing for either candidate is to be the only one quitting Iowa, because it makes that candidate look like a loser. If Clark quits and Lieberman stays in, Lieberman gets a slightly higher payoff than if both stayed in (since Lieberman gets some of Clark's voters), and similarly, Clark gets a slightly higher payoff if only Lieberman drops out. The best thing for both candidates, however, is for both to quit, so they can concentrate on the upcoming New Hampshire primary. So this is the game I come up with:

	Lieberman stays	Lieberman quits
Clark stays	0,0	1,-10
Clark quits	-10,1	5,5

11. In the movie *Return to Paradise* (see the "Media Clips" link), Sheriff, Tony, and Lewis went to Malaysia and did various illegal things. After Sheriff and Tony left, Lewis was charged and scheduled to be executed. If either Sheriff or Tony returns to Malaysia and admits shared responsibility, Lewis's sentence will be reduced and he will live. If both Sheriff and Tony return, then each will have to serve three years in prison in Malaysia. If only one returns, then that person will have to serve six years in prison. The two players, Sheriff and Tony, can each either decide to go back to Malaysia or stay in New York. Model the situation as a game with two players (Sheriff and Tony). Feel free to choose payoffs which make sense to you (that's what makes the problem kind of interesting).
The two players, Sheriff and Tony, can each either decide to go back to Malaysia or stay in New York. What you think the game is depends on your assumptions about how much guilt Sheriff and Tony will feel.
For example, say that Sheriff and Tony care only about avoiding jail themselves, and all other things equal would be happy to have Lewis alive. Then the game would look like this:

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-1,-1	-0,-6
Sheriff goes back to Malaysia	-6,0	-3,-3

In other words, the best thing would be to have the other guy go back and you stay at home. Here you can think of Lewis's life as worth 1 year of jail time to Sheriff and Tony.
Another possibility is that Lewis's life is worth 4 years of jail time—in other words, neither will go back if he has go back alone. Then the game would look like this:

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-4,-4	-0,-6
Sheriff goes back to Malaysia	-6,0	-3,-3

Note that this game is a Prisoners' Dilemma: if you know the other person is going back, you have the incentive to not get on the plane and stay in NYC; however, if you both stay in NYC, Lewis dies and youre worse off than if you both returned to Malaysia.
Another possibility is that Lewis's life is worth 10 years of jail time—having Lewis die is the absolute worst thing. Then the game would look like this:

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-10,-10	-0,-6
Sheriff goes back to Malaysia	-6,0	-3,-3

Note that this game is a game of chicken, where Lewis dying is like the two cars crashing into each other. Still the best thing for each person is to stay home and have the other person to return to Malaysia.
Finally, say that people feel really guilty and basically internalize the others' pain. So if you Tony goes back to Malaysia and Sheriff doesn't, Sheriff feels just as bad as Tony.

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-10,-10	-6,-6
Sheriff goes back to Malaysia	-6,-6	-3,-3

12. Say that you and a friend are meeting for lunch. Both you and your friend can either be late or on time. If both of you are on time, you each get a utility of 3. If one is on time and the other is late, the prompt one gets a utility of 1 (since she has to wait around doing nothing) and the tardy one gets a utility of 4 (since she doesn't have to wait). However, if both are late, you don't find each other and you each get a utility of 0.

a. Model this as a strategic form game and find all (pure strategy and mixed strategy) Nash equilibria.
The game looks like

	On time	Late
On time	3,3	1,4
Late	4,1	0,0

b. Are there strongly or weakly dominated strategies in this game?
There are no strongly or weakly dominated strategies in this game.

c. Find the (pure strategy) Nash equilibria of this game.
The pure strategy Nash equilibria are (Late, On time) and (On time, late).

13. Say you and a friend each privately choose a whole number between 0 and 5 (that is: 0, 1, 2, 3, 4, or 5). If you both choose the same number, I will give you both that number times $100. If your number is exactly one less than your opponent's, however, you will get your opponent's number times $100 plus a bonus $100 and your opponent will get nothing. In any other case, both of you get nothing. So for example, if you both choose the number 5, I will give you both $500. If you choose 4 and your opponent chooses 5, you will get $600 and your opponent nothing. If you choose 3 and your opponent chooses 5, you both get nothing. Model this as a strategic form game.

a. "Solve" this game by iteratively eliminating weakly dominated strategies.
This game looks like this:

	2 chooses 0	2 chooses 1	2 chooses 2	2 chooses 3	2 chooses 4	2 chooses 5
1 chooses 0	0, 0	200, 0	0, 0	0, 0	0, 0	0, 0
1 chooses 1	0, 200	100, 100	300, 0	0, 0	0, 0	0, 0
1 chooses 2	0, 0	0, 300	200, 200	400, 0	0, 0	0, 0
1 chooses 3	0, 0	0, 0	0, 400	300, 300	500, 0	0, 0
1 chooses 4	0, 0	0, 0	0, 0	0, 500	400, 400	600, 0
1 chooses 5	0, 0	0, 0	0, 0	0, 0	0, 600	500, 500

Undercut your opponent

Note that for player 1, choosing 4 weakly dominates choosing 5. The only way player 1 makes money by choosing 5 is if player 2 also chooses 5. But then player 1 is better off choosing 4. Choosing 4 is also better if player 2 chooses 4 also. So choosing 4 is always at least as good as choosing 4. Once we eliminate this strategy, we can use the same reasoning to conclude that player 2 should not choose 5 as well. Then we get the following game:

	2 chooses 0	2 chooses 1	2 chooses 2	2 chooses 3	2 chooses 4
1 chooses 0	0, 0	200, 0	0, 0	0, 0	0, 0
1 chooses 1	0, 200	100, 100	300, 0	0, 0	0, 0
1 chooses 2	0, 0	0, 300	200, 200	400, 0	0, 0
1 chooses 3	0, 0	0, 0	0, 400	300, 300	500, 0
1 chooses 4	0, 0	0, 0	0, 0	0, 500	400, 400

After strategies have been eliminated

Then we can use similar reasoning to conclude that for both players, choosing 3 weakly dominates choosing 4. We can continue to eliminate strategies successively until we get

	2 chooses 0	2 chooses 1
1 chooses 0	0, 0	200, 0
1 chooses 1	0, 200	100, 100

What's left

Now we can say that choosing 0 weakly dominates choosing 1 for both players, so all we have left is that both players will choose 0. Or, if we eliminate player 1's strategy of choosing 1 first, then we get:

	2 chooses 0	2 chooses 1
1 chooses 0	0, 0	200, 0

Now we cannot eliminate player 2's choosing 2. So we have strategy profiles (0, 0) and (0, 1) as what is left. Similarly, if we eliminated player 2's strategy of choosing 2 first, we would get (0, 0) and (1, 0) as the strategy profiles left. This example shows how our result depends on the order in which we eliminated the weakly dominated strategies.

b. Find the (pure strategy) Nash equilibria of this game.
If we find the Nash equilibria of the original game, we find that (0, 0), (1, 0) and (0, 1) are the Nash equilibria. So we can say that the most that I will have to shell out is \$100, which is surprising since the two players could possibly make much more. Since both players have the incentive to undercut each other, they undercut each other (almost) all the way down.

14. Ann and Bob are each trying to win a prize in a school raffle (lottery). Each can buy either 0, 1, 2, or 3 raffle tickets. Ann and Bob are the only two people in the raffle, and each ticket has an equal chance of winning. So for example, if Ann buys 2 tickets and Bob buys 3 tickets, then Ann has a 2/5 chance of winning and Bob has a 3/5 chance of winning (if no one buys any tickets, the raffle is cancelled). The prize is worth \$60, and both Ann and Bob are risk-neutral: for example, if Ann has a 2/5 chance of winning, her expected utility is \$24. Model the following situations with strategic form games.

a. Say that raffle tickets are free. What does the game look like? Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?
We simply do some expected value calculations to derive the following:

	2 buys 0 tickets	2 buys 1 ticket	2 buys 2 tickets	2 buys 3 tickets
1 buys 0 tickets	0, 0	0, 60	0, 60	0, 60
1 buys 1 ticket	60, 0	30, 30	20, 40	15, 45
1 buys 2 tickets	60, 0	40, 20	30, 30	24, 36
1 buys 3 tickets	60, 0	45, 15	36, 24	30, 30

Tickets are free

Here it is easy to see that for both players, the strategy of buying 0 tickets is strongly dominated by any other strategy. Also, for both players, the strategy of buying 1 ticket and the strategy of buying 2 tickets are both weakly dominated by the strategy of buying 3 tickets—this makes sense, since if tickets don't cost anything, there is no reason not to buy a lot. It is easy to see that the only pure strategy Nash equilibrium is where player 1 buys 3 tickets and player 2 buys 3 tickets.

b. Now say that raffle tickets cost \$6 each. What does the game look like? Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?
Now the game looks like this:

	2 buys 0 tickets	2 buys 1 ticket	2 buys 2 tickets	2 buys 3 tickets
1 buys 0 tickets	0, 0	0, 54	0, 48	0, 42
1 buys 1 ticket	54, 0	24, 24	14, 28	9, 27
1 buys 2 tickets	48, 0	28, 14	18, 18	12, 18
1 buys 3 tickets	42, 0	27, 9	18, 12	12, 12

Tickets cost \$6 each

Now, again for both players, the strategy of buying 0 tickets is strongly dominated by any other strategy. But now for both players, the strategy of buying 3 tickets is weakly dominated by the strategy of buying 2 tickets. There are four pure strategy Nash equilibria: (buy 2 tickets, buy 2 tickets), (buy 3 tickets, buy 2 tickets), (buy 2 tickets, buy 3 tickets), and (buy 3 tickets, buy 3 tickets).

c. Now say that raffle tickets cost \$10 each. What does the game look like? Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?
Now the game looks like this:

	2 buys 0 tickets	2 buys 1 ticket	2 buys 2 tickets	2 buys 3 tickets
1 buys 0 tickets	0, 0	0, 50	0, 40	0, 30
1 buys 1 ticket	50, 0	20, 20	10, 20	5, 15
1 buys 2 tickets	40, 0	20, 10	10, 10	4, 6
1 buys 3 tickets	30, 0	15, 5	6, 4	0, 0

Tickets cost \$10 each

Now for both players, the strategy of buying 0 tickets is strongly dominated by the strategy of buying 1 ticket or the strategy of buying 2 tickets. Also, for both players the strategy of buying 3 tickets is strongly dominated by the strategy of buying 1 ticket. For both players, the strategy of buying 2 tickets is weakly dominated by the strategy of buying 1 ticket. There are four pure strategy Nash equilibria: (buy 1 ticket, buy 1 ticket), (buy 1 tickets, buy 2 tickets), (buy 2 tickets, buy 1 tickets), and (buy 2 tickets, buy 2 tickets).

15. Say that Spy 1 is trying to listen in on Spy 2. There are three rooms, A, B, and C, arranged in a line like this: A—B—C. In other words, A is on the left, B is in the middle, and C is on the right. Each spy must decide independently and simultaneously which room to enter. Their payoffs are determined as follows. If they both choose the same room, then they will see each other, a bloody gun battle will ensue, and both get payoff -10. If they are in adjacent rooms (for example, if Spy 1 is in room A and Spy 2 is in room B) then Spy 1 can set up her eavesdropping equipment and can intercept Spy 2's communications; hence Spy 1 gets a payoff of 5 and Spy 2 gets a payoff of -5. If they are not in adjacent rooms and they are not in the same room (for example, if Spy 1 is in room A and Spy 2 is in room C) then the distance between them is too great for the eavesdropping equipment to work; Spy 1 gets no secrets and Spy 2 gets to keep hers, and so both get a payoff of 0.

a. Model this as a strategic form game.
The game looks like this:

	$2A$	$2B$	$2C$
$1A$	$-10,-10$	$5,-5$	$0,0$
$1B$	$5,-5$	$-10,-10$	$5,-5$
$1C$	$0,0$	$5,-5$	$-10,-10$

b. Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?
No strategies are strongly or weakly dominated. It is not hard to see that $(1B, 2A)$ and $(1B, 2C)$ are the Nash equilibria of this game. This makes sense: if Spy 1 chooses room B, Spy 2 has "no choice" but to choose either room A or room C, thus guaranteeing Spy 1 eavesdropping access.

16. Say that persons 1, 2, and 3 each decide whether to go to restaurant A or restaurant B. Person 1 wants the dinner group to be as large as possible. For person 1, the worst thing is if she goes to a restaurant alone, the best thing is if all three go to the same place, and going with one person (it doesn't matter which) is OK, neither best or worst. Person 2 is the exact opposite; she wants the dinner group to be as small as possible. All person 3 cares about is going to the same place as person 1, since he likes person 1.

a. Model this as a strategic form game.
Each person can go to A or go to B. The game looks like this.

	2 goes to A	2 goes to B
1 goes to A	10, 0, 10	5, 10, 10
1 goes to B	0, 5, 0	5, 5, 0

3 goes to A

	2 goes to A	2 goes to B
1 goes to A	5, 5, 0	0, 5, 0
1 goes to B	5, 10, 10	10, 0, 10

3 goes to B

b. **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**
In this game, no strategy is weakly or strongly dominated. The two pure strategy Nash equilibria are $(1A, 2B, 3A)$ and $(1B, 2A, 3B)$. Since person 3 likes person 1, he will always go to where person 1 goes, and person 2 will avoid them both.

c. Now say that person 3 loses interest in person 1 and becomes grouchy like person 2. Model this as a strategic form game.
Now the game looks like this.

	2 goes to A	2 goes to B
1 goes to A	10, 0, 0	5, 10, 5
1 goes to B	0, 5, 5	5, 5, 10

3 goes to A

	2 goes to A	2 goes to B
1 goes to A	5, 5, 10	0, 5, 5
1 goes to B	5, 10, 5	10, 0, 0

3 goes to B

d. **Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?**

Again, no strategy is weakly or strongly dominated. There are four pure strategy Nash equilibria: $(1A, 2B, 3A)$, $(1B, 2B, 3A)$, $(1A, 2A, 3B)$, $(1B, 2A, 3B)$. Since it is not possible for both grouchy people to be alone, one of them has to eat with someone else. Note that the grouchy people will never eat together with person 1 at the other restaurant, because then person 1, who likes a big crowd, would prefer to join them. Of course, everyone eating together is not an equilibrium because one of the grouchy people would like to deviate.

17. Say that there are two people, a security guard and a thief. The security guard can either be vigilant or relax. The thief can either steal or do nothing. If the guard is vigilant, then the thief would rather do nothing than steal. If the guard is relaxed, however, the thief would rather steal than do nothing. If the thief steals, the guard would rather be vigilant than be relaxed. If the thief does nothing, however, the guard would rather be relaxed than vigilant.

a. Model this as a strategic form game. Feel free to choose payoffs which make sense to you. My version of this game looks like this:

	thief steals	thief does nothing
guard is vigilant	5,-10	-2,0
guard is relaxed	-10,10	0,0

b. What are the (pure strategy) Nash equilibria of this game?
In this game, there are no pure strategy Nash equilibria: from every outcome, someone would like to deviate.

18. Read "Running Mates: The Clark-Lieberman Iowa Jailbreak" by William Saletan (at the "Media Clips" link). Model the situation as a strategic form game.

a. Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?

	Lieberman stays	Lieberman quits
Clark stays	0,0	1,-10
Clark quits	-10,1	5,5

In this game, no strategy is strongly or weakly dominated. There are two Nash equilibria: (Clark stays, Lieberman stays), and (Clark quits, Lieberman quits).

19. Look at Clip 1 of *Return to Paradise* on the web site (look under the "Media Clips" link). Model the situation as a game with two players (Sheriff and Tony). Feel free to choose payoffs which make sense to you (that's what makes the problem kind of interesting).

a. In your version of the game, are there strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?

For example, say that Sheriff and Tony care only about avoiding jail themselves, and all other things equal would be happy to have Lewis alive. Then the game would look like this:

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-1,-1	0,-6
Sheriff goes back to Malaysia	-6,0	-3,-3

In other words, the best thing would be to have the other guy go back and you stay at home. Here you can think of Lewis's life as worth 1 year of jail time to Sheriff and Tony.
In this game, staying in NYC strongly dominates going back to Malaysia for both people. The only Nash equilibrium is (NYC, NYC).

Another possibility is that Lewis's life is worth 4 years of jail time—in other words, neither will go back if he has go go back alone. Then the game would look like this:

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-4,-4	0,-6
Sheriff goes back to Malaysia	-6,0	-3,-3

Note that this game is a Prisoners' Dilemma: if you know the other person is going back, you have the incentive to not get on the plane and stay in NYC; however, if you both stay in NYC, Lewis dies and you're worse off than if you both returned to Malaysia. Here, again NYC strongly dominates Malaysia and the only Nash equilibrium is (NYC, NYC).

Another possibility is that Lewis's life is worth 10 years of jail time—having Lewis die is the absolute worst thing. Then the game would look like this:

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-10,-10	0,-6
Sheriff goes back to Malaysia	-6,0	-3,-3

Note that this game is a game of chicken, where Lewis dying is like the two cars crashing into each other. Still the best thing for each person is to stay home and have the other person to return to Malaysia. Here, no strategy is weakly or strongly dominated and there are two Nash equilibria: (NYC, Malaysia) and (Malaysia, NYC).

Finally, say that people feel really guilty and basically internalize the others' pain. So if you Tony goes back to Malaysia and Sheriff doesn't, Sheriff feels just as bad as Tony.

	Tony stays in NYC	Tony goes back to Malaysia
Sheriff stays in NYC	-10,-10	-6,-6
Sheriff goes back to Malaysia	-6,-6	-3,-3

Now Malaysia strongly dominates NYC for both players, and the only Nash equilibrium is (Malaysia, Malaysia).

20. Read "Report Calls Recycling Costlier Than Dumping" by Eric Lipton on the "Media Clips" web site. At the end of the story, Lipton states that if people don't recycle much, then it is too costly to have a recycling program, but if people recycle a lot, it becomes economically worthwhile. Model the situation as a strategic form game (say with just two people for simplicity) and show that there are two Nash equilibria: one in which both people recycle a lot and one in which both people recycle little.

The main point of the article is that New York's recycling program is costly, and hence it is not cost-effective in the sense that the costs of the program are not offset by the money gained by selling recycled materials. It would be cheaper to simply put everything into landfills or incinerators. The costs are mainly collection costs: paying the sanitation workers to pick up the recyclables. However, near the end of the article, it is pointed out that if city residents recycled a significantly larger share of their trash, the program would begin to make economic sense. So for simplicity say that there are two people, who can each recycle a little bit or a lot.

	Recycle little	Recycle lot
Recycle little	0, 0	0, -5
Recycle lot	-5, 0	10, 10

The idea here is that if everyone recycles a lot, the city keeps its recycling program (since it is cost-effective) and everyone is happy. However, if everyone recycles a little bit, then the city loses its recycling program, which is not as nice. If only one person recycles a lot, then the recycling program is not profitable and hence there is no recycling program, and the person who recycles a lot (since he is the only one) has to deliver her recyclables herself to the recycling station, which is very inconvenient. Here there are two pure strategy Nash equilibria: (Recycle little, Recycle little) and (Recycle lot, Recycle lot).

21. Read "Drifter Jailed on Girls' Lies Set Course of Desperation" by H. G. Reza, Christine Hanley, and James Ricci on the "Media Clips" web site. Model the situation as a strategic form game. Is this a Prisoners' Dilemma? Why is it standard procedure for police to interview witnesses separately?

We can simplify the situation to include just two girls. I would say that each girl can choose to lie or tell the truth, and the game looks like this:

	Lie	Truth
Lie	5, 5	-10, 0
Truth	0,-10	1, 1

If both girls lie, then they have an excuse for getting home late from school, which is better for them than if they both tell the truth. If one girl lies and the other tells the truth, the liar gets into big trouble; the one telling the truth does not get into trouble but feels bad for the other girl.

Here each girl wants to lie only if the other lies also. This not a Prisoners' Dilemma, since no strategy is strongly dominated. There are two pure strategy Nash equilibria: (Lie, Lie) and (Truth, Truth).

I think of the situation this way. It is "safer" to tell the truth, since you avoid getting into trouble (the -10 payoff). If both girls lie, the situation is better for both girls and is sustainable, but before one girl lies, she would have to be quite sure that the other will lie also. By allowing the girls to talk to each other, the girls could assure each other than they would all lie (and also have the same story).

22. Read the excerpt from Richard Wright's *Black Boy* on the "Media Clips" web site (down at the bottom of the page under "Text documents"). Model this as a strategic form game and interpret the situation and the outcome in terms of the game and your predictions given the game.
I would model it as a coordination game, something like this:

	Harrison fights	Harrison pretends
Wright fights	-10, -10	0, -100
Wright pretends	-100, 0	5, 5

Here both intend to pretend to fight and earn a few dollars. However, if you pretend to fight and the other person fights for real, then you get a very bad payoff. Hence if you have the slightest doubt that the other person is not pretending, you fight also. Hence you both end up fighting, which is much worse than if you both kept pretending.

23. Read "9 Questions about Syria You Were Too Embarrassed to Ask" by Max Fisher at the "Media Clips" link. On page 6, under the "You didn't answer my question" heading, a situation concerning chemical weapons is described. Model this as a strategic form game.
According to the article, if a country believes its adversary is going to use chemical weapons (CW), it has a strong incentive to use them also. A country will not use chemical weapons if it is certain that its opponent will not use them. Both sides are better off if neither uses chemical weapons. So if we have two countries, we have something like this:

	CW	not
CW	-10, -10	-5, -50
not	-50, -5	0, 0

In this game, there are two (pure strategy) NE: (CW, CW) and (not, not).
Say that country 1 now wants to use chemical weapons. The game now looks like

	CW	not
CW	-10, -10	5, -50
not	-50, -5	0, 0

Now the only (pure strategy) NE is (CW, CW). Country 1 wanting to use chemical weapons makes the situation devolve into one in which both countries use chemical weapons. The norm against chemical weapons is now broken.
Say a superpower punishes any country which uses chemical weapons: whenever a country uses chemical weapons, they get some inbound cruise missiles which adds an additional -20 to their payoff. Then the game looks like this:

	CW	not
CW	-30, -30	-15, -50
not	-50, -25	0, 0

Now we have two (pure strategy) NE again: (CW, CW) and (not, not). Now the norm against chemical weapons is restored because of the superpower's threat.

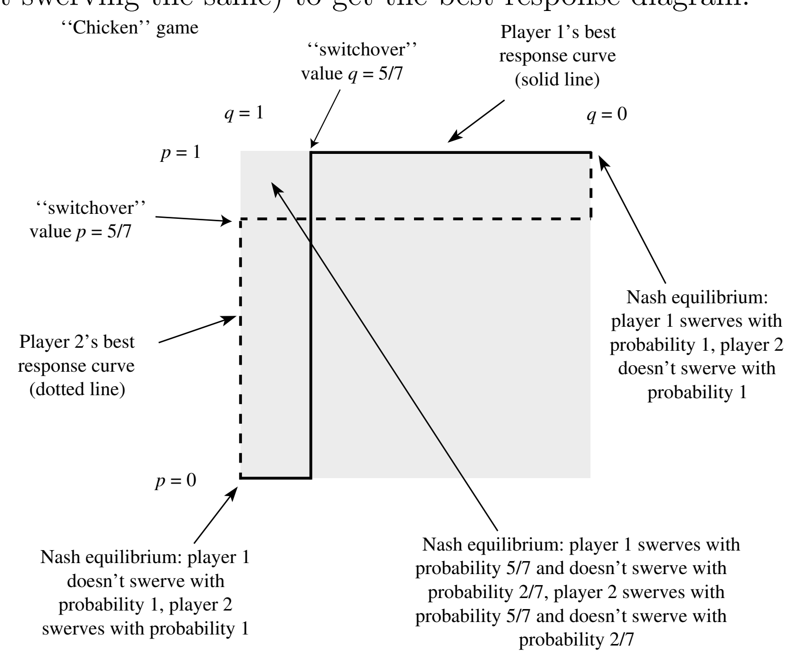
24. Find all Nash (mixed strategy and pure strategy) equilibria to this version of the "Chicken" game:

		$[q]$	$[1-q]$
		2 swerves	2 doesn't
$[p]$	1 swerves	1, 1	0, 5
$[1-p]$	1 doesn't	5, 0	-10, -10

Chicken

We can see that (swerves, doesn't) and (doesn't, swerves) are pure strategy Nash equilibria. To find the mixed strategy Nash equilibria, let p be the probability player 1 swerves and q be the probability player 2 swerves.

If player 2 swerves with probability q, then player 1's expected payoff from swerving is $(1)(q)+(0)(1-q) = q$ and her expected payoff from not swerving is $(5)(q)+(-10)(1-q) = -10+15q$. Now if $q = 1$, we can substitute to find that 1's expected payoff from swerving is 1 and from not swerving is 5. So player 1's best response is to set $p = 0$, that is, to not swerve for sure. Now if $q = 0$, we can substitute to find that 1's expected payoff from swerving is 0 and from not swerving is -10. So player 1's best response is to set $p = 1$, that is, to swerve for sure. We can do this for various values of q and find player 1's best response curve. When q is near 1, player 1's best response is to set $p = 0$. When q is near 0, player 1's best response is to set $p = 1$. At some value of q, player 1 will "switch over" from playing $p = 0$ to $p = 1$. If we set the expected value of swerving equal to the expected value of not swerving, we get $q = -10+15q$. We solve this to find $q = 10/14 = 5/7$. At $q = 5/7$, player 1's expected value for swerving and not swerving are the same. Therefore, any p between 0 and 1 will be a best response (in fact he will get the same expected utility no matter what value of p he chooses). So we can draw player 1's best response curve. We can similarly find player 2's best response curve (check what happens at the extremes when $p = 0$ and $p = 1$, and calculate the "switchover" value of p which makes player 2's expected value from swerving and not swerving the same) to get the best response diagram.



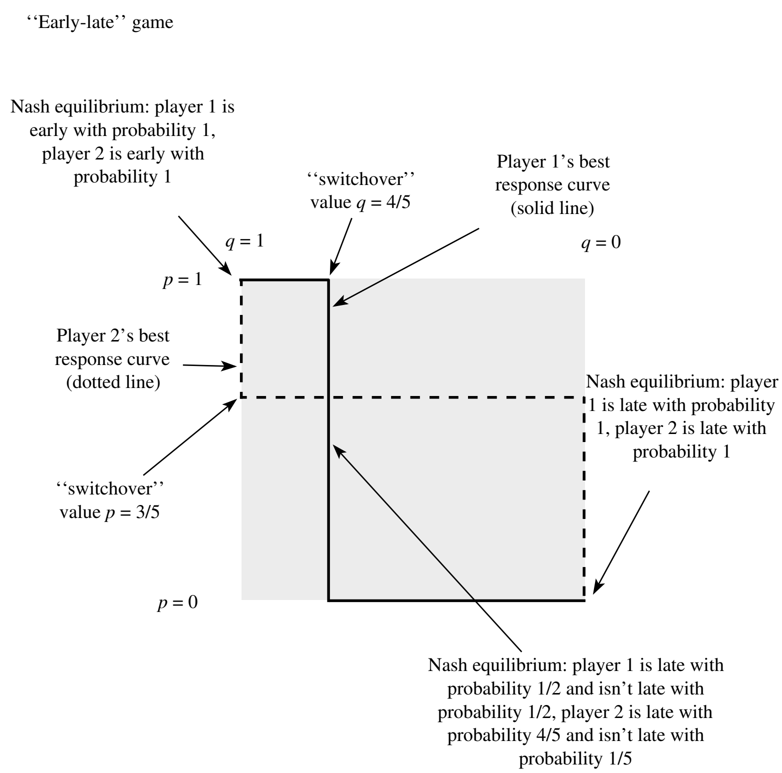
So the three Nash equilibria are where these two graphs intersect. There are the two pure strategy Nash equilibria we found earlier, and a mixed strategy Nash equilibrium in which player 1 swerves with probability 5/7 and doesn't swerve with probability 2/7 and player 2 swerves with probability 5/7 and doesn't swerve with probability 2/7.

25. Find all Nash equilibria to the "Early-late" game, which looks like this:

		$[q]$	$[1-q]$
		2 arrives early	2 arrives late
$[p]$	1 arrives early	1, 1	-5, -1
$[1-p]$	1 arrives late	-1, 0	3, 3

Early-late

We do the same thing as before: find the value of q which makes player 1 indifferent between being early and late: we set $(1)q + (-5)(1-q) = (-1)(q) + 3(1-q)$ and find that $q = 4/5$. We find the value of p which makes player 2 indifferent between being early and late: $(1)(p) + (0)(1-p) = (-1)(p) + (3)(1-p)$ and find that $p = 3/5$. We draw the best response curves (the diagram follows) and find that there are three Nash equilibria: two pure ones, (early, early) and (late, late), and one mixed one in which player 1 is early with probability 3/5 and late with probability 2/5 and player 2 is early with probability 4/5 and late with probability 1/5.



26. Say you have an admirer whom you don't like very much. You can either go to the library or the coffee shop to study. You prefer the coffee shop but you want to avoid your admirer. Your admirer can also go to the library or coffee shop to study. Your admirer prefers the library but wants to be where you are more than anything else. So the game looks like:

	Admirer goes to library	Admirer goes to coffeeshop
You go to library	0, 3	4, 0
You go to coffeeshop	6, 0	0, 1

a. Find all (pure strategy *and* mixed strategy) Nash equilibria of this game.
It is easy to verify that there are no pure strategy Nash equilibria of this game. To find mixed strategy Nash equilibria, let p be the probability you go to the library and let q be the probability that your admirer goes to the library. To find out your best response curve, we need to find the "switchover" value of q, that is, the q which makes you indifferent between going to the library and going to the coffeeshop. So $(0)(q)+(4)(1-q) = (6)(q)+(0)(1-q)$. Hence $4 = 10q$ and so $q = 2/5$. To find out your admirer's best response curve, we need to find the "switchover" value of p, that is, the p which makes your admirer indifferent between going to the library and going to the coffeeshop. So $(3)(p)+(0)(1-p) = (0)(p)+(1)(1-p)$. Hence $4p = 1$ and so $p = 1/4$. The only Nash equilibrium is where you go to the library with probability 1/4 and the coffeeshop with probability 3/4, and your admirer goes to the library with probability 2/5 and the coffeeshop with probability 3/5.

b. Now say that you begin to actually enjoy your admirer's company. The game is now:

	Admirer goes to library	Admirer goes to coffeeshop
You go to library	4, 3	0, 0
You go to coffeeshop	0, 0	6, 1

Find all (pure strategy *and* mixed strategy) Nash equilibria of this game.
It is easy to verify that (library, library) and (coffeeshop, coffeeshop) are the only pure strategy Nash equilibria. To find mixed strategy Nash equilibria, again let p be the probability you go to the library and let q be the probability that your admirer goes to the library. The "switchover" value of q is determined by the equation $(4)(q)+(0)(1-q) = (0)(q)+(6)(1-q)$, so $4q = 6-6q$, so $10q = 6$, and hence $q = 3/5$. The "switchover" value of p is determined by the equation $(3)(p)+(0)(1-p) = (0)(p)+(1)(1-p)$, so $3p = 1-p$, and hence $4p = 1$, so $p = 1/4$. (Note that since your admirer's payoffs did not change, this is just the same as before.) So the mixed strategy Nash equilibrium is where you go the library with probability 1/4 and the coffeeshop with probability 3/4, and your admirer goes to the library with probability 3/5 and the coffeeshop with probability 2/5.

27. [from Spring 2002 midterm] Person 1 and Person 2 are competing for the affections of the extremely attractive Sandy. Person 1 and Person 2 plan to show up at Sandy's house at the same time on Saturday night to ask Sandy out. Since this is southern California, a crucial decision for both Person 1 and Person 2 is what kind of car to drive to Sandy's house. Person 1 is wealthy and can either have the butler prepare the impressive Lamborghini or the cute VW. Person 2 ordinarily drives a pickup truck, but can borrow a Lexus for the night from a roommate. If Sandy sees the Lamborghini and the Lexus pull up, Sandy chooses the Lamborghini because it is of course more impressive. If Sandy sees the Lamborghini and the pickup truck, Sandy chooses the pickup truck because the Lamborghini is clearly trying too hard. If Sandy sees the VW and the Lexus, Sandy chooses the Lexus because it is more impressive than the VW. If Sandy sees the VW and the pickup truck, Sandy chooses the VW because it is obviously more comfortable than the pickup truck.

a. Model this as a strategic form game between Person 1 and Person 2 (assume Sandy is not a player).
Say that taking out Sandy yields a payoff of 1 and not taking Sandy out yields a payoff of 0. We then get the following game.

	Pickup truck	Lexus
Lamborghini	0,1	1,0
VW	1,0	0,1

b. Find all (pure strategy and mixed strategy) Nash equilibria of this game.
It is easy to see that there are no pure strategy Nash equilibria of this game. For mixed strategy Nash equilibria, say that person 1 plays Lamborghini with probability p and VW with probability $1-p$. Say that person 2 plays Pickup truck with probability q and Lexus with probability $1-q$.
If person 1 plays Lamborghini, her expected utility is $0q+1(1-q)=1-q$. If person 1 plays VW, her expected utility is $1q+0(1-q)=q$. To find the "switchover probability," we set these equal: $1-q=q$ and hence $q=1/2$.
If person 2 plays Pickup truck, his expected utility is $p+0(1-p)=p$. If person 2 plays Lexus, his expected utility is $0p+1(1-p)=1-p$. To find the "switchover probability," we set these equal: $p=1-p$ and so $p=1/2$.
Thus in the mixed Nash equilibrium, person 1 plays Lamborghini with probability 1/2 and VW with probability 1/2; person 2 plays Pickup truck with probability 1/2 and Lexus with probability 1/2.

28. [from Spring 2002 midterm] Consider the following game.

	$2a$	$2b$	$2c$
$1a$	$0,9$	$3,0$	$1,5$
$1b$	$1,2$	$5,4$	$4,3$
$1c$	$0,6$	$2,1$	$6,7$

a. Find all pure strategy Nash equilibria of this game.
Using our $*$ and $+$ notation to indicate best responses, we get

	$2a$	$2b$	$2c$
$1a$	$0, 9+$	$3, 0$	$1, 5$
$1b$	$*1, 2$	$*5, 4+$	$4, 3$
$1c$	$0, 6$	$2, 1$	$*6, 7+$

So $(1b, 2b)$ and $(1c, 2c)$ are the pure strategy Nash equilibria.

b. Use the method of iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can't eliminate any more).
It's easy to see that $1b$ strongly dominates $1a$ and hence person 1 will never play $1a$. Next, we iteratively eliminate $2a$ (once $1a$ is eliminated, $2c$ strongly dominates $2a$). We thus have the following game left over.

	$2b$	$2c$
$1b$	$5, 4$	$4, 3$
$1c$	$2, 1$	$6, 7$

c. Find all mixed strategy Nash equilibria of this game. (Note: an answer like "$p = 2/3, q = 2/5$" is not sufficient. Please write down in a sentence which strategies are played with what probability.)
Say that person 1 plays $1b$ with probability p and $1c$ with probability $1-p$. Say that person 2 plays $2b$ with probability q and $2c$ with probability $1-q$.
If person 1 plays $1b$, her expected utility is $5q + 4(1-q)$. If person 1 plays $1c$, her expected utility is $2q+6(1-q)$. To find the "switchover probability," we set these equal: $5q+4(1-q) = 2q + 6(1-q)$, or in other words $4 + q = 6 - 4q$. Hence $5q = 2$ and so $q = 2/5$.
If person 2 plays $2b$, his expected utility is $4p + 1(1-p)$. If person 2 plays $2c$, his expected utility is $3p+7(1-p)$. To find the "switchover probability," we set these equal: $4p+1(1-p) = 3p + 7(1-p)$, or in other words $1 + 3p = 7 - 4p$. Hence $7p = 6$ and so $p = 6/7$.
Thus in the mixed Nash equilibrium, person 1 plays $1b$ with probability $6/7$ and $1c$ with probability $1/7$; person 2 plays $2b$ with probability $2/5$ and $2c$ with probability $3/5$.

29. [from Spring 2002 final] In a simplified version of "Battleship," say that there are four spaces, numbered $1, 2, 3, 4$. Person 1 chooses to fire a missle at one of these four spaces. Person 2 has a ship which is two spaces long, and chooses where to put the ship on the board: she can either put it on spaces 1 and 2, on spaces 2 and 3, or on spaces 3 and 4. The people make their choices simultaneously.

a. Model this as a strategic form game and find all mixed-strategy and pure-strategy Nash equilibria.
Person 1's possible strategies are $1, 2, 3, 4$ (which space to fire the missile at). Person 2's possible strategies are $12, 23, 34$ (put the ship on spaces 1 and 2, put the ship on spaces 2 and 3, or put the ship on spaces 3 and 4). Say that you get a payoff of 1 for winning and 0 for losing. We thus get the following game.

	12	23	34
1	$1,0$	$0,1$	$0,1$
2	$1,0$	$1,0$	$0,1$
3	$0,1$	$1,0$	$1,0$
4	$0,1$	$0,1$	$1,0$

It's easy to see that there are no pure strategy Nash equilibria. To find mixed strategy Nash equilibria, we can simplify the game first. For person 1, firing at 2 weakly dominates firing at 1 and firing at 3 weakly dominates firing at 4. So we can eliminate firing at 1 and firing at 4 and get the following.

	12	23	34
2	$1,0$	$1,0$	$0,1$
3	$0,1$	$1,0$	$1,0$

Now it is easy to see that for person 2, putting the ship at 23 is weakly dominated by putting it at 34 or 12. So we eliminate 23 and get the following game.

	12	34
2	$1,0$	$0,1$
3	$0,1$	$1,0$

To find the mixed strategy Nash equilibria, say that person 1 fires at 2 with probability p and fires at 3 with probability $1-p$. Say that person 2 places the boat at 12 with probability q and places the boat at 34 with probability $1-q$.
If person 1 fires at 2, her expected utility is $1q + 0(1-q) = q$. If person 1 fires at 3, her expected utility is $0q + 1(1-q) = 1-q$. To find the "switchover probability," we set these equal: $q = 1-q$ and hence $q = 1/2$.
If person 2 places the boat at 12, his expected utility is $0p + 1(1-p) = 1-p$. If person 2 places the boat at 34, his expected utility is $1p + 0(1-p) = p$. To find the "switchover probability," we set these equal: $1-p = p$ and so $p = 1/2$.
Thus in the mixed Nash equilibrium of the original game, person 1 fires at 2 with probability 1/2 and fires at 3 with probability 1/2; person 2 places the boat at 12 with probability 1/2 and places the boat at 34 with probability 1/2.

b. Now say that there are 5 spaces, numbered $1,2,3,4,5$. Model this as a strategic form game and find all mixed-strategy and pure-strategy Nash equilibria.
Similarly, we get the following game.

	12	23	34	45
1	$1,0$	$0,1$	$0,1$	$0,1$
2	$1,0$	$1,0$	$0,1$	$0,1$
3	$0,1$	$1,0$	$1,0$	$0,1$
4	$0,1$	$0,1$	$1,0$	$1,0$
5	$0,1$	$0,1$	$0,1$	$1,0$

Again, there are no pure Nash equilibria of this game. As before, for person 1, firing at 1 is weakly dominated by firing at 2, and firing at 5 is weakly dominated by firing at 5. So we get the following.

	12	23	34	45
2	$1,0$	$1,0$	$0,1$	$0,1$
3	$0,1$	$1,0$	$1,0$	$0,1$
4	$0,1$	$0,1$	$1,0$	$1,0$

Now for person 2, placing the boat at 23 is weakly dominated by placing the boat at 12, and placing the boat at 34 is weakly dominated by placing the boat at 45. So we get the following.

	12	45
2	$1,0$	$0,1$
3	$0,1$	$0,1$
4	$0,1$	$1,0$

Now for person 1, firing at 3 is weakly dominated by firing at 2 (or at 4). Hence we have the following.

	12	45
2	$1,0$	$0,1$
4	$0,1$	$1,0$

We can find the mixed strategy Nash equilibrium as before. We find that in the mixed Nash equilibrium of this game (and hence the original game), person 1 fires at 2 with probability 1/2 and fires at 4 with probability 1/2; person 2 places the boat at 12 with probability 1/2 and places the boat at 45 with probability 1/2.

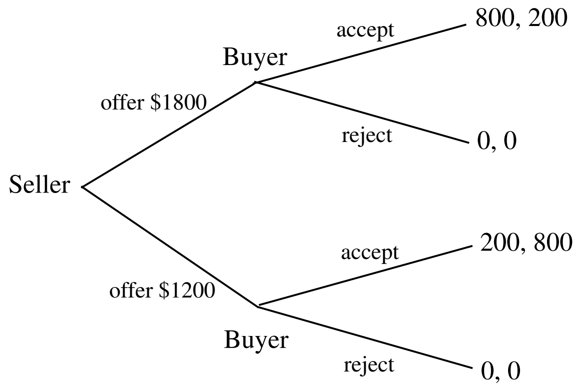
30. Say that a seller tries to sell a car to a buyer. The car is worth \$1000 to the seller and \$2000 to the buyer, and both people know this. First, the seller proposes a price of either \$1800 or \$1200 to the buyer. Given this price, the buyer can either accept or reject the offer. Model this as an extensive form game.

a. Find all Nash equilibria.
The easiest way to find all Nash equilibria of this game is to represent it as a strategic form game:

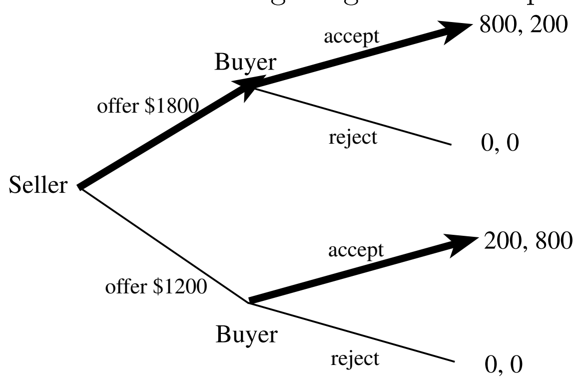
	if \$1800, accept; if \$1200, accept	if \$1800, accept; if \$1200, reject	if \$1800, reject; if \$1200, accept	if \$1800, reject; if \$1200, reject
\$1800	800, 200	800, 200	0, 0	0, 0
\$1200	200, 800	0, 0	200, 800	0, 0

where the seller chooses the row and the buyer chooses the column. Then it is clear that the three (pure strategy) Nash equilibria are (\$1800, if \$1800, accept; if \$1200, accept), (\$1800, if \$1800, accept; if \$1200, reject), and (\$1200, if \$1800, reject; if \$1200, accept). Note that there is a Nash equilibrium where the buyer "holds out" for a price of \$1200 and succeeds.

b. Which Nash equilibria are subgame perfect?
We note that in a subgame perfect Nash equilibrium, the buyer will accept either offer. So the only subgame perfect Nash equilibrium is (\$1800, if \$1800, accept; if \$1200, accept).
If you want to do this with a game tree, the game in extensive form looks like this:

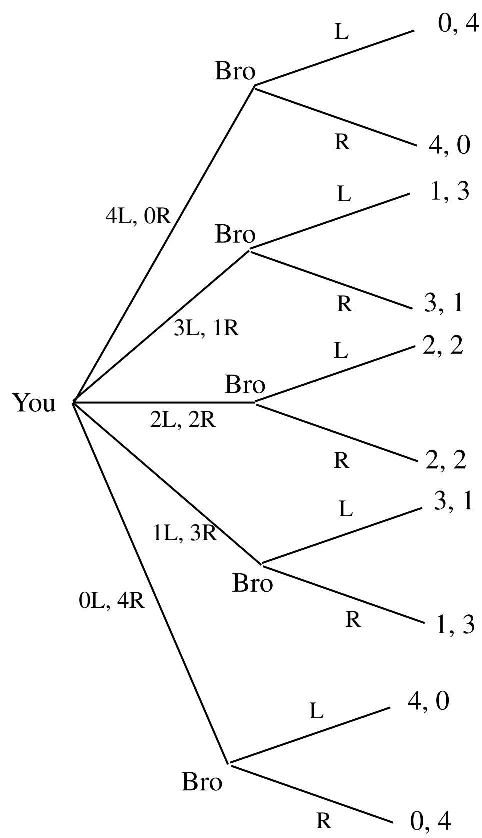


By backward induction, we find the following subgame Nash equilibrium:

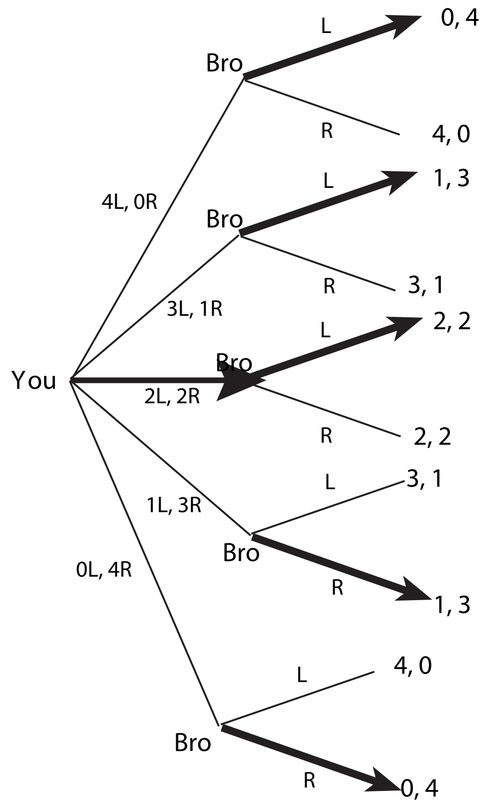


31. Say that you and your brother divide up 4 oranges in the following manner. First, you divide the oranges up into two piles: the left pile and the right pile. For example, you could put one orange in the left pile and three in the right pile. Then, your brother decides which of the two piles he wants; he keeps that pile and you get the other pile. Both of you like oranges.

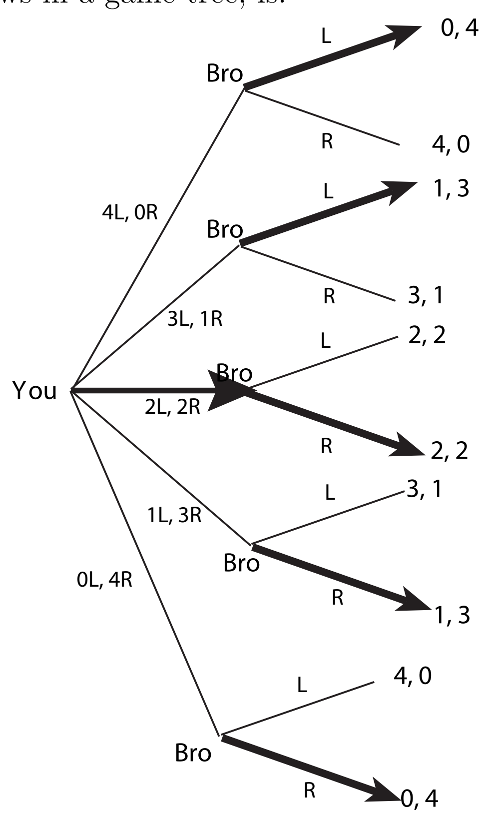
a. Model this as an extensive form game and find the subgame perfect Nash equilibria. How will the oranges be divided?
You have five possible strategies: put 4 oranges in the left pile and 0 in the right, 3 in the left and 1 in the right, 2 in the left and 2 in the right, 1 in the left and 3 in the right, and 0 in the left and 4 in the right (I abbreviate these as 4L, 0R, etc.) Your brother can choose in each of these contingencies whether to choose the left or right pile. We can say that payoffs are just the number of oranges one gets. Hence the extensive form game looks like this:



There are two subgame perfect Nash equilibria. The first, written as arrows in a game tree, is:



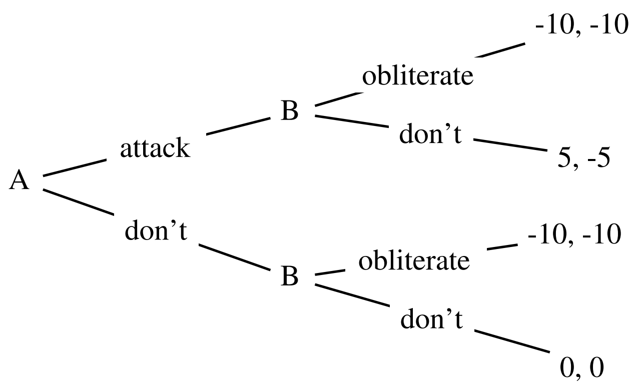
The second, written as arrows in a game tree, is:



If we were to spell these out explicitly, in the first equilibrium, person 1 plays strategy "2L, 2R" and person 2 plays strategy "If 4L, 0R, choose L; if 3L, 1R, choose L; if 2L, 2R, choose L; if 1L, 3R, choose R; if 0L, 4R, choose R". The second equilibrium is similar. In both of these equilibria, you and your brother split the 4 oranges equally. Note that your brother's strategy is the choice of either the left or right pile *conditional* on your division.

32. Say that country A can either attack or not attack country B. Country B cannot defend itself; the only thing it can do is to set off a "doomsday device" which obliterates both countries. The worst thing for both countries is to be obliterated; for country A, the best thing is to attack and not be obliterated; for country B, the best thing is to not be attacked.

a. Think of country A moving first and model this as an extensive form game. Find all pure strategy Nash equilibria and find the subgame perfect Nash equilibria.
The game in extensive form looks like this (of course, your choice of payoffs might be different from mine):

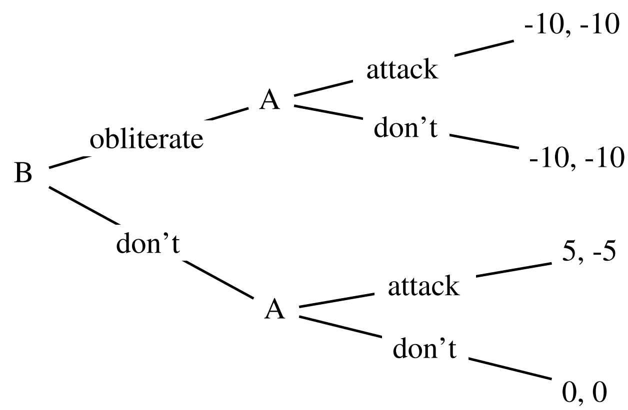


The strategic form looks like this:

	if attack, obliterate; if not, obliterate	if attack, obliterate; if not, don't	if attack, don't; if not, obliterate	if attack, don't; if not, don't
attack	-10,-10	-10,-10	5,-5	5,-5
don't	-10,-10	0,0	-10,-10	0, 0

There are three pure strategy Nash equilibria: (don't, if attack, obliterate; if not, don't), (attack, if attack, don't; if not, obliterate), (attack, if attack, don't; if not, don't). By backwards induction or whatever other method, the subgame perfect Nash equilibrium is (attack, if attack, don't; if not, don't).

b. Now think of country B moving first and model this as an extensive form game. Find all pure strategy Nash equilibria and find the subgame perfect Nash equilibria.
The game in extensive form looks like this (even though now B moves first, I write country A's payoffs first and country B's payoffs second, so to remain consistent with part a. above).



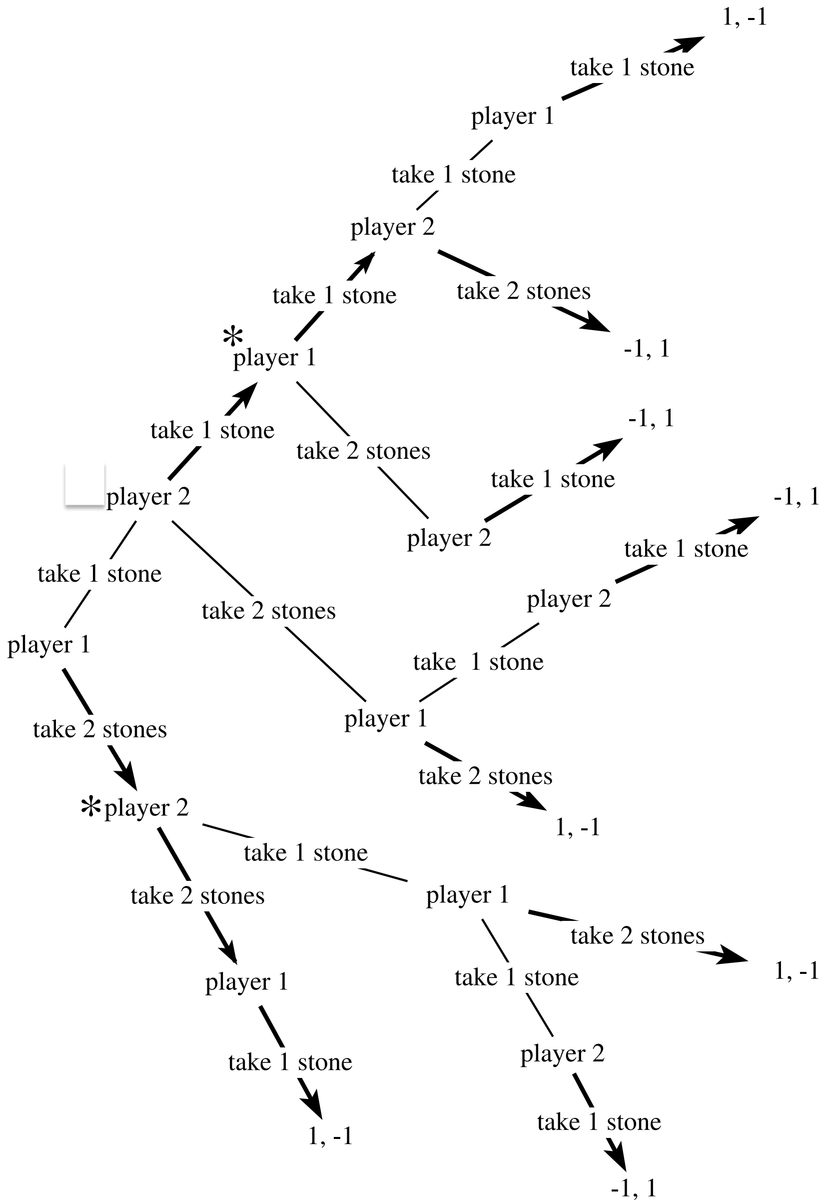
The strategic form looks like this:

	obliterate	don't
if obliterated, attack; if not, attack	-10,-10	5,-5
if obliterated, attack; if not, don't	-10,-10	0,0
if obliterated, don't; if not, attack	-10,-10	5,-5
if obliterated, don't; if not, don't	-10,-10	0,0

The two Nash equilibria are (if obliterated, attack; if not, attack, don't) and (if obliterated, don't; if not, attack, don't). These are both subgame perfect Nash equilibria.

33. The game called "Nim" goes like this: there are a pile of five stones on the ground. Player 1 can take either 1 or 2 stones. Then player 2 can take either 1 or 2 stones. They continue taking either 1 or 2 stones in turn until all the stones are gone. The player who takes the last stone (or stones) wins.

a. Model this as an extensive form game and find the subgame perfect Nash equilibria.
The extensive form game is shown below.



Arrows in the game tree show one of the subgame perfect Nash equilibria. There are actually four subgame perfect Nash equilibria, because there are ties at the starred nodes. I won't actually spell out all of the four subgame perfect Nash equilibria, since it would be tedious. In all of the subgame perfect Nash equilibria, player 1 initially takes 2 stones, and then no matter what player 2 then does, player 1 will eventually win.

b. What happens when you start with n stones instead of five?
You can show that if n is a multiple of 3, then in any subgame perfect Nash equilibrium, player 2 wins. Player 2 plays the following strategy: if player 1 picks one stone, then player 2 picks two stones; if player 1 picks two stones, then player 2 picks one stone. This way player 2 ensures that the remaining stones left after she plays is always a multiple of three.

If n is not a multiple of 3, then in any subgame perfect Nash equilibrium, player 1 wins. Player 1 initially picks up whatever number of stones which makes the remaining number of stones a multiple of three. From then on, if player 2 picks two stones, player 1 picks one stone, and if player 2 picks one stone, player 1 picks two stones.

34. Say we have a version of the "centipede" game:

$$\begin{array}{ccccccccc} 1 & -\,a\,- & 2 & -\,a\,- & 1 & -\,a\,- & 2 & -\,a\,- & 8,8 \\ | & & | & & | & & | & & \\ d & & d & & d & & d & & \\ | & & | & & | & & | & & \\ 2,2 & & 1,8 & & 4,4 & & 2,16 & & \end{array}$$

a. Find the subgame perfect Nash equilibrium.
We can write down the subgame perfect equilibrium in the game tree below:

$$\begin{array}{ccccccccc} 1 & -\,a\,- & 2 & -\,a\,- & 1 & -\,a\,- & 2 & -\,a\,- & 8,8 \\ | & & | & & | & & | & & \\ d & & d & & d & & d & & \\ \blacktriangledown & & \blacktriangledown & & \blacktriangledown & & \blacktriangledown & & \\ 2,2 & & 1,8 & & 4,4 & & 2,16 & & \end{array}$$

At each "node," each player plays down. Person 1's strategy is "First play down; if person 2 played across, play down." Person 2's strategy is "If person 1 played across, play down; if person 1 plays across again, then play down."

b. Represent this game in strategic (matrix) form.
Remember that a strategy is a complete contingent plan. Person 1 can move either across or down, and in the contingency that person 2 moves across, can move either across or down. So we will represent his strategies as *aa*, *ad*, *da*, and *dd*. If Person 1 moves across, Person 2 can move either across or down, and in the contingency that person 1 moves across again, can move either across or down. So we will represent his strategies also as *aa*, *ad*, *da*, and *dd*. So the strategic form game looks like this:

	aa	*ad*	*da*	*dd*
aa	8, 8	2, 16	1, 8	1, 8
ad	4, 4	4, 4	1, 8	1, 8
da	2, 2	2, 2	2, 2	2, 2
dd	2, 2	2, 2	2, 2	2, 2

c. Solve this strategic form game using iterated elimination of weakly dominated strategies. Show the order of elimination.
By inspection, we can see that for person 2, *dd* and *ad* weakly dominate *aa*. So we are left with

	ad	da	dd
aa	$2, 16$	$1, 8$	$1, 8$
ad	$4, 4$	$1, 8$	$1, 8$
da	$2, 2$	$2, 2$	$2, 2$
dd	$2, 2$	$2, 2$	$2, 2$

Now for person 1, both da and dd weakly dominate aa. So we are left with

	ad	da	dd
ad	$4, 4$	$1, 8$	$1, 8$
da	$2, 2$	$2, 2$	$2, 2$
dd	$2, 2$	$2, 2$	$2, 2$

Now it is clear that for person 2, both da and dd weakly dominate ad. Hence we are left with

	da	dd
ad	$1, 8$	$1, 8$
da	$2, 2$	$2, 2$
dd	$2, 2$	$2, 2$

Now for person 1, both da and dd weakly dominate ad. So finally, we have

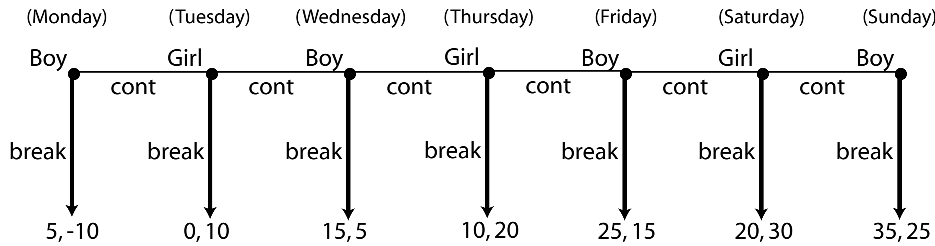
	da	dd
da	$2, 2$	$2, 2$
dd	$2, 2$	$2, 2$

It is clear that there can be no more domination. So the prediction is that person 1 will play either da or dd, and person 2 will play either da or dd. In any of these cases, the outcome will be that they both get a payoff of 2.

35. [from Spring 2004 midterm] Boyfriend and Girlfriend are on a 7 day cruise, which starts on Monday and goes until Sunday. Both know that this cruise is their last hurrah. Both know that once Sunday comes around and the cruise ends, if they are still together, the game ends because Boyfriend will be forced to dump Girlfriend (because otherwise he would be disowned by his wealthy parents).
On Monday evening, Boyfriend decides whether to break up or continue in the relationship. If Boyfriend decides to continue, then on Tuesday evening, Girlfriend decides whether to break up or to continue. If Girlfriend decides to continue, then on Wednesday evening, Boyfriend decides whether to continue. The game continues like this, with Boyfriend and Girlfriend taking turns deciding whether to continue or to break up (Boyfriend gets to decide on Monday, Wednesday, and Friday, and Girlfriend decides on Tuesday, Thursday, and Saturday). If anyone decides to break up, the game is immediately over. If no one decides to break up, then they make it to Sunday, and Girlfriend is left heartbroken.
Each person enjoys the others company, and gets a payoff of 5 for every day the relationship continues. However, no person wants to be dumped; if a person breaks up the relationship, the other person (the dumped one) gets 10 added to her payoff. For example, if they make it all the way to Sunday, Boyfriend's payoff is 35 and Girlfriend's payoff is 25.

a. Represent this as an extensive form game.

It's just like the centipede game above. Note that on Sunday, Boyfriend has only one move. One could have written it so that Girlfriend has the last move, on Saturday.



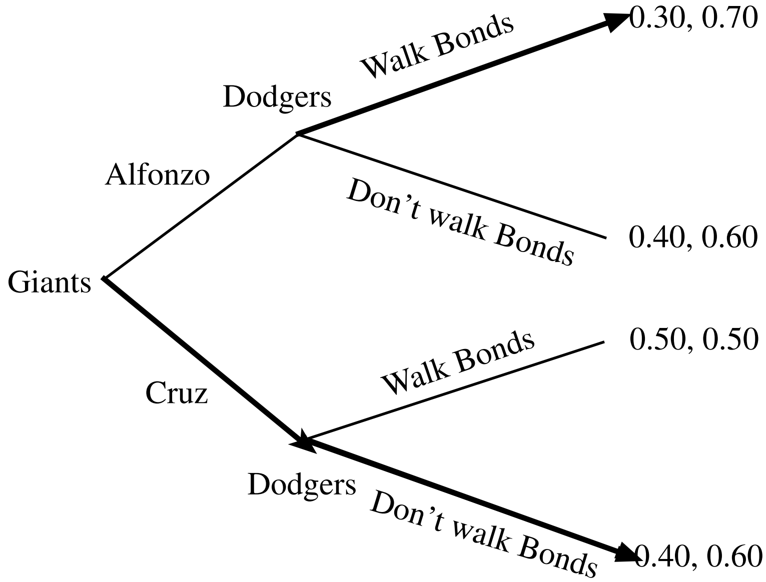
b. Find the subgame perfect Nash equilibrium.
The subgame perfect Nash equilibrium is shown by the arrows above. Even though they enjoy each other's company, Boyfriend and Girlfriend want to dump before being dumped. Hence Boyfriend dumps immediately, which is a sad thing for all.

36. Consider the "cross-out game." In this game, one writes down the numbers 1, 2, 3. Person 1 starts by crossing out any one number or any two adjacent numbers: for example, person 1 might cross out 1, might cross out 1 and 2, or might cross out 2 and 3. Then person 2 also crosses out either one number or two adjacent numbers. For example, starting from 1, 2, 3, say person 1 crosses out 1. Then person 2 can either cross out 2, cross out 3, or cross out both 2 and 3. Play continues like this. Once a number is crossed out, it cannot be crossed out again. Also, if for example person 1 crosses out 2 in her first move, person 2 cannot then cross out both 1 and 3, because 1 and 3 are not adjacent (even though 2 is crossed out). The winner is the person who crosses out the last number.

a. Model this as an extensive form game. Show a subgame perfect Nash equilibrium of this game by drawing appropriate arrows in the game tree.
We did this in class.

b. Now instead of just three numbers, say that you start with m numbers. In other words, you have the numbers $1, 2, 3, \ldots, m$. Can person 1 always win this game? (Hint: look at $m = 4$, $m = 5$, etc. first to get some ideas.)
Person 1 can always win this game. His strategy is follows. If m is even, then person 1 starts by crossing out the two middle numbers. If m is odd, then person 1 starts out by crossing out the middle number. Henceforth, whatever number(s) person 2 crosses out, person 1 responds by crossing out the "mirror image" of those number(s) on the "other side" (if person 2 crosses out the number x, person 1 crosses out $m - x + 1$). Play continues like this, with each time person 1 responding in this manner to person 2. It is easy to see that person 1 is guaranteed to cross out the last number.
For example, say that $m = 7$. Person 1 crosses out 4, the middle number. If person 2 crosses out 1, then person 1 responds by crossing out 7. If person 2 then crosses out 5 and 6, then person 1 responds by crossing out 2 and 3.
Notice that this game is not related to the Nim game in the homework (there is no issue of multiples of 3, for example). Answers of the form "Person 1 will always win by crossing out the middle numbers" receive no credit without a full explanation like the one above.

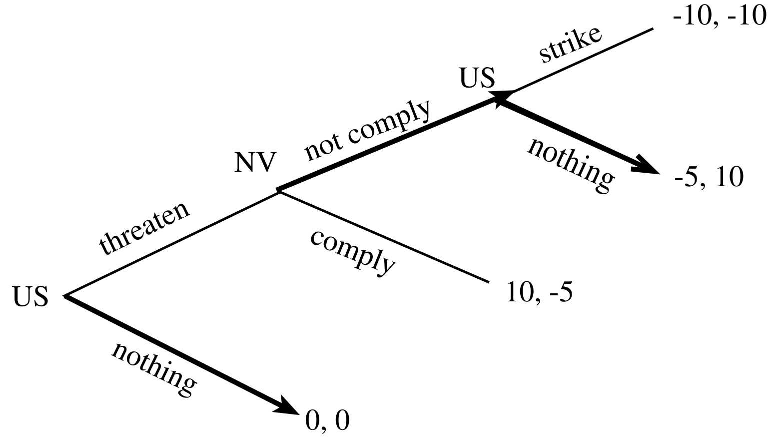
37. Read "It's Not Just About Bonds, It's About Who's on Deck" by Jack Curry on the "Media Clips" website. Model the situation using extensive form games and use the games to show that who bats behind Barry Bonds influences a pitcher's decision of how to pitch to Bonds.
I model the situation in this way.



Here the Giants can either put in Alfonzo or Cruz behind Bonds. The Dodgers can either intentionally walk Bonds or not. If Alfonzo bats behind Bonds, and the Dodgers intentionally walk Bonds, putting him on base, they run the risk that Alfonzo will advance Bonds. But Alfonzo is hitting poorly enough these days that there is only a 30 percent chance that Bonds will advance. If the Dodgers do not intentionally walk Bonds and pitch to him, then Bonds will get on base with a 40 percent chance. If the Giants put in Cruz behind Bonds, then if the Dodgers intentionally walk Bonds, they would have to face Cruz, who is on a hot streak and will advance Bonds with a 50 percent chance. As before, if the Dodgers pitch to Bonds, then Bonds will get on base with a 40 percent chance.
The subgame perfect Nash equilibrium is shown in the game tree also. If the Giants put Alfonzo behind Bonds, then the Dodgers will walk Bonds, knowing that Alfonzo is slumping. If the Giants put Cruz behind Bonds, then the Dodgers will pitch to Bonds. Hence the Giants put Cruz behind Bonds.

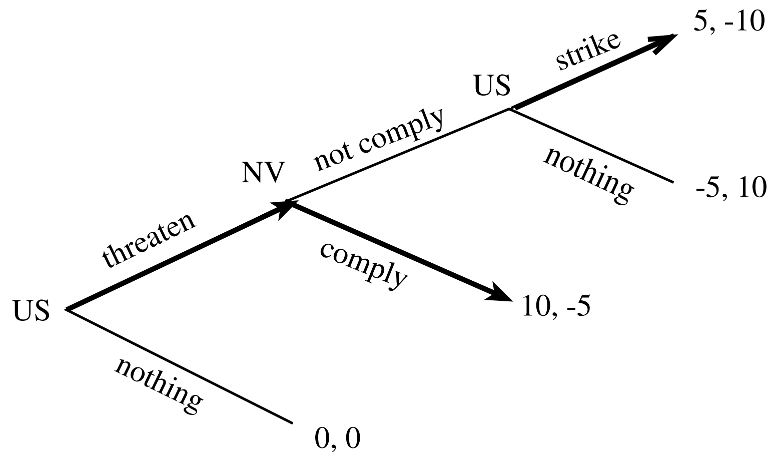
38. Richard Nixon subscribed to a foreign-policy principle which he called the "madman theory" or the "theory of excessive force" (see "The Trials of Henry Kissinger" on the Media Clips website, or read "Nixon's Nuclear Ploy" by William Burr and Jeffrey Kimball, also on the Media Clips website). We will explain this theory using an extensive form game.
Say that the US is facing an adversary (for example North Vietnam). The US makes the first move: it can either threaten North Vietnam or do nothing. If the US does nothing, then the game ends and "nothing happens." If the US makes a threat, North Vietnam can either comply or not comply. If North Vietnam complies, then the game ends. If North Vietnam does not comply, then the US can either strike militarily or not.

a. Say the payoffs are like this: if the US does nothing, then both get payoff 0. If the US threatens and North Vietnam complies, then the US gets payoff 10 and North Vietnam gets payoff -5. If the US threatens, North Vietnam does not comply, and the US strikes, then the US gets payoff -10 and North Vietnam gets payoff -10. If the US threatens, North Vietnam does not comply, and the US does not strike, then the US gets payoff -5 and North Vietnam gets payoff 10.
The idea here is that the best thing for the US is for the US to make a threat and North Vietnam to comply; this way the US doesn't have to perform the military strike. The worst thing for the US is to have to perform the military strike, since this would risk expanding the war. The best thing for North Vietnam is to openly defy the US and "call the bluff" of the US; the worst thing is the military strike. The second-worst thing is to comply to the US threat, and the second-best thing is to not be threatened at all.
Model this as an extensive form game and find the subgame perfect Nash equilibrium.
The game looks like this.



The subgame perfect Nash equilibrium is shown with the arrows. Since a military strike is costly to both sides, the US will not strike if North Vietnam does not comply. Hence North Vietnam will not comply if threatened, and hence the US will not threaten. In a sense, if the US threatens, then North Vietnam will "call the bluff" because it knows that the US is not prepared to face the losses of a military strike.

b. Now say that Nixon, by purposefully showing the world that he is a "madman," convinces everyone that the US payoff from a military strike is 5, not -10. In other words, Nixon tells the world that as far as he is concerned, he would enjoy a military strike. Model this as an extensive form game (all other payoffs are the same) and find the subgame perfect Nash equilibrium.
Now the game looks like this.



Now the US would love to engage in the military strike if North Vietnam does not comply. Hence North Vietnam complies, and hence the US can threaten.

c. Compare the two situations and explain why Nixon wants to show the world that he is a "madman."
Nixon is better off in the second scenario. If people believe that the second scenario is the correct one (that Nixon is a "madman" in the sense that he wants to attack even if high casualties result) then Nixon can threaten and get his way without ever actually having to resort to military action.

39. Say we have two basketball players, Nash and Yao. Nash has the ball 20 feet from the basket and Yao is defending. Nash can either take a jump shot or drive the basket. Yao can either come out and contest the shot, or can stay in and protect the basket. The game looks like this.

	Yao comes out	Yao protects
Nash takes jump shot	0.3, 0.7	0.4, 0.6
Nash drives	0.5, 0.5	0.3, 0.7

For example, if Nash takes a jump shot and Yao comes out, then Nash has a 30 percent chance of scoring and Yao has a 70 percent chance of successfully defending.

a. Find the mixed strategy Nash equilibrium of this game. In the mixed Nash equilibrium, what is Nash's overall probability of scoring?
It's easy to see that there are no pure strategy Nash equilibria. To find mixed strategy Nash equilibria, let's set up our p and q:

		$[q]$	$[1-q]$
		Yao comes out	Yao protects
$[p]$	Nash takes jump shot	0.3, 0.7	0.4, 0.6
$[1-p]$	Nash drives	0.5, 0.5	0.3, 0.7

First solve for q. If Nash takes a jump shot, he gets payoff $0.3(q)+0.4(1-q)$. If Nash drives, he gets payoff $0.5(q)+0.3(1-q)$. To find the switchover value of q, we set these equal and get $0.3(q)+0.4(1-q) = 0.5(q)+0.3(1-q)$, or in other words $0.4-0.1q = 0.3+0.2q$. Hence $0.1 = 0.3q$ and we have $q = 1/3$.

Now solve for p. If Yao comes out, he gets payoff $0.7p + 0.5(1-p)$. If Yao protects, he gets $0.6p + 0.7(1-p)$. To find the switchover value of p, we set these equal and get $0.7p + 0.5(1-p) = 0.6p + 0.7(1-p)$, or in other words $0.5 + 0.2p = 0.7 - 0.1p$. Hence $0.3p = 0.2$ and we have $p = 2/3$.
So the mixed Nash equilibrium is that Nash takes a jump shot with probability 2/3 and drives with probability 1/3 and Yao comes out with probability 1/3 and protects with probability 2/3.
If Nash takes a jump shot, his overall probability of scoring is $0.3(1/3) + 0.4(2/3) = 1.1/3 = 0.37$. If Nash drives, his overall probability of scoring is $0.5(1/3) + 0.3(2/3) = 1.1/3 = 0.37$. Notice these numbers are the same because otherwise, Nash would not want to randomize. Hence Nash's overall probability of scoring is 0.37.

b. Now say that Nash practices over the summer and improves his jump shot. Now the game looks like this.

	Yao comes out	Yao protects
Nash takes jump shot	0.4, 0.6	0.5, 0.5
Nash drives	0.5, 0.5	0.3, 0.7

Find the mixed strategy Nash equilibrium of this new game. In the mixed Nash equilibrium, what is Nash's overall probability of scoring?
Again, there are no pure strategy Nash equilibria. To find mixed strategy Nash equilibria, let's set up our p and q:

		$[q]$	$[1-q]$
		Yao comes out	Yao protects
$[p]$	Nash takes jump shot	0.4, 0.6	0.5, 0.5
$[1-p]$	Nash drives	0.5, 0.5	0.3, 0.7

First solve for q. If Nash takes a jump shot, he gets payoff $0.4(q) + 0.5(1-q)$. If Nash drives, he gets payoff $0.5(q) + 0.3(1-q)$. To find the switchover value of q, we set these equal and get $0.4(q) + 0.5(1-q) = 0.5(q) + 0.3(1-q)$, or in other words $0.5 - 0.1q = 0.3 + 0.2q$. Hence $0.2 = 0.3q$ and we have $q = 2/3$.
Now solve for p. If Yao comes out, he gets payoff $0.6p + 0.5(1-p)$. If Yao protects, he gets $0.5p + 0.7(1-p)$. To find the switchover value of p, we set these equal and get $0.6p + 0.5(1-p) = 0.5p + 0.7(1-p)$, or in other words $0.5 + 0.1p = 0.7 - 0.2p$. Hence $0.3p = 0.2$ and we have $p = 2/3$.
So the mixed Nash equilibrium is that Nash takes a jump shot with probability 2/3 and drives with probability 1/3 and Yao comes out with probability 2/3 and protects with probability 1/3.
If Nash takes a jump shot, his overall probability of scoring is $0.4(2/3) + 0.5(1/3) = 1.3/3 = 0.43$. If Nash drives, his overall probability of scoring is $0.5(2/3) + 0.3(1/3) = 1.3/3 = 0.43$. Notice these numbers are the same because otherwise, Nash would not want to randomize. Hence Nash's overall probability of scoring is 0.43.

c. After Nash improves his jump shot, does he go to his jump shot more often? After Nash improves his jump shot, is it more successful? After Nash improves his jump shot, is his drive more successful? Why does an improved jump shot improve other parts of Nash's game?

When Nash improves his jump shot, he doesn't go to it any more often; he still goes to it 1/3 of the time. Nash's jump shot improves from an overall 0.37 scoring probability to 0.43. Interestingly, his drive is more successful, also improving to an overall scoring probability of 0.43. His drive is more successful even though he hasn't practiced it (he only practiced his jump shot). The reason his drive is more successful is because Yao has to come out more often (with probability 2/3 instead of 1/3) because of his improved jump shot. So it really is true that improving one aspect of your game improves the other aspects of your game, because it makes the defense react. Nash's better jump shot makes Yao come out more, which improves the chances of Nash's drive.

40. Consider the "Battle of the Sexes" game below.

	$2a$	$2b$
$1a$	$2, 1$	$0, 0$
$1b$	$0, 0$	$1, 2$

a. Find all Nash equilibria (pure strategy and mixed strategy) of this game.
The pure Nash equilibria are $(1a, 2a)$ and $(1b, 2b)$. To find the mixed Nash equilibria, let person 1 play $1a$ with probability p and $1b$ with probability $1-p$. Let person 2 play $2a$ with probability q and $2b$ with probability $1-q$.
Given person 2's strategy, if person 1 plays $1a$, he gets an expected payoff of $2q+0(1-q) = 2q$. If person 1 plays $1b$, he gets an expected payoff of $0q + 1(1-q) = 1-q$. To find person 1's "switchover" probability, we set $2q = 1-q$ and get $q = 1/3$.
Given person 1's strategy, if player 2 plays $2a$, she gets an expected payoff of $1p+0(1-p) = p$. If person 2 plays $2b$, she gets an expected payoff of $0p + 2(1-p) = 2-2p$. To find person 2's "switchover" probability, we set $p = 2-2p$ and get $p = 2/3$.
Thus in the mixed strategy Nash equilibrium, person 1 plays $1a$ with probability 2/3 and plays $1b$ with probability 1/3 and person 2 plays $2a$ with probability 1/3 and $2b$ with probability 2/3.

b. Are any strategies in this game weakly or strongly dominated?
No strategies are weakly or strongly dominated.

41. Consider the following game.

	$2a$	$2b$	$2c$	$2d$	$2e$
$1a$	$63, -1$	$28, -1$	$-2, 0$	$-2, 45$	$-3, 19$
$1b$	$32, 1$	$2, 2$	$2, 5$	$33, 0$	$2, 3$
$1c$	$54, 1$	$95, -1$	$0, 2$	$4, -1$	$0, 4$
$1d$	$1, -33$	$-3, 43$	$-1, 39$	$1, -12$	$-1, 17$
$1e$	$-22, 0$	$1, -13$	$-1, 88$	$-2, -57$	$-3, 72$

a. Find all pure strategy Nash equilibria of this game.
We can use our stars and pluses:

	$2a$	$2b$	$2c$	$2d$	$2e$
$1a$	$*63,-1$	$28,-1$	$-2,0$	$-2,45+$	$-3,19$
$1b$	$32,1$	$2,2$	$*2,5+$	$*33,0$	$*2,3$
$1c$	$54,1$	$*95,-1$	$0,2+$	$4,-1$	$0,4$
$1d$	$1,-33$	$-3,43+$	$-1,39$	$1,-12$	$-1,17$
$1e$	$-22,0$	$1,-13$	$-1,88+$	$-2,-57$	$-3,72$

So the only pure strategy Nash equilibrium is $(1b, 2c)$.

b. Make a prediction in this game by iteratively eliminating (strongly or weakly) dominated strategies.
First we note that $1c$ strongly dominates $1d$ and $1e$.

	$2a$	$2b$	$2c$	$2d$	$2e$
$1a$	$63,-1$	$28,-1$	$-2,0$	$-2,45$	$-3,19$
$1b$	$32,1$	$2,2$	$2,5$	$33,0$	$2,3$
$1c$	$54,1$	$95,-1$	$0,2$	$4,-1$	$0,4$

Next we note that $2c$ strongly dominates $2a$ and $2b$.

	$2c$	$2d$	$2e$
$1a$	$-2,0$	$-2,45$	$-3,19$
$1b$	$2,5$	$33,0$	$2,3$
$1c$	$0,2$	$4,-1$	$0,4$

Next we note that $1b$ strongly dominates $1a$ and $1c$.

	$2c$	$2d$	$2e$
$1b$	$2,5$	$33,0$	$2,3$

Finally, we note that $2c$ strongly dominates $2d$ and $2e$.

	$2c$
$1b$	$2,5$

So the prediction is that person 1 plays $1b$ and person 2 plays $2c$.

42. [from Spring 2002 midterm] Mother can ask either Sister or Brother to do the dishes while she goes out shopping. If Mother asks Sister, she can either do it or not do it. If Mother asks Brother, he can either do it or not do it. Since Sister does a better job, Mother prefers Sister doing the dishes over Brother doing the dishes. However, Mother prefers Brother doing the dishes over them not being done.
Both Sister's and Brother's preferences are like this: the best thing is for the other person to do the dishes; the second best thing is for Mother to ask the other person and have the other person not do it (since then the other person will get blamed). The third best thing is to do the dishes, and the worst thing is to be asked to do the dishes but then not do it (since you will get in trouble).
The main thing here is to set up the players and strategies correctly. It is crucial that this is represented as a 3 person game.

	You do it	You don't
Mom asks you	5, 3, 9	0, 0, 7
Mom asks brother	4, 9, 3	4, 9, 3

Brother does it

	You do it	You don't
Mom asks you	5, 3, 9	0, 0, 7
Mom asks brother	0, 7, 0	0, 7, 0

Brother doesn't

a. Find all (pure strategy) Nash equilibria.
By checking all the strategy profiles, we can see that (Mom asks you, You do it, Brother does it), (Mom asks you, You do it, Brother doesn't), and (Mom asks brother, You don't, Brother does it) are the three Nash equilibria.

43. The simplest kind of game has two players, who each have two possible actions. We call these games "2×2 games."

a. Write down a 2×2 game which has exactly one pure strategy Nash equilibrium and no mixed strategy Nash equilibrium. Solve for the equilibrium.
A Prisoners' Dilemma works here.

	$2a$	$2b$
$1a$	$3, 3$	$0, 4$
$1b$	$4, 0$	$1, 1$

There is only one pure strategy Nash equilibrium $(1b, 2b)$ and no mixed strategy Nash equilibrium (since person 1 will never play $1a$ and person 2 will never play $2a$).

b. Write down a 2×2 game which has no pure strategy Nash equilibrium and exactly one mixed strategy Nash equilibrium. Solve for the equilibrium.
The penalty kick game works here.

	$2a$	$2b$
$1a$	$1, 0$	$0, 1$
$1b$	$0, 1$	$1, 0$

As we did in class, in the mixed strategy Nash equilibrium, person 1 plays $1a$ with probability $1/2$ and $1b$ with probability $1/2$, and person 2 plays $2a$ with probability $1/2$ and $2b$ with probability $1/2$.

c. Write down a 2×2 game which has exactly three total (pure and mixed) Nash equilibria. Solve for the equilibria.
The chicken game works here.

	$2a$	$2b$
$1a$	$3, 3$	$1, 4$
$1b$	$4, 1$	$0, 0$

Here $(1b, 2a)$ and $(1a, 2b)$ are pure strategy Nash equilibria. If we compute the mixed strategy Nash equilibrium, we find that person 1 plays $1a$ with probability $1/2$ and $1b$ with probability $1/2$, and person 2 plays $2a$ with probability $1/2$ and $2b$ with probability $1/2$.

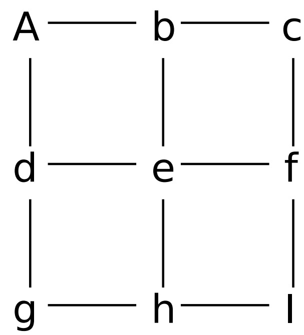
d. Write down a 2×2 game in which the total number of (pure and mixed) Nash equilibria is neither one nor three. Solve for the equilibria.

The game in which "everything ties" works here.

	$2a$	$2b$
$1a$	$3,3$	$3,3$
$1b$	$3,3$	$3,3$

Here there are four pure strategy Nash equilibria: $(1a, 2a), (1a, 2b), (1b, 2a), (1b, 2b)$. In fact, if person 1 plays any mixed strategy and person 2 plays any mixed strategy, then that is a mixed strategy Nash equilibrium.

44. Say that country A and country I are at war. The two countries are separated by a system of rivers, as shown below.



Country I sends a naval fleet with just enough supplies to reach A. The fleet must stop for the night at intersections (for example, if the fleet takes the path IhebA, it must stop the first night at h, the second at e, and the third at b). If unhindered, on the fourth day the fleet will reach A and destroy country A. Country A can send a fleet to prevent this. Country A's fleet has enough supplies to visit three contiguous intersections, starting from A (for example Abcf). If it catches Country I's fleet (that is, if both countries stop for the night at the same intersection), it destroys the fleet and wins the war. Model this as a strategic form game, assuming that the winner gets payoff 1 and the loser gets payoff -1. Iteratively eliminate weakly dominated strategies and make some sort of prediction.
Please see the attached page.

2.14 A Military Strategy Game

First we can eliminate all Country I strategies that don't arrive at A. This leaves six strategies, which we can label fcb, feb, fed, hed, heb, and hgd. We can also eliminate all Country A strategies that stay at A at any time, or that hit h or f. This leaves the six strategies bcb,beb,bed,ded,deb,dgd. Here is the payoff matrix:

	bcb	beb	bed	ded	deb	dgd
fcb	−1	−1	1	1	−1	1
feb	−1	−1	−1	−1	−1	1
fed	1	−1	−1	−1	−1	−1
hed	1	−1	−1	−1	−1	−1
heb	−1	−1	−1	−1	−1	1
hgd	1	1	−1	−1	1	−1

Now feb is weakly dominated by fcb, as is heb. Moreover, we see that fed and hed are weakly dominated by hgd. Thus there are two remaining strategies for Country I, "south" (hgd) and "north" (fcb).

Also bcb is dominated by beb and dgd is dominated by ded, so we may drop them. Moreover, beb and deb are the same "patrol north", while bed and ded are the same "patrol south." This gives us the following reduced game:

	patrol north	patrol south
attack north	−1,1	1,−1
attack south	1,−1	−1,1

So this complicated game is just the heads-tails game, which we will finish solving when we do mixed strategy equilibria!

45. [from Spring 2002 final] There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.
The game looks like this:

	$2A$	$2B$		2A	2B
$1A$	$3,3,3$	$0,4,0$	1A	0, 0, 4	4, 0, 0
$1B$	$4,0,0$	$0,0,4$	1B	0, 4, 0	0, 0, 0
	$3A$			3B	

The pure strategy Nash equilibria are $(1B, 2A, 3A)$, $(1A, 2B, 3A)$, $(1B, 2B, 3A)$, $(1A, 2A, 3B)$, $(1B, 2A, 3B)$, and $(1A, 2B, 3B)$.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability p and B with probability $1 - p$. Find p.
Say person 2 plays $2A$ with probability p and $2B$ with probability $1 - p$ and person 3 plays $3A$ with probability p and $3B$ with probability $1 - p$. If person 1 plays $1A$, her expected payoff is $3p^2 + 4(1 - p)^2$. If person 1 plays $1B$, her expected payoff is $4p^2$. To find the switchover probability, we set $3p^2 + 4(1 - p)^2 = 4p^2$ and hence get $4(1 - p)^2 = p^2$. Thus $4 - 8p + 4p^2 = p^2$ and so $3p^2 - 8p + 4 = 0$. Factoring, we get $(3p - 2)(p - 2) = 0$ and hence $p = 2/3$ ($p = 2$ is not possible because p is a probability). If you do the same thing with persons 2 and 3, you will get the same answer for p since the game is so symmetric.
Hence in the mixed Nash equilibrium, each person plays A with probability $2/3$ and B with probability $1/3$.

46. Say that Townsville is deciding how many coal-fired energy plants to build to supply its energy needs. Some people are more environmentally oriented and thus prefer fewer plants, and some people think that the jobs and electricity that the plants provide are more important. Hence people differ on how many plants they feel are necessary. An opinion poll is taken asking each person how many plants she or he prefers. The results are that 14 percent of the population prefer 0 plants, 16 percent prefer 1 plant, 18 percent prefer 2 plants, 6 percent prefer 3 plants, 30 percent prefer 4 plants, and 16 percent prefer 5 plants.

a. Say that there are two candidates running for office, and the only relevant issue is how many plants to build. Each candidate takes a position on how many plants to build, and then each voter votes for the candidate which is closest to her own position or "ideal point." For example, if candidate 1 is for 3 plants and candidate 2 is for 0 plants, a voter who prefers 2 plants will vote for candidate 1. If there is a "tie" (if two candidates are equally close to a voter's ideal point), then half of the votes go to each candidate. For example, if candidate 1 is for 2 plants and candidate 2 is for 0 plants, then half of the people who prefer 1 vote for candidate 1 and half vote for candidate 2. Each candidate wants to maximize the total number of votes she gets.

Model this as a strategic form game (the candidates move simultaneously) as in the Downsian model. Find the pure strategy Nash equilibrium. Predict what positions the candidates will take and how many plants the town will build.
The game looks like this:

	0	1	2	3	4	5
0	$50, 50$	$14, 86$	$22, 78$	$30, 70$	$39, 61$	$48, 52$
1	$86, 14$	$50, 50$	$30, 70$	$39, 61$	$48, 52$	$51, 49$
2	$78, 22$	$70, 30$	$50, 50$	$48, 52$	$51, 49$	$54, 46$
3	$70, 30$	$61, 39$	$52, 48$	$50, 50$	$54, 46$	$69, 31$
4	$61, 39$	$52, 48$	$51, 49$	$46, 54$	$50, 50$	$84, 16$
5	$52, 48$	$49, 51$	$46, 54$	$31, 69$	$16, 84$	$50, 50$

The only pure strategy Nash equilibrium is $(3, 3)$. Both candidates support building 3 plants. Note that the median voter (the 50th percentile voter) favors building 3 plants.

b. Now say that there are three candidates. Is there a pure strategy Nash equilibrium which is similar to what you found in part a.?
If there were three candidates, and all picked 3, then each would get a payoff of 33.3. If one of them deviated and picked (for example) 4, then the person deviating would get 46, which is better than 33.3. Hence $(3, 3, 3)$ would not be a Nash equilibrium of the three person game.

c. Now say that opinions shift. A new poll is taken, and it is found that 4 percent of the population prefer 0 plants, 10 percent prefer 1 plant, 78 percent prefer 2 plants, 2 percent prefer 3 plants, 2 percent prefer 4 plants, and 4 percent prefer 5 plants. Say there are two candidates. Predict what positions the candidates will take and how many plants the town will build.
Now the game looks like this:

	0	1	2	3	4	5
0	$50, 50$	$4, 96$	$9, 91$	$14, 86$	$53, 47$	$92, 8$
1	$96, 4$	$50, 50$	$14, 86$	$53, 47$	$92, 8$	$93, 7$
2	$91, 9$	$86, 14$	$50, 50$	$92, 8$	$93, 7$	$94, 6$
3	$86, 14$	$47, 53$	$8, 92$	$50, 50$	$94, 6$	$95, 5$
4	$47, 53$	$8, 92$	$7, 93$	$6, 94$	$50, 50$	$96, 4$
5	$8, 92$	$7, 93$	$6, 94$	$5, 95$	$4, 96$	$50, 50$

The only pure strategy Nash equilibrium is $(2, 2)$. Both candidates support building 2 plants. Note that the median voter (the 50th percentile voter) favors building 2 plants.

d. Now again say that there are three candidates. Is there a pure strategy Nash equilibrium which is similar to what you found in part c.?
If there were three candidates, and all picked 2, then each would get a payoff of 33.3. If one of them deviated and picked 0, then the person deviating would get 7.33. If a person deviated to 1, she would get 14. If a person deviated to 3, she would get 8. If a person deviated to 4, she would get 6.67. If a person deviated to 5, she would get 6. Hence a person cannot gain by deviating. Hence $(3, 3, 3)$ is a Nash equilibrium.

Note that in part a, all three candidates taking the median voter position was not a Nash equilibrium. Here, because there are so many people at the "center" (78 percent prefer 2 plants), a candidate cannot gain by taking a more left or right position, even if there are three candidates.

47. Say that Gotham City is deciding how many skate parks and dog walks to build in the city. A survey is done and it is found that 40 percent of the population are cranky taxpayers who dislike public expenditures and prefer 0 skate parks and 0 dog walks; 22 percent are hard core skate punks who prefer 6 skate parks and 0 dog walks; 30 percent are yuppie golden retriever owners who prefer 0 skate parks and 6 dog walks; and 8 percent are consensus-minded Buddhists who prefer 2 skate parks and 2 dog walks.
Say that there are two candidates running for office who take positions on both issues. As in the Downsian model, each voter votes for the candidate which is closest to her own position or "ideal point." For example, if candidate 1 favors 1 skate park and 1 dog walk, and candidate 2 favors 4 skate parks and 2 dog walks, then candidate 1 gets 78 percent of the vote (the cranky taxpayers, the yuppies, and the Buddhists) and candidate 2 gets 22 percent (the skate punks). Each candidate wants to maximize the total number of votes she gets.

a. Let's try to make a prediction in this game by eliminating weakly and strongly dominated strategies. First, simplify the game a lot by considering only the following strategies: (0,0), (0,3), (0,6), (1,1), (2,2), (3,0), (3,3), (6,0). Here (3,0) means for example 3 skate parks and 0 dog walks. Each of the two candidates thus has eight possible strategies. Write this as a strategic form game and make a prediction by eliminating weakly and strongly dominated strategies.
The game looks like this (we show only person 1's payoffs; person 2's payoff is 100 minus person 1's payoff).

	$(0,0)$	$(0,3)$	$(0,6)$	$(1,1)$	$(2,2)$	$(3,0)$	$(3,3)$	$(6,0)$
$(0,0)$	50	62	70	40	40	70	40	78
$(0,3)$	38	50	70	30	30	54	70	78
$(0,6)$	30	30	50	30	30	30	30	54
$(1,1)$	60	70	70	50	40	78	44	78
$(2,2)$	60	70	70	60	50	78	48	78
$(3,0)$	30	46	70	22	22	50	62	78
$(3,3)$	60	30	70	56	52	38	50	78
$(6,0)$	22	22	46	22	22	22	22	50

In this game, the strategies (0,0), (0,6), (1,1), (3,0), (6,0) are weakly or strongly dominated and we are left with (0,3),(2,2),(3,3). No further elimination is possible.

	$(0,3)$	$(2,2)$	$(3,3)$
$(0,3)$	50	30	70
$(2,2)$	70	50	48
$(3,3)$	30	52	50

b. Now say that most of the yuppies see Richard Gere movies and decide to become Buddhists (and take the Buddhist position of supporting 2 skate parks and 2 dog walks). Now there are 5 percent yuppies, 33 percent Buddhists, 40 percent cranky taxpayers, and 22 percent skate punks. Again, find all strategies for both candidates which are not weakly dominated. Now we get the following game (again, for simplicity only candidate 1's payoffs are given).

	$(0,0)$	$(0,3)$	$(0,6)$	$(1,1)$	$(2,2)$	$(3,0)$	$(3,3)$	$(6,0)$
$(0,0)$	50	62	95	40	40	45	40	78
$(0,3)$	38	50	95	5	5	41.5	45	78
$(0,6)$	5	5	50	5	5	5	5	41.5
$(1,1)$	60	95	95	50	40	78	56.5	78
$(2,2)$	60	95	95	60	50	78	73	78
$(3,0)$	55	58.5	95	22	22	50	62	78
$(3,3)$	60	55	95	43.5	27	38	50	78
$(6,0)$	22	22	58.5	22	22	22	22	50

In this game, the weakly or strongly dominated strategies are (0,0), (0, 3), (0, 6), (1,1), (3,0), (3,3), and (6,0). Only (2,2) remains. The shift in the electorate toward central positions causes the candidates to take more centrist positions.

48. The city council of Asbestosville wants to improve its image by bringing in the Palookaville Pirates, a minor league baseball franchise which is currently located in Palookaville. Asbestosville has built a brand new baseball field and is now trying to come up with other enticements for the Pirates, such as how much cash to give to the team. The city has already agreed to give the team \$1 million, but some council members want to give the team more money. There are 11 council members. One member does not want to give the team any more money and prefers to give only \$1 million total to the Pirates, one member prefers to give \$2 million in total, one member prefers to give \$3 million, one member prefers to give \$4 million, and so forth; the eleventh council member wants to give the Pirates \$11 million in total. As you can see, the median member of the council wants to give the Pirates \$6 million.

a. The city council chairperson is a baseball fanatic and wants to give the Pirates \$11 million in total. The chairperson controls the city agenda and thus decides what proposal to bring to the council. When a proposal is brought to the council, all council members simply vote yes or no (the chairperson also votes). If the vote fails, then the policy remains at the status quo (giving \$1 million total). Like in the Downsian model, a council member wants the final policy to be as close to her own "ideal point" as possible. What proposal will the chairperson make? How much money will the Pirates receive?
We can model this as an extensive form game and find the subgame perfect Nash equilibria, but it is easy enough to simply reason through it. If the chair proposes \$6 million for example, council member 4 will vote for it (since 6 is closer to 4 than the status quo, 1, is). For that matter, council members 4 through 11 will vote for it. Council member 3 will vote against it (since the status quo 1 is closer than 6), and so will council members 1 and 2. Hence a \$6 million proposal will pass by majority. Since the chairperson wants to give as much money as possible to the team, she will propose the highest possible amount such that a majority will vote for it over the status quo. If the chairperson proposes \$10 million, council members 6 through 11 will vote for it (6 votes in favor) and council members 1 through 5 will vote

against (5 votes against). Hence this proposal will pass. If the chairperson proposes the more extreme \$11 million, council members 7 through 11 will vote for it (5 votes in favor) and council members 1 through 5 will vote against it (5 votes against); council member 6 will be indifferent and might vote either way (or "split" her vote evenly among the two). Since council member 6 is "shaky," we'll say that the chairperson will propose \$10 million and it will pass for certain. So the Pirates will receive \$10 million.

b. Now say that the mayor can veto the city council decision. The mayor's ideal point is to give the Pirates a total of \$2.6 million. If the council's decision is farther away than the status quo from the mayor's ideal point, then the mayor will veto the council's decision and the status quo will be implemented. The sequence of decisions is like this: the council chairperson first makes a proposal, then the council members vote, and then the mayor can veto. Now what proposal will the chair person make? How much money will the Pirates receive?
Again, we can model this as an extensive form game, but let's just reason through it. For the mayor, the status quo (1) is 1.6 away from his ideal point (2.6). So he will veto any proposal which is more than 1.6 away (any proposal higher than 4.2). The only proposals which he will not veto are 1, 2, 3, and 4. Given this, the council chairperson will propose \$4 million, which will pass by a vote of 9 to 2, and will not be vetoed by the mayor. The Pirates will receive \$4 million. If we allow the chairperson to propose fractional amounts, the chairperson will propose \$4.2 million.

c. Now say that if the mayor vetoes the city council decision, the city council can override the veto with two thirds of the council vote (in this case, 8 of the 11 council members). If the council's proposal is vetoed, then if the council overrides the veto, it can implement its original proposal. So now the sequence of decisions is like this: the council chairperson first makes a proposal, then the council members vote, and then the mayor can veto; if the mayor vetoes, then the council can override. Now what proposal will the chair person make? How much money will the Pirates receive?
If eight council members prefer a proposal to the status quo, then the council will override the mayor's veto in favor of the proposal. Note that the \$6 million proposal would get eight votes (council members 4 through 11) in favor, three against. The \$7 million proposal will get seven votes in favor (members 5 through 11), three against (members 1 through 3) and member 4 will be indifferent between the two proposals. An \$8 million or higher proposal will get seven votes or fewer, and hence a proposal of \$8 million would not have support of two thirds of the council over the status quo. So \$6 million is the highest proposal which will get two thirds support for certain. A \$7 million proposal might get two thirds, but is shaky because of the indifference of member 4. Since the chair prefers the highest possible proposal, she will propose \$6 million, it will pass, and not be vetoed.

49. Say that there are three people and five candidates $\{a, b, c, d, e\}$. Say person 1's order of preference (from best to worst) is c, b, e, d, a. Person 2's order is d, c, a, b, e. Person 3's order is e, a, b, d, c.

a. Show that for each candidate, there is an order of voting (an "agenda") in which that candidate wins.
The easiest way to do this problem is first figure out which candidate beats which by majority. It's easy to find that $a > b$, $b > d$, $b > e$, $c > a$, $c > b$, $c > e$, $d > a$, $d > c$, $e > a$, $e > d$, where $a > b$ means that a beats b by majority rule.
There are many possible agendas which make a win. The easiest way to find one is to first figure out which candidate a beats (in this case b). Then we figure out which candidate b beats (say d). Then we figure out which candidate d beats (in this case c). Then we figure out which candidate c beats (in this case e). So we have the sequence a, b, d, c, e.
Thus our agenda goes like this: First, vote over a or not. If not, then vote over b or not. If not, vote over d or not. If not, vote between c and e.
If the last (fourth) stage is reached, c will be chosen (c beats e by majority). If the third stage is reached, d wins (d beats c). If the second stage is reached, b wins (b beats d). In the first stage, a wins since it beats b.

b. Say that there are three people and four candidates $\{a, b, c, d\}$. Say person 1's order of preference (from best to worst) is c, b, d, a. Person 2's order is b, a, d, c. Person 3's order is a, c, d, b. Show that there is no order of voting in which candidate d wins. Why is this?
There is no agenda in which d wins simply because d does not beat any other candidate by majority rule. Regardless of the agenda, at some point d will have to be compared with some other candidate, and when this happens, d will lose.

50. Say that eleven people vote over four candidates $\{a, b, c, d\}$. Three people have preference order (from best to worst) b, a, c, d. Three people have preference order b, a, d, c. Three people have preference order a, c, d, b. Two people have preference order a, d, c, b.

a. Is there a candidate which beats all others by majority rule (a Condorcet winner)?
It is easy to find that b beats all other candidates by majority rule. Hence b is a Condorcet winner. Candidate a beats both c and d but is beaten by b.

b. Say that instead of standard majority rule voting, that each person votes 3 points for their first choice, 2 points for their second choice, 1 point for their third choice, and no points for their last choice. This procedure is called the "Borda count." Which candidate wins now?
Now a gets 27 points, b gets 18 points, c gets 11 points, and d gets 10 points. Candidate a wins by the Borda count.

c. Which procedure (majority rule or Borda count) is more reasonable or more fair in your opinion?
One could argue that a is fairer overall because it is everyone's first or second choice. Candidate b beats a by majority rule, but for many people (5 out of 11) b is the worst choice.

51. Another system of voting is "approval voting," in which each voter can place vote for as many candidates as she wishes: for example, one person might put one vote on candidate a, and another person might put one vote on candidate a and one vote on candidate b. Say we have approval voting, and each person votes for his top two candidates.

a. Say that candidate a receives more votes by approval voting than the other two candidates. Is it possible for a to not be a Condorcet winner? If so, write down the preference orderings for each person which make this possible.
Of course, there are many possible examples. The one I thought of has five people and three candidates. Persons 1, 2, and 3 rank the candidates from best to worst as b, a, c. Persons 4 and 5 rank the candidates as a, c, b. Here b beats all other candidates by majority rule and is thus the Condorcet winner.
However, under approval voting, candidate a gets 5 votes, candidate b gets 3 votes, and candidate c gets 2 votes. Thus candidate a wins under approval voting. In this sense, like the Borda count, approval voting is "fairer" than majority rule.

b. Say that a is a Condorcet winner. Is it possible for a to receive fewer approval votes than some other candidate? If so, write down the preference orderings for each person which make this possible.
One can use a similar example. Say persons 1, 2, and 3 rank the candidates from best to worst as a, b, c and persons 4 and 5 rank the candidates as b, c, a. Here a beats all other candidates by majority rule and is thus the Condorcet winner, but b wins under approval voting.

52. Say that there are three people who are deciding over three alternatives, a, b, c. Is it possible for the Borda count winner to be different from the Condorcet winner? If so, write down the preference orderings for each person which make this possible.
Note that since each person casts 3 Borda points, there are a total of 9 Borda points. Hence it is impossible to be a Borda count winner with just 3 points (either someone else will get 4 points or more, or the other two people will get 3 points each, in which case everyone will tie for first place). Thus, to be a Borda count winner, you must receive at least 4 points.
If you receive 6 Borda count points, you are everyone's first choice and hence you are obviously the Condorcet winner. If you receive 5 Borda count points, you are the first choice of two people and the second choice of a third. Since a majority (two people) have you as their first choice, you must be the Condorcet winner. If you receive 4 Borda count points, you must either be two people's first choice and one person's last choice, or one person's first choice and two people's second choice. If you are two people's first choice, you must be the Condorcet winner. If you are one person's first choice and two people's second choice, the only way that another candidate can beat you by majority is if that other candidate is two people's first choice. But then that other candidate would receive 4 Borda count points at least, which must mean you are not a Borda count winner.
Hence in this particular example (3 candidates, 3 voters), it is impossible for a Borda count winner to not be a Condorcet winner.

53. Say that there are three groups in society. Group X's preferences (from best to worst) are a, b, c. Group Y's preferences (from best to worst) are c, b, a. Group Z's preferences (from best to worst) are b, c, a. There are 13 people in group Y and 7 people in group Z. There are x people in group X.

a. Depending on the value of x, what is the Condorcet winner? Is there some value of x such that there is no Condorcet winner?
I should have said this in the problem, but we assume throughout that the number of people in society is odd to avoid ties. Since the total number of people in society is $x + 20$, assume x is odd.
In a vote between a and b, a gets x votes (group X) and b gets 20 votes (groups Y and Z). Similarly, in a vote between a and c, a gets x votes and c gets 20 votes. In a vote between b and c, b gets $x + 7$ votes (groups X and Z) and c gets 13 votes. Thus if $x \geq 21$, then a beats both b and c by majority, and a is the Condorcet winner. If $x \leq 19$, then a loses to both b and c, and the only contest is between b and c. If $7 \leq x \leq 19$, then b beats a and c by majority, and hence b is the Condorcet winner. If $x \leq 5$, then c beats a and b and hence c is the Condorcet winner.
As long as we assume x is odd (and hence there are no ties), there is always a Condorcet winner in this example.

b. Say society uses the Borda count system. For what values of x does the outcome of the Borda count differ from what the society would choose if they chose the Condorcet winner?
Under the Borda count system, a gets $2x$ points (each person in X gives a 2 points), b gets $27 + x$ points (each person in X gives b one point, each person in Y gives b one point, and each person in Z gives b 2 points), and c gets 33 points (each person in Y gives c 2 points and each person in Z gives c one point).
Thus if $2x > 27 + x$ and $2x > 33$, then a wins. Doing some algebra, this is true when $x > 27$ (or in other words, when $x \geq 29$, since x is assumed odd). When $x = 27$, then a and b tie for first place in the Borda count. If $7 \leq x \leq 25$, then b wins in the Borda count because then $27 + x > 2x$ and $27 + x > 33$. If $x \leq 5$, then c wins in the Borda count.
Comparing this with part a. above, we find that if $x \geq 29$, then a is both the Condorcet winner and the Borda count winner. If $x = 27$, then a is the Condorcet winner but ties with b in the Borda count. If $x = 21, 23, 25$ then a is the Condorcet winner but b is the Borda count winner. If $7 \leq x \leq 19$, then b is the Condorcet winner and the Borda count winner. If $x \leq 5$, then c is both the Condorcet winner and the Borda count winner.
So in this example, the Condorcet winner and the Borda count winner are different when $x = 21, 23, 25$.

c. Say society uses approval voting in which each person votes for her top two candidates. For what values of x does the outcome of this approval voting system differ from what the society would choose if they chose the Condorcet winner?
When each person votes for her top two candidates, a gets x votes, b gets $20 + x$ votes, and c gets 20 votes. Notice that everyone now votes for b, since it is everyone's first or second choice. Hence b always wins.
Comparing this with part a. above, we find that when $x \geq 21$, a is the Condorcet winner while b is the approval voting winner. When $7 \leq x \leq 19$, b is the Condorcet winner and the

approval voting winner. When $x \leq 5$, c is the Condorcet winner but b is the appoval voting winner.
Hence the approval voting winner differs from the Condorcet winner when $x \geq 21$ or $x \leq 5$.

54. Say that there are three people deciding by majority rule over four candidates a, b, c, d. Person 1's preferences (from best to worst) are a, b, c, d. Person 2's preferences (from best to worst) are c, d, b, a. Person 3's preferences (from best to worst) are d, a, c, b. Consider voting agendas in which people vote on candidates sequentially.
We can write $a > b$, $a > c$, $c > b$, $c > d$, $d > a$, $d > b$, where $a > b$ means a beats b by majority rule.

a. Is there an agenda in which they decide on a? If there is, show it. If not, explain why.
How about this agenda: first a or not; if not, c or not; if not, d or b. Going backward from the end: d beats b, so if the last vote is reached, d will win. Given this, c will win on the second vote. Given this, a will win on the first vote.

b. Is there an agenda in which they decide on b? If there is, show it. If not, explain why.
There is no agenda in which b wins because b does not beat any other candidate by majority.

c. Is there an agenda in which they decide on c? If there is, show it. If not, explain why.
How about this agenda: first c or not; if not, d or not; if not, a or b. Going backward from the end: a beats b, so if the last vote is reached, a will win. Given this, d will win on the second vote. Given this, c will win on the first vote.

d. Is there an agenda in which they decide on d? If there is, show it. If not, explain why.
How about this agenda: first d or not; if not, a or not; if not, b or c. Going backward from the end: c beats b, so if the last vote is reached, c will win. Given this, a will win on the second vote. Given this, d will win on the first vote.

55. Say that there are three people deciding by majority rule over eight candidates $\{a, b, c, d, e, f, g, h\}$. Person 1's preferences (from best to worst) are f, c, g, e, b, h, a, d. Person 2's preferences (from best to worst) are a, f, e, g, h, b, d, c. Person 3's preferences (from best to worst) are g, a, b, c, h, d, e, f. Consider voting agendas in which people vote on candidates sequentially.

a. Find the top cycle.
It is not hard to see that a beats everything but g, f beats everything but a, and g beats everything but f. In other words, a is beaten only by g, f is beaten only by a, and g is beaten only by f. Thus the top cycle is a, f, g. No other candidate is in the top cycle. For example, if b were in the top cycle, it would have to beat something which beats something, and so forth, which eventually beats a. But the only thing which beats a is g, and the only thing which beats g is f, and b does not beat f or g (or a for that matter).

b. For every candidate in the top cycle, find a voting agenda in which that candidate wins.
An agenda which implements a is as follows. First vote on a or not. If a loses, vote on f or not. If f loses, vote on g or not. Then consider the rest of the candidates in any order.

This agenda works because g would beat any of the candidates other than a or f. When they vote on f, then f is chosen because f beats g. Thus when they vote for a, they vote for a because a beats f.
Agendas which implement f and g are similar.

56. Say that we have a threshold model in which there are 5 people. If the total number of other people who participate is greater or equal to a person's threshold, the person wants to participate also. If the total number of other people who are participating is less than a person's threshold, the person does not want to participate.

a. Say that one person has threshold 1, two people have threshold 2, and two people have threshold 4. Find all of the pure strategy Nash equilibria.
One Nash equilibrium is (n, n, n, n, n), in which no one participates—since no one participates, no one's threshold is met. Another is (p, p, p, p, p), in which everyone participates—everyone's threshold is met (for example, the threshold 4 people see 4 other people participating). Another is (p, p, p, n, n)—the threshold 1 and the two threshold 2 people participate because their thresholds are met.

b. Now say that one of the threshold 2 people becomes a threshold 0 person. Find all of the pure strategy Nash equilibria. Does this change guarantee some level of participation?
The threshold 0 person participates for sure. Hence the threshold 1 joins in, and then the threshold 2 person joins in. However, a threshold 4 person does not join in because there are only three other people participating. So one Nash equilibrium is (p, p, p, n, n). Another is (p, p, p, p, p)—if everyone starts off by participating, then everyone wants to continue to participate. The outcome (n, n, n, n, n), however, is no longer a Nash equilibrium. Hence the change guarantees at least some level of participation.

57. Say that you have a group of 50 people who can either buy a color fax machine or not buy. No one wants to buy a color fax machine if no one else has one (because there would be no one to exchange color faxes with). In fact, each person will buy one only if at least 6 other people buy them. Thus each person has a threshold of 6.

a. Find the two pure strategy Nash equilibria.
The pure strategy Nash equilibria are $(p, \ldots, p)$ (everyone participating) and $(n, \ldots, n)$ (no one participating).

b. Now say that you are a sales rep for the color fax machine company. You can offer discount coupons to potential customers. If you give 1 discount coupon to someone, that decreases their threshold by 1. For example, if you give 6 discount coupons to a single person, you can make that person have threshold 0. If you give 2 discount coupons to a single person, you can make that person have threshold 4. Obviously, you can guarantee that everyone will buy a color fax machine by giving all 50 people six coupons each, but that would be silly (the company would get no profits). Using the fewest possible number of coupons, how can you guarantee that everyone will buy a color fax machine?
To guarantee that someone will participate for sure, you need to have one person who has threshold 0 (you give that person 6 coupons). Given that this person participates, you need to make one person have threshold 1 (you give that person 5 coupons). Similarly, you give 4

coupons to another person to make that person have threshold 2, you give 3 coupons to make another person have threshold 3, and so forth. Hence you give out 6 coupons to 1 person, 5 coupons to another person, 4, to another person, 3 to another person, 2 to another person, and 1 to another person. So the resulting thresholds are $0, 1, 2, 3, 4, 5, 6, 6, 6, \ldots, 6$. The first six people (with thresholds 0 to 5) will revolt, and then everyone else's threshold will be met and then everyone else will revolt. You give out a total of $6+5+4+3+2+1 = 21$ coupons.

58. [from Spring 2003 final] Say that we have 10 people. Each person is thinking about whether or not to join a revolt or not. Each person has a threshold: five people have threshold 4 and five people have threshold 6.

a. Find all pure strategy Nash equilibria of this game.
Everyone not revolting is a Nash equilibrium; if no one revolts, no one wants to revolt. If someone revolts, at least five people must revolt (since the lowest threshold is threshold 4). If at least five people revolt, then all of the threshold 4 people revolt. If all five threshold 4 people revolt, this alone is not enough to get the threshold 6 people to revolt. So another Nash equilibrium is the threshold 4 people revolting and the threshold 6 people not revolting. Say one of the threshold 6 people revolts; then all of the threshold 6 people want to revolt. Hence another Nash equilibrium is everyone revolting.
So there are three Nash equilibria: no one revolts, only the threshold 4 people revolt, and everyone revolts.

b. Say that you have some discount coupons which lower the cost of revolting and hence lower a person's threshold. For example, if I give 3 coupons to a person with threshold 4, she now has threshold 1. If I give 1 coupon to a person with threshold 6, he now has threshold 5. By giving out coupons, I can change the game so that the only Nash equilibrium is one in which everyone revolts. I want to do this by giving out the fewest number of coupons. How do I distribute the coupons (who gets coupons, and how many does each person get)?
To get rid of the Nash equilibrium in which no one revolts, someone must have threshold zero. The "cheapest" way to make someone have threshold zero is to give 4 coupons to a threshold 4 person. To make another person revolt for sure, we need to make her have threshold 1; so we can give 3 coupons to a threshold 4 person. Similarly, we make another threshold 4 person have threshold 2 by giving him 2 coupons and make another threshold 4 person have threshold 3 by giving him 1 coupon. Finally, we give a threshold 6 person 1 coupon and make him a threshold 5 person. So in other words, we started with thresholds $4, 4, 4, 4, 4, 6, 6, 6, 6, 6$ and we end up with thresholds $0, 1, 2, 3, 4, 5, 6, 6, 6, 6$. We give out $4+3+2+1+0+1 = 11$ coupons.

c. Now say that you can give tickets which raise the cost of revolting and hence raise a person's threshold. For example, if I give 3 tickets to a person with threshold 4, she now has threshold 7. If I give 2 tickets to a person with threshold 6, he now has threshold 8. By giving out tickets, I can change the game so that the only Nash equilibrium is one in which no one revolts. I want to do this by giving out the fewest number of tickets. How do I distribute the tickets (who gets tickets, and how many does each person get)?
To eliminate the equilibrium in which everyone revolts, there must be at least one person with threshold 10. Given that this person never revolts, you can make another person

never revolt by making them have threshold 9, and so forth. So we start with thresholds $4, 4, 4, 4, 4, 6, 6, 6, 6, 6$ and end up with thresholds $4, 4, 4, 4, 5, 6, 7, 8, 9, 10$. We give out $1+0+1+2+3+4 = 11$ coupons.

59. Say that there are four people: Alicia, Betsy, Carlos, and Davis. Each can choose whether to wear platform sandals or not. Alicia is very fashion-forward and will wear them even if no one else wears them; in fact, if more than one other person wears them, she won't wear them anymore because she hates being part of a crowd. Betsy also likes fashion but is not as cutting-edge: she will wear them if at least one other person wears them, but like Alicia, hates being "part of the crowd" and will not wear them if more than two other people wear them. Carlos thinks of himself as hip, but is kind of slow on the uptake and will wear them if at least two others wear them. Still, Carlos has at least some fashion pride and will not wear them if everyone else wears them. Finally, Davis gets his fashion tips from the JC Penney catalog and will wear them if everyone else wears them.

a. Say that at the beginning, no one wears platform sandals (they have just hit the market). Show that at first the sales of platform sandals steadily grow, but eventually sales "cycle" between high and low in a never-ending "fashion cycle."
Starting from (n, n, n, n) (no one buys), first Alicia only wears platform sandals. Hence we get (p, n, n, n). Given this, Betsy jumps in and we get (p, p, n, n). Given this, Carlos jumps in and we get (p, p, p, n). Now Davis jumps in but Alicia drops out because they are becoming too popular; hence we get (n, p, p, p). Now Davis drops out and we get (n, p, p, n). Now Carlos drops out and we get (n, p, n, n). Now Alicia jumps back in and Betsy drops out, and we get (p, n, n, n). Then the cycle starts all over again.

b. Are there any pure strategy Nash equilibria of this game?
There are sixteen possibilities to consider (p, p, p, p), (p, n, n, n), and so forth. To avoid having to go through all sixteen, we first realize that the only way that Davis participates is if everyone else participates. It is easy to show that (p, p, p, p) is not a Nash equilibrium (Alicia wants to drop out). Hence we can assume that Davis never participates. Given this, Carlos participates only if Alicia and Betsy participate: (p, p, p, n). It is easy to see that this is not a Nash equilibrium (Alicia wants to drop out). Hence we can assume that Carlos does not participate. So there are only four possibilities: (p, p, n, n), (p, n, n, n), (n, p, n, n), and (n, n, n, n). It is easy to check that none of these is a Nash equilibrium. So there are no pure strategy Nash equilibria of this game.

60. Say that there are three men A, B, and C and three women X, Y, and Z. Each of these six people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman X best, woman Y next, and woman Z least. Man B prefers woman Y best, woman Z next, and woman X least. Man C prefers woman Y best, woman X next, and woman Z least. Woman X prefers man B best, man A next, and man C least. Woman Y prefers man A best, man B next, and man C least. Woman Z prefers man A best, man C next, and man B least.

a. Say that man A is matched with woman X, man B is matched with woman Y, and man C is matched with woman Z. Is this matching stable?

Yes, this match is stable. Men A and B get their first choice. Man C would like to call X or Y, but both X and Y like their current partners over C. Woman X would like to call B, but B is attached to his first choice. Women Y and Z would like to call A, but A is attached to his first choice.

b. Write down all possible matchings and determine which of them are stable and which are not stable.
(AX, BY, CZ) is stable, as mentioned above. (AX, BZ, CY) is not stable—B and Y would like to dump their current partners and join with each other. (AY, BX, CZ) is stable. (AY, BZ, CX) is not stable—A and X would like to get together. (AZ, BX, CY) is not stable—A and Y would like to get together. (AZ, BY, CX) is not stable—A and Y would like to get together.

c. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?
All of the men prefer the stable matching (AX, BY, CZ) at least as much as the other stable matching, (AY, BX, CZ). All of the women prefer the stable matching (AY, BX, CZ) at least as much as the other stable matching, (AX, BY, CZ).

61. Say that there are four men, A, B, C, and D and four women W, X, Y, and Z. Each of these eight people is considering matching up with a member of the opposite sex. Each person would rather be matched with someone than not have a partner at all. Man A prefers woman W best, X next, Y, next, and Z least. Man B's preference ordering (from best to worst) is Y, X, W, Z. Man C's preference ordering is Y, Z, X, W. Man D's ordering is Y, Z, X, W. Woman W's preference ordering (from best to worst) is D, C, B, A. Woman X's preference ordering is C, B, A, D. Woman Y's ordering is D, A, C, B. Woman Z's ordering is B, C, D, A.

a. Among the set of stable matchings, which matching is most preferred by the men? Among the set of stable matchings, which matching is most preferred by the women?
To find the stable matching most preferred by the men, we use the "men ask" algorithm. First everyone asks their first choice: A asks W, B asks Y, C asks Y, and D asks Y. Woman Y likes D best out of all the men who ask her, and thus B and C are rejected. So B and C ask their second choices: B asks X and C asks Z. As it stands, A is matched with W, B with X, C with Z, and D with Y, and there are no women who are asked by more than one man. Hence (AW, BX, CZ, DY) is a stable matching.
Now let's use the "women ask" algorithm. First W asks D, X asks C, Y asks D, and Z asks B. Man D is asked by both W and Y, and so D rejects W and pairs up with Y. So W then asks C. Now C is asked by two people (W and X), and C chooses to pair up with X. So the current matches are XC, YD, and ZB.
Since W is rejected again, W asks B. Now B is asked by W and Z, and B chooses to pair up with W. So the current matches are now WB, XC, and YD. Since Z is rejected, she asks C. Now C is asked by X and Z, and C chooses Z. Now the current matches are WB, YD, and ZC. Since X is rejected, she asks B. Now B is asked by both W and X, and B chooses X. Now the current matches are XB, YD, and ZC. Now W asks A. Since no man is now asked by more than one woman, the matching (WA, XB, YD, ZC) is stable.

Note that this matching is the same as what we obtained in the "men ask" algorithm. Since the best matching for the women is the same as the best matching for the men, in this case this matching is the only stable matching.

62. Say that Person 1 and Person 2 each decide whether to go to the auto racing match or the ballet. If they go to different places, both get utility zero. If they both go to the auto racing match, person 1 gets a utility of \$4 and person 2 gets a utility of \$1. If they both go to the ballet, person 1 gets a utility of \$1 and person 2 gets a utility of \$4. This is a "battle of the sexes" game. Now say that before playing this battle of the sexes game, person 1 can either burn \$2 or not burn it. Person 2 can see whether person 1 burns the money or not.

a. Represent this as a strategic form game (each person has four strategies) and find all (pure strategy) Nash equilibria.

b. Show that by iteratively eliminating both strongly and weakly dominated strategies, one can predict that person 1 does not burn the money and that both go to the auto racing match.

63. Say that you and I are playing a game in which we both simultaneously yell out either a or b. If we say the same letter, then you get \$1 and I get nothing. If we say different letters, than I get \$1 and you get nothing.

a. Model this as a game and find the Nash equilibrium.

b. Now say that I just had my wisdom teeth pulled out and my mouth is still quite numb. So it's more difficult for me to say b: when I say b, I have to pay the cost r, where $r > 0$. Now we play the same game above (my payoffs have changed but yours have not). Model this as a game and find the Nash equilibrium.

c. How does my expected utility in the Nash equilibrium change as r changes? What happens when $r = 1$? When $r > 1$?

64. Say that person 1 and person 2 are playing a drinking game which goes like this. There are m beers in the refrigerator. Person 1 goes first by drinking either 1 or 2 beers. Then person 2 can drink either 1 or 2 beers. Then person 1 can drink either 1 or 2 beers, and so forth. In other words, when it is a person's turn to drink, she can drink either 1 or 2 beers. Whoever drinks the last beer wins the game. Winning the game yields a payoff of 1 and losing yields a payoff of 0. However, there is an additional feature to the game: there is a "magic number" x (which is greater than 0 and less than m). If after your turn, there are exactly x beers left, then you lose the game and also have to go out and buy more beer; this has a payoff of -3 for the loser and a payoff of 1 for the winner.

a. Say that $m = 6$ and $x = 4$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

b. Say that $m = 6$ and $x = 3$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

c. Now let m and x be any number. Find a subgame perfect Nash equilibrium. For what values of m and x can person 1 guarantee a win? For what values of m and x can person 2 guarantee a win?

65. [from Spring 2002 final] There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability p and B with probability $1-p$. Find p.

66. [from Spring 2003 final] Say that we have 10 people. Each person is thinking about whether or not to join a revolt or not. Each person has a threshold: five people have threshold 4 and five people have threshold 6.

a. Find all pure strategy Nash equilibria of this game.

b. Say that you have some discount coupons which lower the cost of revolting and hence lower a person's threshold. For example, if I give 3 coupons to a person with threshold 4, she now has threshold 1. If I give 1 coupon to a person with threshold 6, he now has threshold 5. By giving out coupons, I can change the game so that the only Nash equilibrium is one in which everyone revolts. I want to do this by giving out the fewest number of coupons. How do I distribute the coupons (who gets coupons, and how many does each person get)?

c. Now say that you can give tickets which raise the cost of revolting and hence raise a person's threshold. For example, if I give 3 tickets to a person with threshold 4, she now has threshold 7. If I give 2 tickets to a person with threshold 6, he now has threshold 8. By giving out tickets, I can change the game so that the only Nash equilibrium is one in which no one revolts. I want to do this by giving out the fewest number of tickets. How do I distribute the tickets (who gets tickets, and how many does each person get)?

67. Say that we have three men, A, B, and C, and three women X, Y, and Z. Each person has preferences about which member of the opposite sex they would like to be matched with. For example, say woman X likes A the best, B second best, and C worst. Assume that there are no ties (i.e. it cannot be that woman Y likes A the best and B and C are tied for worst). Note that there are six possible matchings.

a. Is it possible for people's preferences to be such that there exists exactly one stable matching? If so, write down the preferences which make this possible. If not, explain why not.

b. Is it possible for people's preferences to be such that there exists exactly two stable matchings? If so, write down the preferences which make this possible. If not, explain why not.

c. Is it possible for people's preferences to be such that there exists exactly three stable matchings? If so, write down the preferences which make this possible. If not, explain why not.

d. Is it possible for people's preferences to be such that there exists exactly four stable matchings? If so, write down the preferences which make this possible. If not, explain why not.

68. Say that there are five people choosing among three candidates x, y, z. Persons 1 and 2's preference ordering from best to worst is x, z, y. Persons 3 and 4 have preference ordering y, z, x. Person 5 has preference ordering z, x, y.

a. Say that they make their decision using a runoff procedure: first everyone votes for their first choice, and the two alternatives which get the most votes go to a runoff. In the runoff, each person votes for one of these two candidates, and whoever gets the most votes in the runoff wins. Which candidate wins in this procedure?

b. Is the candidate who wins the runoff the Condorcet winner? Is there a Condorcet winner?

69. Say that there are three people choosing among five candidates s, t, w, x, y. Person 1's preference ordering from best to worst is w, x, t, y, s. Person 2's preference ordering is y, w, x, t, s. Person 3's preference ordering is s, x, t, y, w.

a. Say that people decide using majority rule according to some agenda. For example, one agenda might be to vote on w first; if w loses, then vote on x; if x loses, then vote on s; if s loses, then vote on t versus y. Can you find an agenda in which t is chosen?

b. What is the top cycle? Remember that the candidates in the top cycle are those which win given some suitably chosen agenda. Candidates not in the top cycle are those which are never chosen regardless of the agenda.

70. Say that the town of Mollusk Beach is deciding how many oil refineries to build. Forty percent of voters prefer no refineries, 36 percent prefer 1 refinery, and 24 percent prefer two refineries. Say that there are two candidates A and B, and each has to take a position on this issue. Given their positions, each voter will vote for the candidate whose position is closest to their own (if there are two candidates who are equally far away, assume that the vote is split equally among the two candidates). Each candidate wants to maximize the total number of votes she receives.

a. Say that candidates A and B choose their positions simultaneously. Which positions will they take?

b. Say now that candidate A can choose her position first and then candidate B chooses her position. Which positions will they take?

c. Now say that there are three candidates, A, B, and C. Say that candidate A chooses her position first, then candidate B, and then finally candidate C. Which positions will they take?