THEORY OF GAMES AND ECONOMIC BEHAVIOR

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PRINCETON

PRINCETON UNIVERSITY PRESS

PRINTED IN THE UNITED STATES OF AMERICA
Second Edition, 1947
Third Edition, 1953

LCC: 53-4426
ISBN: 0-691-04183-0
ISBN: 0-691-00362-9 pbk.

FIRST PRINCETON PAPERBACK printing, 1980

PREFACE TO FIRST EDITION

This book contains an exposition and various applications of a mathematical theory of games. The theory has been developed by one of us since 1928 and is now published for the first time in its entirety. The applications are of two kinds: On the one hand to games in the proper sense, on the other hand to economic and sociological problems which, as we hope to show, are best approached from this direction.

The applications which we shall make to games serve at least as much to corroborate the theory as to investigate these games. The nature of this reciprocal relationship will become clear as the investigation proceeds. Our major interest is, of course, in the economic and sociological direction. Here we can approach only the simplest questions. However, these questions are of a fundamental character. Furthermore, our aim is primarily to show that there is a rigorous approach to these subjects, involving, as they do, questions of parallel or opposite interest, perfect or imperfect information, free rational decision or chance influences.

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PRINCETON, N. J.
January, 1943.

PREFACE TO SECOND EDITION

The second edition differs from the first in some minor respects only. We have carried out as complete an elimination of misprints as possible, and wish to thank several readers who have helped us in that respect. We have added an Appendix containing an axiomatic derivation of numerical utility. This subject was discussed in considerable detail, but in the main qualitatively, in Section 3. A publication of this proof in a periodical was promised in the first edition, but we found it more convenient to add it as an Appendix. Various Appendices on applications to the theory of location of industries and on questions of the four and five person games were also planned, but had to be abandoned because of the pressure of other work.

Since publication of the first edition several papers dealing with the subject matter of this book have appeared.

The attention of the mathematically interested reader may be drawn to the following: *A. Wald* developed a new theory of the foundations of statistical estimation which is closely related to, and draws on, the theory of choose heads less frequently, since the premium makes this choice plausible and therefore dangerous. The direct threat of extra loss by being matched on heads influences player 2 in the same way. This verbal argument has some plausibility but is certainly not stringent. Our formulae which yielded this result, however, were stringent.

18.4.3. (c) Matching Pennies, where matching on heads gives a double premium but failing to match on a choice (by player 1) of heads gives a triple penalty. Thus the matrix of Figure 27 is modified as follows:

	1	2
1	2	−3
2	−1	1

Figure 28*b*.

The diagonals are separated (1 and 2, are > than −1, −3), hence the good strategies are unique and mixed (cf. as before). The formulae used before give the value

$$v' = -\tfrac{1}{7},$$

and the good strategies

$$\vec{\xi} = \{\tfrac{2}{7}, \tfrac{5}{7}\}, \qquad \vec{\eta} = \{\tfrac{4}{7}, \tfrac{3}{7}\}.$$

We leave it to the reader to formulate a verbal interpretation of this result, in the same sense as before. The construction of other examples of this type is easy along the lines indicated.

18.4.4. (d) We saw in 18.1.2. that these variants of Matching Pennies are, in a way, the simplest forms of zero-sum two-person games. By this circumstance they acquire a certain general significance, which is further corroborated by the results of 18.2. and 18.3.: indeed we found there that this class of games exhibits in their simplest forms the conditions under which strictly and not-strictly determined cases alternate. As a further addendum in the same spirit we point out that the relatedness of these games to Matching Pennies stresses only one particular aspect. Other games which appear in an entirely different material garb may, in reality, well belong to this class. We shall give an example of this:

The game to be considered is an episode from the Adventures of Sherlock Holmes.[1,2]

[1] *Conan Doyle:* The Adventures of Sherlock Holmes, New York, 1938, pp. 550–551.

[2] The situation in question is of course again to be appraised as a paradigm of many possible conflicts in practical life. It was expounded as such by *O. Morgenstern:* Wirtschaftsprognose, Vienna, 1928, p. 98.

The author does not maintain, however, some pessimistic views expressed id. or in "Vollkommene Voraussicht und wirtschaftliches Gleichgewicht," Zeitschrift für Nationalökonomie, Vol. 6, 1934.

Accordingly our solution also answers doubts in the same vein expressed by *K. Menger:* Neuere Fortschritte in den exacten Wissenschaften, "Einige neuere Fortschritte in der exacten Behandlung Socialwissenschaftlicher Probleme," Vienna, 1936, pp. 117 and 131.

Sherlock Holmes desires to proceed from London to Dover and hence to the Continent in order to escape from Professor Moriarty who pursues him. Having boarded the train he observes, as the train pulls out, the appearance of Professor Moriarty on the platform. Sherlock Holmes takes it for granted—and in this he is assumed to be fully justified—that his adversary, who has seen him, might secure a special train and overtake him. Sherlock Holmes is faced with the alternative of going to Dover or of leaving the train at Canterbury, the only intermediate station. His adversary—whose intelligence is assumed to be fully adequate to visualize these possibilities—has the same choice. Both opponents must choose the place of their detrainment in ignorance of the other's corresponding decision. If, as a result of these measures, they should find themselves, *in fine*, on the same platform, Sherlock Holmes may with certainty expect to be killed by Moriarty. If Sherlock Holmes reaches Dover unharmed he can make good his escape.

What are the good strategies, particularly for Sherlock Holmes? This game has obviously a certain similarity to Matching Pennies, Professor Moriarty being the one who desires to match. Let him therefore be player 1, and Sherlock Holmes be player 2. Denote the choice to proceed to Dover by 1 and the choice to quit at the intermediate station by 2. (This applies to both τ_1 and τ_2.)

Let us now consider the $\mathcal{H}$ matrix of Figure 27. The fields (1, 1) and (2, 2) correspond to Professor Moriarty catching Sherlock Holmes, which it is reasonable to describe by a very high value of the corresponding matrix element,—say 100. The field (2, 1) signifies that Sherlock Holmes successfully escaped to Dover, while Moriarty stopped at Canterbury. This is Moriarty's defeat as far as the present action is concerned, and should be described by a big negative value of the matrix element—in the order of magnitude but smaller than the positive value mentioned above—say, −50. The field (1, 2) signifies that Sherlock Holmes escapes Moriarty at the intermediate station, but fails to reach the Continent. This is best viewed as a tie, and assigned the matrix element 0.

The $\mathcal{H}$ matrix is given by Figure 29:

	1	2
1	100	0
2	−50	100

Figure 29.

As in (b), (c) above, the diagonals are separated (100 is > than 0, −50); hence the good strategies are again unique and mixed. The formulae used before give the value (for Moriarty)

$$v' = 40$$