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POWER DISTRIBUTION IN THE ELECTORAL BODY WITH AN APPLICATION TO THE RUSSIAN PARLIAMENT

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Power distribution in the electoral body
with an application to the Russian Parliament

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This paper presents several new approaches to evaluate power distribution in an electoral body. We define the index of consistency of two groups’ positions (briefly, the consistency index) which is used to separate possible coalitions in the Parliament. This allows to analyze power distribution within restricted coalition formations. Then we provide several new power indices for the case in which the intensity of factions to coalesce is taken into account. Our analysis of the power distribution model extends the one proposed by Shapley-Owen. A new consistency index is given allowing to construct such an extension. We illustrate these approaches via the analysis of power distribution among factions in the Russian Parliament (Duma) from 1993 to 2005.

1. Introduction

In legislative bodies decisions are made by voting. The decision is accepted if the number of votes in favour exceeds a certain quota, which is defined by the voting procedure. For instance, in the Russian Parliament the quota for federal laws is equal to 226 votes (50% +1vote). If a parliament consists of three or more parties, there is a possibility that none of them possesses the votes which exceed a quota, so to make a decision the parties should coalesce. The coalitions which guarantee the necessary number of votes are of particular importance.

Consider two examples.

Example 1. Let three parties *A*, *B* and *C*, with 33, 33 and 33 seats respectively, be presented in a parliament, where the voting rule is the simple majority, i.e., 50 votes in favour.

Then the winning coalitions are *A+B*, *A+C*, *B+C*, *A+B+C* and *A*, *B* and *C* are pivotal[1] in all but the grand coalition. If one measures the power of a party as a number of winning coalitions in which it is pivotal, then all parties in this example have the same power.

Let us now change the distribution of seats. Assume that the parties *A*, *B* and *C* have 48, 48 and 3 seats respectively, and the voting rule is the same, i.e., 50 votes in favour. Then the winning coalitions are the same and the parties are pivotal in the same winning coalitions. Thus they have the same distribution of power.

The distribution of power is studied using power indices. Below we will use the Banzhaf index [6] which is evaluated as

$$b_i = \frac{b_i}{\sum_j b_j},$$

b_i is the number of winning coalitions in which agent i is pivotal. This form of Banzhaf index is called the normalized one. Note that a non-normalized index of this kind was first introduced in [19].

Example 2. Let us now consider the parliament with 100 seats in which three parties A, B and C are represented, with the distributions of seats among them being 50, 49, 1. Let the decision making rule be the simple majority, i.e., 51 votes in favour. Then the winning coalitions are A+B, B+C, A+B+C. The party A is pivotal in all three coalitions, B is pivotal only in the coalition A+B, and C is pivotal in A+C. Then

$$b(A) = \frac{3}{3+1+1} = \frac{3}{5},$$

$$b(B) = b(C) = \frac{1}{3+1+1} = \frac{1}{5}.$$

Assume now that parties *A* and *B* never coalesce together, i.e. coalition *A+B* is impossible. However, let us assume that the coalition *A+B+C* can be implemented, i.e. in the presence of a \`moderator' *C*, parties *A* and *B* can coalesce. Then the winning coalitions are *A+C* and *A+B+C*, and *A* is pivotal in both coalitions while *C* is in one; *B* is pivotal in none of the winning coalitions. In this case $b(A) = 2/3$, $b(B) = 0$ and $b(C) = 1/3$, i.e. although *B* has almost half of the seats in the parliament, its power is equal to 0.

If *A* and *B* never coalesce even in the presence of a moderator *C*, then the only winning coalition is *A+C*, in which both parties are pivotal. Then, $b(A) = b(C) = 1/2$.

Such situations are found in real political life. For instance, the Russian Communist Party in the second Duma (1997-2000) had about 35% of seats; however, its power during that period was always almost equal to 0 [1].

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[1] An agent *i* is called pivotal for a coalition if the latter becomes a loosing coalition when agent *i* leaves it.

We study the problem of power distribution over time in groups and factions of the Russian Parliament (Duma) using the Banzhaf index. The analysis is made under two main assumptions about coalition formation. First, we consider the case when all coalitions are admissible, and after we study several scenarios of coalition formation. To evaluate the possibility of two groups to coalesce we use different versions of the index of consistency of two groups' positions. The index is based on the similarity of voters in one act of voting or on the closeness of their positions on the political map. We study several qualitative scenarios of coalition formation, one of which is considered as real.

This paper gives a short description of a study on distribution of power in the Russian Parliament between 1994-2005. The complete study can be found in [1-3,5], on the sites www.hse.ru, www.ipu.ru/rcpp and www.indem.ru. It contains a complete analysis of the power distribution in each electoral period (1994-1995, 1996-1999, 2000-2003, 2004-2005) with two decision rules (simple majority for federal laws and qualified majority - 300 out of 450 - for constitutional laws) using two power indices – the Banzhaf and Shapley-Shubik indices [21].

The structure of the paper is as follows. Section 2 presents main notions and data used. Section 3 provides an analysis of power distribution in the Russian Parliament using the standard Banzhaf index. In Section 4 we define the index of consistency of two groups' positions (briefly, the consistency index) and analyze power distribution within a restricted coalition formation. Section 5 provides several new power indices for the case in which the intensity of factions to coalesce is taken into account. We consider an analysis of power distribution in the Russian Parliament for two out of many intensity functions introduced in [1,2]. Section 6 contains the analysis of the power distribution model proposed by Shapley-Owen in a spatial context when the agents have ideal (bliss) points on a political map. We define here a new consistency index and use it to propose an extension of the Shapley-Owen index. Then we give an analysis of power distribution in the Russian Parliament using this extension. Section 7 concludes.

2. Main notions and data used

The set of agents (parties, factions) is denoted as N, $N=\{1,\dots,n\}$, $n>1$. A coalition w is a subset of N, $w \subseteq N$. We consider the situation in which the decision of a body is made by voting procedure; agents who do not vote `yes' vote against it, i.e. abstention is not allowed.

Each agent has a predefined number of votes, $v_i > 0$, $i=1,\dots,n$. It is assumed that a quota q is predetermined as well and that the voting with quota is used as a decision making rule, i.e. the decision is made if the number of votes for it is not less than q,

$$\sum_i v_i \geq q .$$

The model describes a voting by simple and qualified majority, voting with veto (as in the Security Council of UN), etc.

A coalition w is called winning if the sum of votes in the coalition is not less than q. An agent i is called pivotal in the coalition w if the coalition $w \setminus \{i\}$ is a loosing one.

For such a voting rule the set of all winning coalitions W possesses the following three properties:

$$f \notin W,\ N \in W,\ w \in W,\ w' \supseteq w \Rightarrow w' \in W .$$

Sometimes, one additional condition is applied as well

$$w \in W \Rightarrow N \setminus w \notin W,$$

implying $q \geq \lceil n/2 \rceil$, where $\lceil x \rceil$ is the smallest integer greater or equal to x.

The system of winning coalitions constitutes an n-person simple game in the form of characteristic function, i.e. every coalition $S \subseteq N$ gets a payoff equal to 0 or 1 [21].

The data. Between 1993-2007 the Russian Parliament consisted of 450 members, half of them being elected by majority voting and the other half by party lists[2]. Factions had been created by electoral blocks which passed by proportional representation scheme. Moreover, there was a possibility to create MP groups with no less than 35 members (until 2004). Decision making rules are simple majority (226 votes) for federal laws and 2/3 (300) votes for constitutional laws.

We have considered the structure of factions and groups on 16th of each month for each of the three parliaments separately (1994-1995, 1996-1999, 2000-2003) and for part of the 4th Parliament (2004-2005).

Using this structure we have calculated the Banzhaf index for federal laws. These evaluations were first made under the assumption that all coalitions are equally feasible, and then after excluding unfeasible coalitions. As source of data we have used that of the foundation INDEM (http://www.indem.ru/indemstat/index.htm)

3. Power distribution in the Russian parliament without restrictions on coalition formation

In the case considered in this Section the changes in power distribution are observed only due to the transfers of MPs from one group or faction to the other. Moreover, essential changes will be observed at the moment of huge transfers of MPs which are usually connected with the formation, sometimes unsuccessful, of new factions or groups.

We have used the following scheme to distribute independent MPs to factions and groups: we distribute them to those factions to which they will transfer afterwards or to which they had belonged before. If none of these situations holds, we studied their political interests and attributed each MP to the group with the closest interests.

In fact, we also evaluated the power index for independent MPs separately. The difference between the two approaches led to a difference in power indices of less than 1%.

Let us now discuss the results. In all three parliaments the following three parties were represented

- Agrarians (Agrarian Group of Russia, APG)
- Communists (Communist Party of the Russian Federation, CPRF)
- Liberal-Democrats (Liberal-Democratic Party of Russia, LDPR)
- Yabloko.

Additionally, in the second and third parliaments the group “Regions of Russia” was represented, and in the third Duma pro-presidential parties Edinstvo and OVR and liberal party SPS were also represented.

The changes in power distribution are shown in Fig.1. Communists as well as Yabloko had the maximal of their power in the second Duma; however, the power of Communists had been decreasing from the beginning of the second parliament through the third one. Agrarians in general had a power of about 9.3%. Liberal-Democrats had been losing their power during all those years. In the first parliament it was one of the most powerful parties, while in the third parliament it was one of the weakest parties. The group Regions of Russia had almost stable power of about 10%.

[2] From 2007 the Russian Parliament is elected by party lists only.

The power of groups and the share of their votes for the first and third parliaments were consistent, i.e. for factions and groups their power values and share of seats were almost equal.

Another picture can be seen in the second parliament, in which there had been one strong faction (Communists) and 6 small groups. On average the power of Communists exceeded its share of votes by 26%, and had the maximum at the beginning of the second parliament when this difference was 30%. For Our Home – Russia this difference was on average 33%, Liberal-Democrats – 19%, for Yabloko –15%, etc. In other words, in the second Duma – compared to the first and third ones - the distribution of power did not correspond to the distribution of seats.

4. The consistency index

Now we will study the approach allowing to separate admissible coalitions from those which are not. The relation between two groups of MPs is naturally reflected by the results of voting. Groups with similar political positions having common political interests initiate consistent bills and support them in voting. On the contrary, groups with opposite interests vote in a different way. This point of view is supported by the observation of voting behavior in the Russian parliament.

Let q_1 and q_2 be the share of 'ay' votes in two groups of MPs.

The consistency index is calculated as

$$c(q_1, q_2) = 1 - \frac{|q_1 - q_2|}{\max(q_1, 1-q_1, q_2, 1-q_2)}.$$

In other words, if in two groups the share of 'ay' votes is equal, then these groups are considered as consistent. The groups are totally inconsistent (*c= -1*) if one group votes 'ay' while another group votes 'no'. The properties of the consistency index were studied in [7].

In our study we use the mean value of the consistency index taken from m monthly observations, i.e.

$$\bar{c} = \frac{1}{m}\sum_{i=1}^{m} c(q_{1i}, q_{2i}).$$

To evaluate this mean value we select votings on the basis of several criteria which express different information about voting and division among factions and MPs. We assume that the abstention of a MP usually indicates disagreement with the bill. In general, the selection of the result of voting is made in two stages. First, we select those results in which even with very few votes against a bill one can obtain the essential difference of votes for and against in at least two factions. For each voting result we calculate the difference between maximal and minimal over factions' share of "ay" votes, and then choose those results for which this measure exceeds some predetermined threshold. Then the 'non-important' votings are excluded from the list of results, for instance those for which the bill is supported by no less than 30 votes or by at least 320 votes "ay". Finally, we did not take into consideration the results in which the difference in voting among factions is caused by some technical reasons, and when furthermore such bills are voted anew.

The analysis of the parliament when not all coalitions are feasible. The assumption that all coalitions are feasible is too strong. For instance, in the First Russian Parliament the coalition between Choice of Russia (main pro-presidential party) and Communists was hardly possible. Obviously, the real power distribution was not stable in all three parliaments in the light of many changes that had been happening during all those years. At that time many bills dealing with the most important changes in the country, from constitutional reforms to the reforms of natural monopolists, were approved.

To construct a power distribution more adequate to the real situation, it is necessary to measure the possibility of coalition formation depending on the relations among groups of MPs. We constructed the model of coalition formation depending on a threshold value of the consistency index.

At the beginning the impossible coalitions were excluded by the introduction of different threshold values of the consistency index. The consistency index introduced above had been calculated for all pairs of groups and factions in the parliament from 1994 to 2005 for all results of voting described above. According to this approach, a coalition is considered to be impossible if the value of the consistency index for two groups in the coalition is less than the threshold value of that index.

It is assumed that under a certain threshold level the evaluated power index should be close to its real value. We consider three values of threshold for each consistency index and, thus, three different distributions of power index. The three threshold values are[3] $? \geq 0.4$, $? \geq 0.5$, $? \geq 0.6$.

The choice of the threshold 0.5 is obvious and does not need any additional explanation. The choice of the other levels needs some explanation. The evaluation of the consistency index for explicit ideological contestants shows that in this case the value of ? does not exceed 0.4. The value of the consistency index between 0.5 and 0.7 corresponds to the relations from potential allies to full allies. The threshold value 0.6 gives from one point of view a minimal level of allies relations and, from the other point of view, preserves enough possibilities for coalition formation.

So the key question is which value of the consistency index generates power distribution reflecting real power distribution in the Russian Parliament? The answer to this question has been given on the basis of a scenario approach applying to coalition formation mechanisms.

To construct scenarios, a scale was suggested for the evaluation of relations among groups and factions in the parliament. This scale includes four grades: explicit “contestants”, potential contestants, potential allies, explicit allies. Using these grades three scenarios were constructed:

- “mild” scenario (coalitions with explicit contestants are excluded);
- “average” scenario (coalitions with explicit and potential contestants are excluded);
- “rigid” scenario (only coalitions with the closest allies are allowed).

The scenario in which all coalitions are admissible can be called null-scenario.

Mild scenario is by definition a real one. Indeed, the strategy that allows potential contestants to be included in a coalition seems to be in some sense optimal. It leaves enough freedom for coalition formation but excludes uncompensated losses a party could meet if it coalesced with explicit contestants, which cannot be forgiven from the point of view of the electorate. One may expect that experienced politicians managing political factions and groups in the Russian Parliament follow this optimal strategy.

The average scenario is interesting in that it allows to evaluate the abilities of the participants in the political process. For heavy players which fill the extreme positions it is an ability to attract the majority of voters; for the players at the center of the political field it is an ability to participate in winning coalitions.

In the rigid scenario, situations in which coalitions are formed only with the closest allies are of particular importance.

The change over time in the consistency index for pair of party factions in the third Duma is given in Fig.2.

As can be seen, Communists and Agrarians are the closest allies, their consistency index is about 0.85. On the contrary, the relation between Communists and Edinstvo is worsening over time, achieving a level of about 0.1. It is important to note that the minimum of the index for this pair is seen in July 2001, the moment when the most crucial decisions were made.

[3] In [3] the analysis for threshold values less than 0.4 is made as well.

The dynamics of the consistency index for Edinstvo and OVR reflect the process of organization of the largest party in the parliament - Edinaya Rossiya. After January 2002 both parties are the closest allies, their consistency index is greater than 0.8.

We show the dynamics of power indices for large parties in Figs. 4-6 below, for the scenarios c=0.4, 0.5, and 0.6, respectively. In Figs. 4-5 the standard Banzhaf index values are given as well.

The share of votes for the faction of Communists was on average 18% while their value of the Banzhaf index was much smaller and from July 2001 it did not exceed 3%. The factions Edinstvo and Narodny Deputat had a power greater than their share of votes.

In general, the following conclusions can be derived from the results obtained:

In scenario c=0.4, the centrist factions increase their power since they do not have explicit contestants. Thus, they possess the same possibilities as in the null-scenario (all coalitions are admissible). On the contrary, groups expressing extreme positions and having large contestants should expect serious losses.

In scenario c=0.5, maximal losses should be expected by those factions which coalesce with potential contestants. The groups which can create majority using explicit and potential allies preserve or even increase their power.

In scenario c=0.6, those groups which can create majority leaning only on explicit allies can preserve their power.

The distribution observed for scenario c=0.4 is the closest to real power distribution.

Let us discuss it in detail. As can be seen from Fig. 4, the share of votes of the CPRF faction in the third Duma is on average equal to 19% while the power index value for this faction equals only to 3%. A similar situation is seen for Agrarians, the closest ally of Communists. It exactly fits the hypothesis that factions with extreme views suffer more than centrist factions.

On the contrary, the power index values for the groups Narodny Deputat, OVR and Regions of Russia are higher than their shares of votes, which fits the hypothesis about the centrist factions.

The faction of Edinstvo also has more power than its share of votes. It is due to the fact that a centrist majority had been formed around this very faction.

5. Intensity functions, ordinal and cardinal power indices

Here we introduce new indices based on an idea similar to the Banzhaf power index, taking however into account agents' preferences to coalesce.

In these indices we used the information about agents' preferences over other agents. These preferences are assumed to be linear orders. Since these preferences may not be symmetric, the desire of agent 1 to coalesce with agent 2 can be different from the desire of agent 2 to coalesce with agent 1. The indices take into account this asymmetry of preferences differently and are constructed on the following basis: the intensity of connection $f(i,w)$ of the agent with other members of w is defined. Then for an agent i the value c_i is evaluated as

$$c_i = \sum_w f(i,w),$$

i.e., the sum of intensities of connections of i over those coalitions w in which i is pivotal. Naturally, other functions can be considered instead of summation.

Then the power indices are constructed as

$$a(i) = \frac{c_i}{\sum_j c_j}.$$

The very idea of $a(i)$ is the same as for the Banzhaf index, with the difference that in the Banzhaf index we evaluate the number of coalitions in which i is pivotal, i.e. $f(i,w)$ in the definition of the Banzhaf index is equal to 1; on the contrary, in our case it can be less than 1.

The main question now is how to construct the intensity functions $f(i,w)$. Below we give two different forms of these functions.

Each agent i is assumed to have a linear order[4] P_i revealing her preferences over other agents in the sense that i prefers to coalesce with agent j rather than with agent k if P_i contains the pair (j, k). Obviously, P_i is defined on the Cartesian product $(N \setminus \{i\}) \times (N \setminus \{i\})$.

Since P_i is a linear order, the rank p_{ij} of the agent j in P_i can be defined. We assume that $p_{ij} = |N|$-1 for the most preferable agent j in P_i.

The value p_{ij} shows how many agents less preferable than j are in P_i. For instance, if $N = \{A,B,C,D\}$ and $P_A : B \succ C \succ D$, then $P_{AB} = 3$, $P_{AC} = 2$ and $P_{AD} = 1$.

Using these ranks, one can construct different intensity functions.

A second way of constructing $f(i,w)$ is based on the idea that the values p_{ij} of connection of i with j are somehow predetermined. In general, it is not assumed that $p_{ij} = p_{ji}$. Then the intensity function can be constructed as above.

Below we give three different ways to construct $f(i,w)$ in an ordinal case and only one way of constructing the cardinal function $f(i,w)$. Other forms of intensity functions can be found in [1].

Ordinal indices. For each coalition w and each agent i now construct an intensity $f(i,w)$ of connections in this coalition. In other words, f is a function which maps $N \times W$ $(=2^N \setminus \{\text{Ø}\})$ into R^1, $f : N \times W \to R^1$. This very value $f(i,w)$ is evaluated using the ranks of members of the coalition. Three different ways to evaluate f using different information about agents' preferences are provided:

a) Intensity of i's preferences. In this form only the preferences of i's agent over other agents are evaluated, i.e.

$$f^+(i,w) = \sum_{j \in w} \frac{p_{ij}}{|w|}$$

b) Intensity of preferences for i. In this case consider the sum of ranks of i given by other members of coalition w

$$f^-(i,w) = \sum_{j \in w} \frac{p_{ji}}{|w|}$$

c) Average intensity with respect to i's agent

$$f(i,w) = \frac{f^+(i,w) + f^-(i,w)}{2}$$

Let's now consider several examples.

Example 3. Let n=3, N={A, B, C}, $n(A) = 33$, $n(B) = n(C) = 33$, q=50. Consider the two preference profiles given in Tables 1 and 2.

[4] i.e. irreflexive, transitive and connected binary relation. We often denote it as $\succ$.

P_A	P_B	P_C
C	C	A
B	A	B

Table 1. First preference profile

P_A	P_B	P_C
B	C	A
C	A	B

Table 2. Second preference profile

For both preference profiles there are three winning coalitions in which agents are pivotal. These coalitions are *A+B*, *A+C* and *B+C*.

Let us calculate the functions *f* as above for each agent in each winning coalition. The preferences from Tables 1 and 2 can be re-written in the matrix form as

$$\|p_{ij}\| = \begin{matrix} & \mathrm{A} & \mathrm{B} & C \\ \mathrm{A} & 0 & 1 & 2 \\ \mathrm{B} & 1 & 0 & 2 \\ \mathrm{C} & 2 & 1 & 0 \end{matrix}, \quad \|p'_{ij}\| = \begin{matrix} & \mathrm{A} & \mathrm{B} & C \\ \mathrm{A} & 0 & 2 & 1 \\ \mathrm{B} & 1 & 0 & 2 \\ \mathrm{C} & 2 & 1 & 0 \end{matrix}$$

Now, for the profile given in Table 1, we can calculate the values of intensities a)-c) obtained by each agent *i* in each winning coalition w. Using these intensity functions we can now define the corresponding power indices $a(i)$. Let i be a pivotal agent in a winning coalition w. Denote by c_i the number equal to the value of the intensity function for a given coalition w and agent *i*. Then the power index is defined as follows

$$a(i) = \frac{\sum\limits_{\substack{w, \\ i \text{ is pivotal in } w}} c_i}{\sum\limits_{j \in N} \sum\limits_{\substack{w, \\ j \text{ is pivotal in } w}} c_j}$$

The indices $a(i)$ will be denoted by $a_1(i), \ldots, a_3(i)$.

Let us now evaluate the values $a_1(\cdot), a_2(\cdot)$ for all agents for the preference profile from Table 1.

The agent *A* (as well as agents *B* and *C*) is pivotal in two coalitions; the sum of the values $f^+(i,w)$ for each *i* is equal to 3/2. Then

$$a_1(A) = \frac{3/2}{3/2+3/2+3/2} = \frac{1}{3} = a_1(B) = a_1(C).$$

The value $a_2(\cdot)$ is evaluated differently. The sum of values $f^-(i,w)$ from Table 3 for all i and w is equal to 9/2. However, for A $\sum_w f(A,w) = 3/2$, $\sum_w f(B,w) = 1$ and $\sum_w f(C,w) = 2$. Then $a_2(A) = \frac{3}{9} = \frac{1}{3}$; $a_2(B) = \frac{2}{9}$ and $a_2(C) = \frac{4}{9}$.

The values of the indices $a_1 - a_3$ for both preference profiles are given in Table 3 as well as the values of the Banzhaf index b

	First profile (Table 1)			Second profile (Table 2)		
	A	B	C	A	B	C
a_1	1/3	1/3	1/3	1/3	1/3	1/3
a_2	1/3	2/9	4/9	1/3	1/3	1/3
a_3	1/3	5/18	7/18	1/3	1/3	1/3
b	1/3	1/3	1/3	1/3	1/3	1/3

Table 3. Power indices values

Let's now consider another example.

Example 4. Consider the case where 3 parties A, B and C have 50, 49 and 1 seats respectively. Assume that the simple majority is the decision making rule, i.e. 51 votes. Then the winning coalitions are A+B, A+C and A+B+C. Note that A is pivotal in all three coalitions, B and C are pivotal in one coalition each. Then $b(A) = 3/5$, $b(B) = b(C) = 1/5$.

Now consider the case with the preferences of agents given below: $P_A : C \succ B$; $P_B : C \succ A$ and $P_C : A \succ B$.

Then the values of a_1 and a_2 (constructed by $f^+(i,w)$ and $f^-(i,w)$) are as follows

$$a_1(A) = 5/12, \quad a_1(B) = 1/4, \quad a_1(C) = 1/3,$$

$$a_2(A) = 5/12, \quad a_2(B) = 7/36, \quad a_2(C) = 7/18.$$

Consider another preference profile: $P_A : C \succ B$, $P_B : C \succ B$; $P_C : B \succ A$, i.e. only agent C changes her preferences. Then one can easily evaluate $a_1(A) = 5/11$, $a_1(B) = 3/11$, $a_1(C) = 3/11$; $a_2(A) = 10/33$, $a_2(B) = 3/11$, $a_2(C) = 14/33$.

In the second type of power index the information about the intensity of preferences is taken into account as well, i.e. we extend the former type of power index to cardinal information about agents' preferences.

Cardinal indices. Let us now assume that the desire of party i to coalesce with party j is given as real number p_{ij}, $\sum_j p_{ij} = 1$, $i, j = 1, \ldots, n$. In general, it is not assumed that $p_{ij} = p_{ji}$.

We can call the value p_{ij} as an intensity of connection of i with j. It may be, for instance, interpreted as a probability for i to form a coalition with j.

As in the previous case, we now define several intensity functions:

a) average intensity of i in connection with other members of coalition w

$$f^+(i,w) = \frac{\sum_{j \in w} p_{ij}}{|w|};$$

b) average intensity of connection of other members of coalition w with i

$$f^-(i,w) = \frac{\sum_{j \in w} p_{ji}}{|w|};$$

c) average intensity for i

$$f(i,w) = \frac{1}{2}(f^+(i,w) + f^-(i,w)).$$

Using the consistency index defined above as the cardinal intensity function we can construct the power distribution for the Russian Parliament. The value of the index a_1 for the third Duma is given in Fig. 6. One can see that the graphs are smoother than in the previous case when the 'threshold' model of coalition formation was used. In fact, such a model can be used here as well.

In [1,2] an axiomatic construction of the first cardinal intensity function is given. Other intensity functions can be constructed in an analogous way.

6. Extended Shapley-Owen index and power distribution in the Russian Parliament

In the Shapley-Owen index the power of an agent depends not only on the voting rule of decision making, but on the position of agents in the spatial context, or in the political space [20], i.e. on ideology as well. This index (the Shapley-Owen value, for short SOV [17]) gives a special role to an ideology in coalition formation, i.e. only the ideologically close agents will coalesce.

Let each player has her own ideal point $P_i \in \Re^m$ in m-dimensional Euclidean space. The ideal points reflect the preferential political outcomes of each player. Let $\Psi \subseteq \Re^m$ be a set of all the outcomes of voting. Each outcome is a vector $x \in \Psi$.

Assume that function $u_i(x)$ such as $u_i : \Psi \to \Re^m$ exists for each player and measures the level of the player's attitude to the outcome x. Using the values of this function, a strict order $\succ$ can be defined on the set ***N***, thus, $j \succ i$, if $u_j(x) - u_i(x) \geq 0$. This relation indicates that player *j* likes the outcome *x* more than *i*.

Define $Y_{ij} = u_i(x) - u_j(x)$. If $Y_{ij} \leq 0$, then *j* joins a coalition of players supporting the outcome *x* more willingly than *i*. Owen and Shapley introduced the player's power index in the spatial context. They considered unit vectors $x \in \Re^m$ on the unit-sphere H^{m-1}, $\forall x \in \Psi$ $\langle x, x \rangle = 1$. Each vector defines a direction in space. It was proposed that the function values are determined by the inner product $u_i(x) = \langle x, P_i \rangle$. Then each unit vector *x* randomly chosen from uniform distribution induces an order relation $\succ$ as $i \succ_U j \Leftrightarrow \langle U, P_i \rangle \geq \langle U, P_j \rangle$.

So the power index for player *i* can be written as ratio

$$SOV_i = \frac{q_i}{n!},$$

where q_i is the number of orderings, for which player *i* is pivotal, *n!* is the total number of all possible orderings. Effective computational scheme for the evaluation of SOV is given in [11].

We now extend the Shapley-Owen index using the notion of the consistency of the players' positions.

Let d_{ij} be the Euclidean distance between ideal points of players *i* and *j* in a normalized two-dimensional political space. Consider an index of consistency of two players

$$k_{ij} = \frac{1}{\sqrt{2}}\left(\frac{1+\sqrt{2}}{1+d_{ij}} - 1\right) \qquad (1)$$

In the Shapley-Owen model a player is pivotal if she occupies the median position in the linear order obtained on each step, i.e. the pivotal player splits the set of players *N* to two disjoint sets, where one of them is a winning coalition.

Denote as *S* the coalition located on the left of the pivotal player in linear order obtained on a certain step, and as *T* the one on the right of the pivotal player (see Fig. 7). The pivotal player can make each of these coalitions winning after joining them.

Then we introduce the weight of player *i*, which is pivotal,

$$w_{im} = \frac{1}{l}\sum_j k_{ij}, \; i \neq j, \qquad (2)$$

It is computed as the sum of indices of consistency for each step *m=1,..,t*, i.e. for each increment of angle of line rotation about origin of the considered political space. Summation in (2) includes that parties *j* enter this coalition, which *i* can make winning, and *l* is the number of players of this coalition.

So two numbers of the weight value are computed, both by the sum of the index of consistency of pivotal position and the players of coalition *S* positions and by the sum of the index of consistency of pivotal position and the players of coalition *T* positions. The largest weight is chosen, meaning that the pivotal player enters the coalition with players being more consistent with him.

Then the average value of *i*'s weight is computed, here *t* is the number of steps:

$$v_i(t) = \frac{\sum_{m=1}^{t} w_{im}}{t} \quad (3)$$

The power index of player *i* is determined as

$$PI_1(i) = \frac{v_i(t) \cdot l_i}{\sum_{j=1}^{n} v_j(t) \cdot l_j}, \quad (4)$$

where $l_i = n_i \Big/ \sum_j n_j$ is the share of votes, and n_i is the number of votes of party *i*.

Two more indices can be constructed based on this very idea: one based on the consistency of the players' positions without taking into account the share of votes of each agent and another one based on the consistency of the player's ideal point to the system of players 'center of mass' [5]

Let us now compute the power for the political parties in the III State Duma of the Russian Federation using the power index introduced above. The data about players' preferences covers the State Duma of the Russian Federation for each month from 2000 to 2003.

The issue space consists of two dimensions defined as "Liberal – State oriented" (horizontal axis) and "Antireform – Pro-reform oriented" (vertical axis). Each dimension is measured using a floating-point scale ranging from -1 to 1. The preferences of players are Euclidean. The decision rule is the simple majority rule. This political map (issue space) has been obtained using factor analysis of votings in that Duma [1].

Fig. 8-11 represent the average distribution of power PI_1 of all parties for the period under study. As we can see in Fig. 9, for instance, the party "Narodny deputat" was the leader at the beginning of 2000, its $\overline{PI_1} \approx 0.29$. But in 2001 the value of its power became lower, $\overline{PI_1} \approx 0.25$. In 2002 the average value of power index of "Narodny deputat" declined, $\overline{PI_1} \approx 0.16$. This effect can be explained by the fact that at the beginning of the third Duma this party 'started' as centrist, its motion pass occupied a considerable area [1]. But by 2002 it is noted that ideal points of this party migrated from the center to the top left corner of the political map and the area of motion path has decreased leading to the reduction of frequency of events in which this party was pivotal.

The party "Regions of Russia" in 2000 had $\overline{PI_1} \approx 0.16$, it was the second with respect to power distribution, and the average value of its power grew constantly during all the period under study. In 2003 its $\overline{PI_1} \approx 0.33$ and as we can see this value was twice as large by the end of 2003. This shows that the frequency of events in which this party was pivotal had increased. The political map of "Regions of Russia" motion paths shows that the party ideal points movement was active, and the ideal points area was considerably wide.

Agrarians were in the third place with $\overline{PI_1} \approx 0.14$, but the average value of the index $\overline{PI_1}$ strongly decreased by 2002 ($\overline{PI_1} \approx 0.058$), and by the end of the period it was almost the same ($\overline{PI_1} \approx 0.078$). Communists and OVR were the next in our rating with $\overline{PI_1} \approx 0.12$.

Communists had an almost constant average value of index during the whole period of 2000-2003.

The average value of the index $\overline{PI}_1$ of group OVR changed strongly: in 2001 $\overline{PI}_1 \approx 0.183$, in 2002 this value was 0.077. "Edinstvo" had strong changes in its power for all the period of 2000-2003. In 2000 the average value of its power was approximately 0.08, but by 2002 the power of this party had increased strongly, more than 50 percent, $\overline{PI}_1 \approx 0.189$. All of these four factions have strong party discipline.

Till then the OVR group unified with "Edinstvo" in December of 2001. Factions SPS, LDPR, "Yabloko" were tiny groups and they were at the end of the power distribution rating. The average values of their power were less than 0.05 and there were not any important changes in these values during all this period.

Thus, it may be concluded that both the greatest power values and strong power changes in time have been shown by those parties which change their political position permanently. It means that these groups did not have a fixed political position, they could maneuver in order to receive strategic advantages. Such power is called payoff-power (or P-power) [9]. Therefore, our power index measures the degree of player's ability to predict and adjust to outcome. This hypothesis is confirmed by political maps of ideal point motion paths, presented month by month for each year [1]. The ideal points of such groups migrate throughout the political space.

One of the findings obtained in the work is that if we construct a trajectory of faction positions on the political map, each point corresponding to the position of a faction in a chosen month, we can see to which extent the overall behavior of a party was volatile or stable. The graphs in Figs. 12-15 present these very trajectories for the third Duma. As one can see, Communists and Edinstvo have rather tight positions during all these years. On the other hand, one of the main liberal parties in Russia – Yabloko – drifted to Communist position for half a year before elections in 2003. Narodny deputat passed through almost all planes during these 4 years.

The parties with a small power index have exact political views, firm politics, and try to find the way to effect the outcome of voting. Such power is called influence-power (or I-power) [9].

Let us go back to Figs. 8-11. As we can see in Fig. 8 there is a peak of Communists PI_1 in autumn of 2000. This peak is associated with their behavior during the discussion in the Parliament of the bill for children benefits supported by Communists, Agrarians (the power value of APG has also increased to 0.2), Narodny deputat and Regions of Russia. Majority (263 votes) voted for this bill, but it was not passed because of the Federation Council veto (300 votes were necessary to override the veto).

The next power value peak, observed in May of 2002, corresponds to the alternative military service federal law. The leftists assemble the majority to defeat this bill. From May to September of 2003 the power of CPRF declined. It can be explained by the adoption of certain bills, for example the bill of local government reform or federal budget bill, as well as by the non-confidence vote to government. In all these votes CPRF was always in minority.

In Fig. 8 the power distribution of Edinstvo is presented. Examining the most important power value changes, it should be noted that there are repeated falls in power to the zero value during the whole period. A first such fall is observed in December of 2000 and January-February of 2001, when laws concerning nuclear exhaust problems had been considered. In 2001 the amendments to these acts allowed the import in Russia of nuclear exhausts for technological storage as well as for waste disposal. Edinstvo and LDPR had consolidated for the law to pass (Fig. 11) and had power value fall at the same time. In addition the decrease of power value for Edinstvo and LDPR can be explained by the fact that they were in majority when amendments to pension federal law had been considered since they voted against those amendments. The next fall in Edinstvo power value is observed in November-December of 2001, when the questions of judicial authority reform were discussed. Centrists had to reconcile their viewpoints with leftists and liberal factions for the adoption of this bill. The left liberal factions took an advantage of the law liberalization. Thus, increments of power value of SPS, LDPR, APG are observed (power value for SPS increased to 0.107, this value was the highest for SPS, APG had $PI_1 = 0.138$, LDPR had $PI_1 = 0.05$). The power value peak of Edinstvo can be observed in spring of 2002, $PI_1 = 0.4$. This peak can be aligned with the break of the so-called package agreement, accepted at the beginning of 2000. The package agreement break was initiated by centrist factions and had been supported by SPS and Yabloko. Edinstvo was a key player in that voting. The next most important power value peak of Edinstvo is observed in September-November of 2002, it can be explained by the adoption of the federal referendum law of the Russian Federation. This draft law proposed by Edinstvo was supported by all factions except Communists and Agrarians.

In Fig. 9 changes in the centrist factions power distribution, namely Narodny deputat and OVR, are given. There are strong changes of power observed for these factions. In January-February of 2001 a power value peak is observed that can be explained by alteration in the federal law of pensions. Narodny deputat was the pivotal player in this voting when the veto had been negotiated. There is the power growth observed for Narodny deputat and OVR in September-November of 2001. Narodny deputat had $PI_1 = 0.45$ and OVR had $PI_1 = 0.266$, one of the most important peaks of this party. It can be associated with the adoption of the most important bill of 2001, the Russian Federation labor code. All factions except Communists and Agrarians voted for this law, and Narodny deputat was pivotal in that voting. Faction OVR had another peak, observed in December of 2000. This is one of the highest values at that period, when the Russian Federation national symbol legislative package was supported by all parties except SPS and Yabloko. The reason of this peak appearance is that OVR was decisive in that voting.

In Fig. 10 power distribution curves of small groups, namely Regions of Russia and APG, are represented. Regions of Russia had the most interesting and important results in May-June of 2001, when its power increased to 0.42. This was the highest value at this period. This peak can be explained by the law of political party consideration. Edinstvo, OVR, LDPR, Narodny deputat, Yabloko and Regions of Russia voted for this law. The votes of Regions of Russia were decisive in this voting, and that was the reason of the power value growth. There is one more peak in the Regions of Russia power value observed in January-February of 2002, its value $PI_1 \approx 0.52$. At this time interval certain bills had been considered, namely the act of nationalization, the termination of broadcasting of TV6 act, the nationality law, the act of electric and heat energy rate management. Regions of Russia was in majority in voting for these laws, and the power value peak can be explained by the fact that Regions of Russia was the pivotal player in these votes. The next peak in the Regions of Russia power value is observed in March-April of 2003, $PI_1 \approx 0.51$, when problems of housing and communal services reform had been examined. The act of housing and communal services reform was accepted in third reading after some amendments to this act, and Regions of Russia was a pivotal player.

As one can see the extended power index differs from SOV results. The most important changes are observed for CPRF and its ally APG. The power value of these parties is higher than SOV. On the contrary, for parties Narodny deputat and LDPR this value is lower than SOV.

The results obtained from the extended Shapley-Owen power index and SOV computed for political parties of the III State Duma correlate badly with the results computed in Section 3. Those results show that the most powerful groups were the Edinstvo and CPRF parties, Narodny deputat and OVR took the third and fourth places accordingly, and the last were Regions of Russia, APG and SPS, respectively.

Results coincide for tiny groups of III Duma, namely for SPS, LDPR and Yabloko. Power analysis on the basis of both standard Banzaf and extended Shapley-Owen indices shows that the power of these groups is very low.

Similarly the results coincide for OVR and APG, both analyses pointing out that these parties took average positions in power rating.

7. Conclusion

We conclude the paper with several remarks.

Remark 1. The suggested approach is valid when the following assumption holds. First, factions vote homogeneously. This assumption seems to be true for the French Parliament, but not for the Russian one. Then the assumption above should be substituted by another assumption – the deviation of faction discipline is the same in all factions with respect to the member of MPs in each faction. This assumption seems to be strong as well.

Then our assumption can be re-formulated as follows: the deviation of homogeneous behavior in each faction is small, contingent and independent from factions and votings. Then the balance of power among opposing coalitions will on average be stable over time.

One can expect that the latter case is the closest to the real behavior of parties. On the other hand, the results obtained can be used as indirect proof of this assumption.

Remark 2. Among other problems considered in [3], there was a problem of analysis of power distribution when factions express their opinions on different issues. Issues such as social policy, attitude to the government and oil/gas sectors of economy were chosen. The results obtained for these cases were different from the case in which all votings are considered altogether. However, the difference was not so crucial as one might expect.

Remark 3. In the fourth and also in the newly-elected fifth Duma in December 2007, there is only one 'power holder', the party Edinaya Rossiya, which possesses the majority sufficient for constitutional laws passage. However, it is well known that this party consists of several 'wings' representing different opinions, from liberal to centrist and even conservative. On the other hand, the regional interests of groups in this party are also different. One of the directions of research in the analysis of power distribution in the fifth Duma is to study the power distribution among regional groups of different parties. It is a very complex computational problem which may be solved using a particular approach.

Remark 4. One of the routes to overcome the high complexity of evaluation of power indices for large societies is the use of generating functions – a special type of polynomials which are widely used in combinatorial theory. It has been shown how to use these functions for evaluating power indices in the case of unrestricted coalition formation [8] as well as restricted coalition formation [25] and of the coalition formation taking into account agents' preferences to coalesce [23]. For some cases it turns out that this technique allows to obtain results which can not be even thought about using direct algorithms.

Remark 5. The results and technique obtained allow us to study power distribution in large organizations such as International Monetary Fund and United Nations Organizations as well as many other institutions. For European Union and IMF several works have been done [10,12-15, 18,22,24] including the one in which we studied some models of coalition formation on the basis of their regional proximity, economic relations, etc. [4] However, this work only takes a first step in this direction.

Remark 6. Another interesting direction of research seems to be an analysis of power distribution surveying MPs in their desire to coalesce with their colleagues on different issues. We are going to start such surveys in one of the regional parliaments of the Russian Federation.

Remark 7. In [16] another index was introduced taking into account agents' preferences to coalesce. It will be interesting to compare the results produced by these approaches on the same data.

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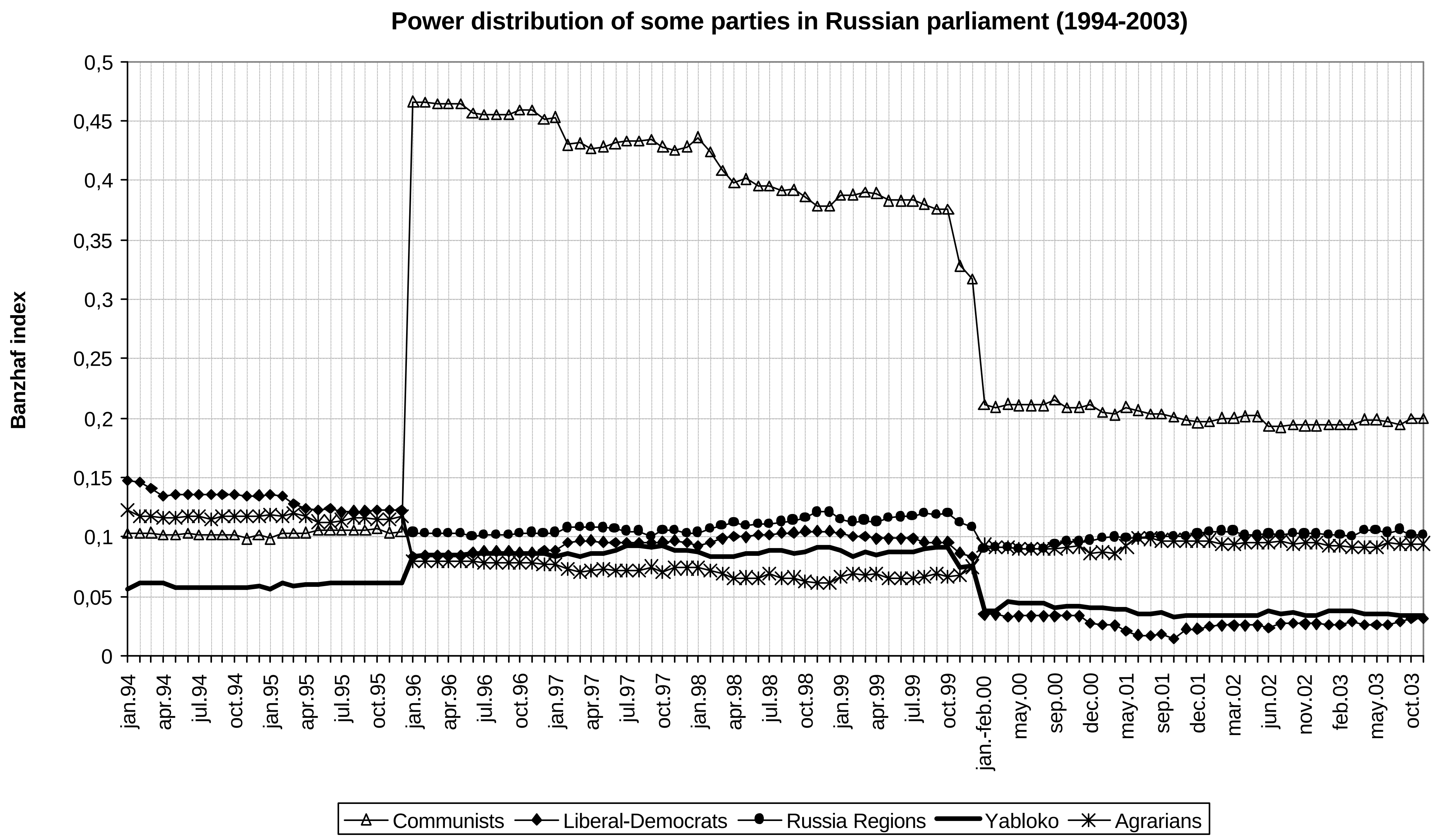

Power distribution of some parties in Russian parliament (1994-2003)
Banzhaf index
0,5
0,45
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
jan.94
apr.94
jul.94
oct.94
jan.95
apr.95
jul.95
oct.95
jan.96
apr.96
jul.96
oct.96
jan.97
apr.97
jul.97
oct.97
jan.98
apr.98
jul.98
oct.98
jan.99
apr.99
jul.99
oct.99
jan.-feb.00
may.00
sep.00
dec.00
may.01
sep.01
dec.01
mar.02
jun.02
nov.02
feb.03
may.03
oct.03
Communists
Liberal-Democrats
Russia Regions
Yabloko
Agrarians

Figure 1.

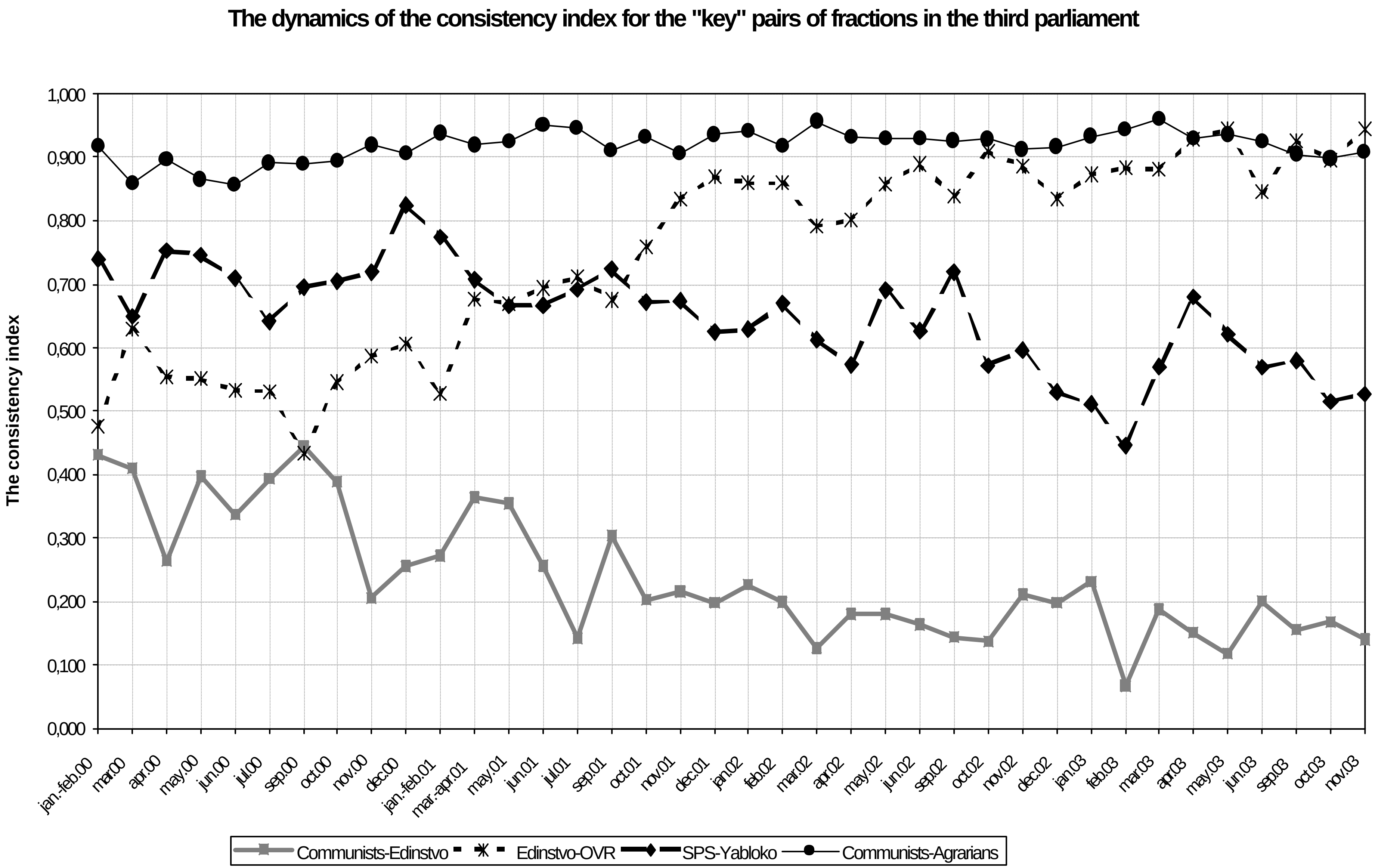
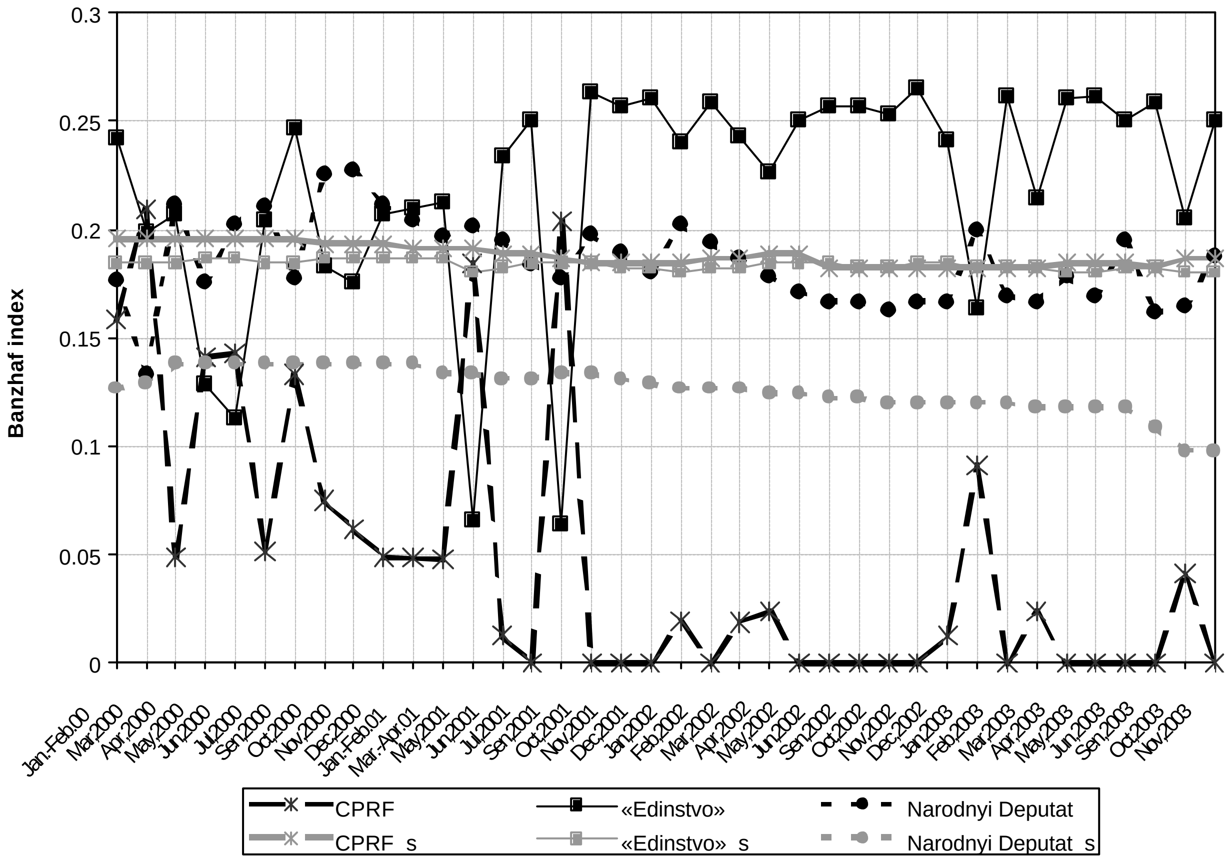
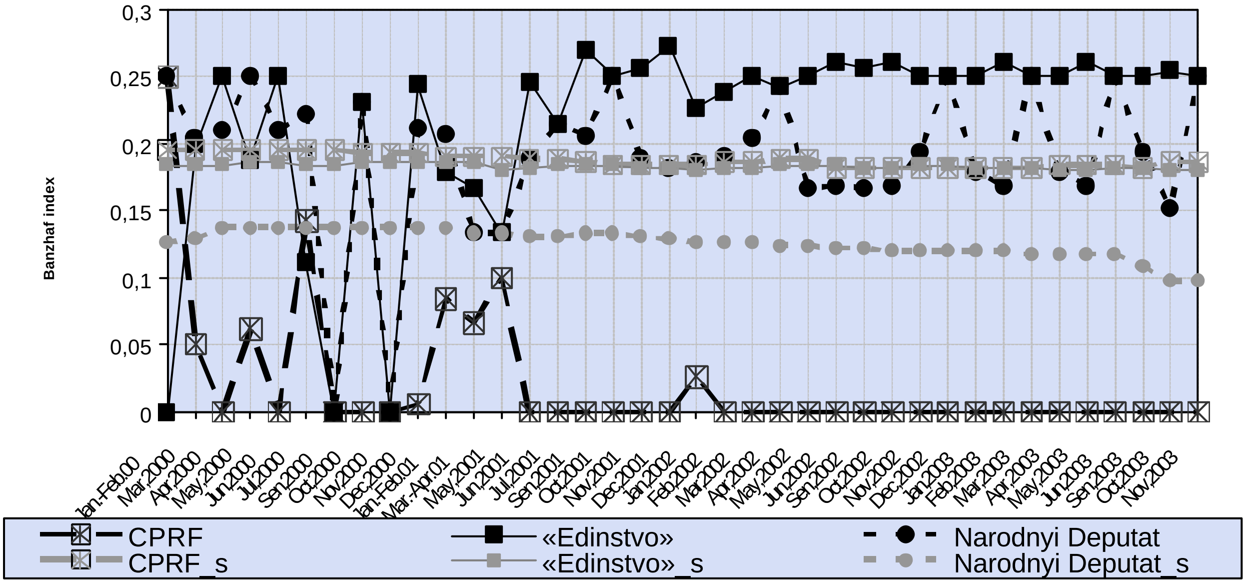

The dynamics of the consistency index for the "key" pairs of fractions in the third parliament
The consistency index
1,000
0,900
0,800
0,700
0,600
0,500
0,400
0,300
0,200
0,100
0,000
jan.-feb.00
mar.00
apr.00
may.00
jun.00
jul.00
sep.00
oct.00
nov.00
dec.00
jan.-feb.01
mar.-apr.01
may.01
jun.01
jul.01
sep.01
oct.01
nov.01
dec.01
jan.02
feb.02
mar.02
apr.02
may.02
jun.02
sep.02
oct.02
nov.02
dec.02
jan.03
feb.03
mar.03
apr.03
may.03
jun.03
sep.03
oct.03
nov.03
Communists-Edinstvo
Edinstvo-OVR
SPS-Yabloko
Communists-Agrarians

Figure 2



Distribution of power for large factions *(CPRF, «Edinstvo», Narodnyi Deputat)* *at the III State Duma (scenario 0,4)*

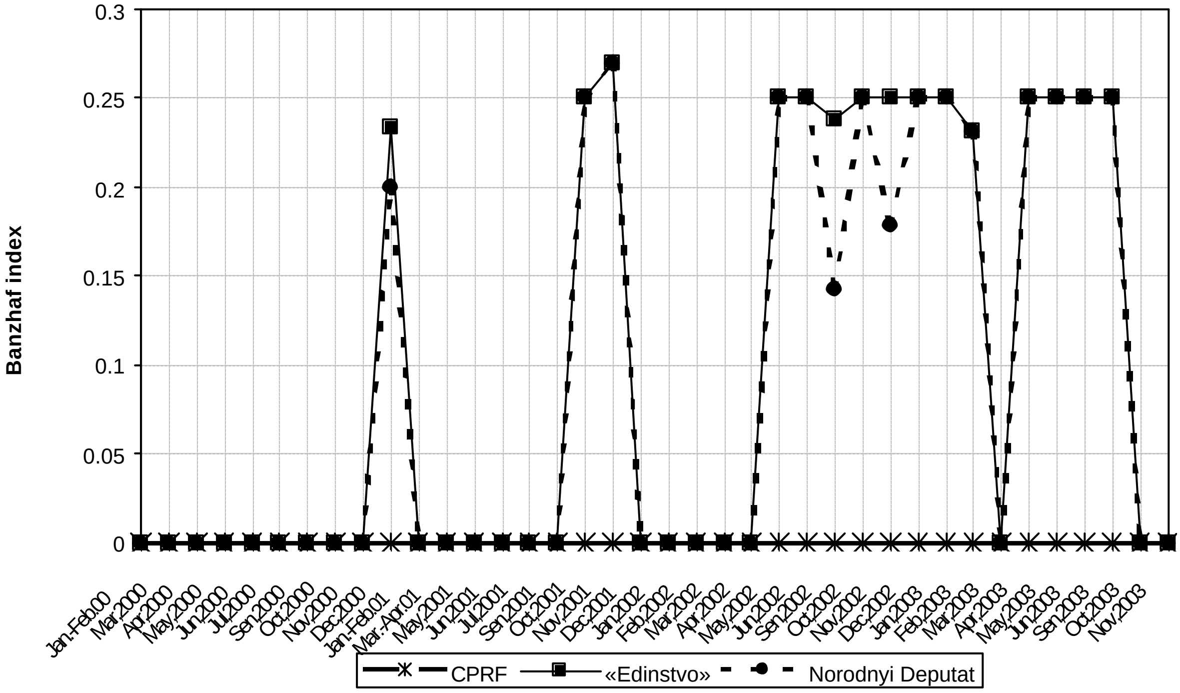
Figure 3



Distribution of power of large factions *(CPRF, «Edinstvo», Narodnyi Deputat)*

in the III State Duma (scenario 0,5)

Figure 4



Distribution of power of large factions *(CPRF, «Edinstvo», Narodnyi Deputat)*

in the III State Duma (scenario 0,6)

Figure 5

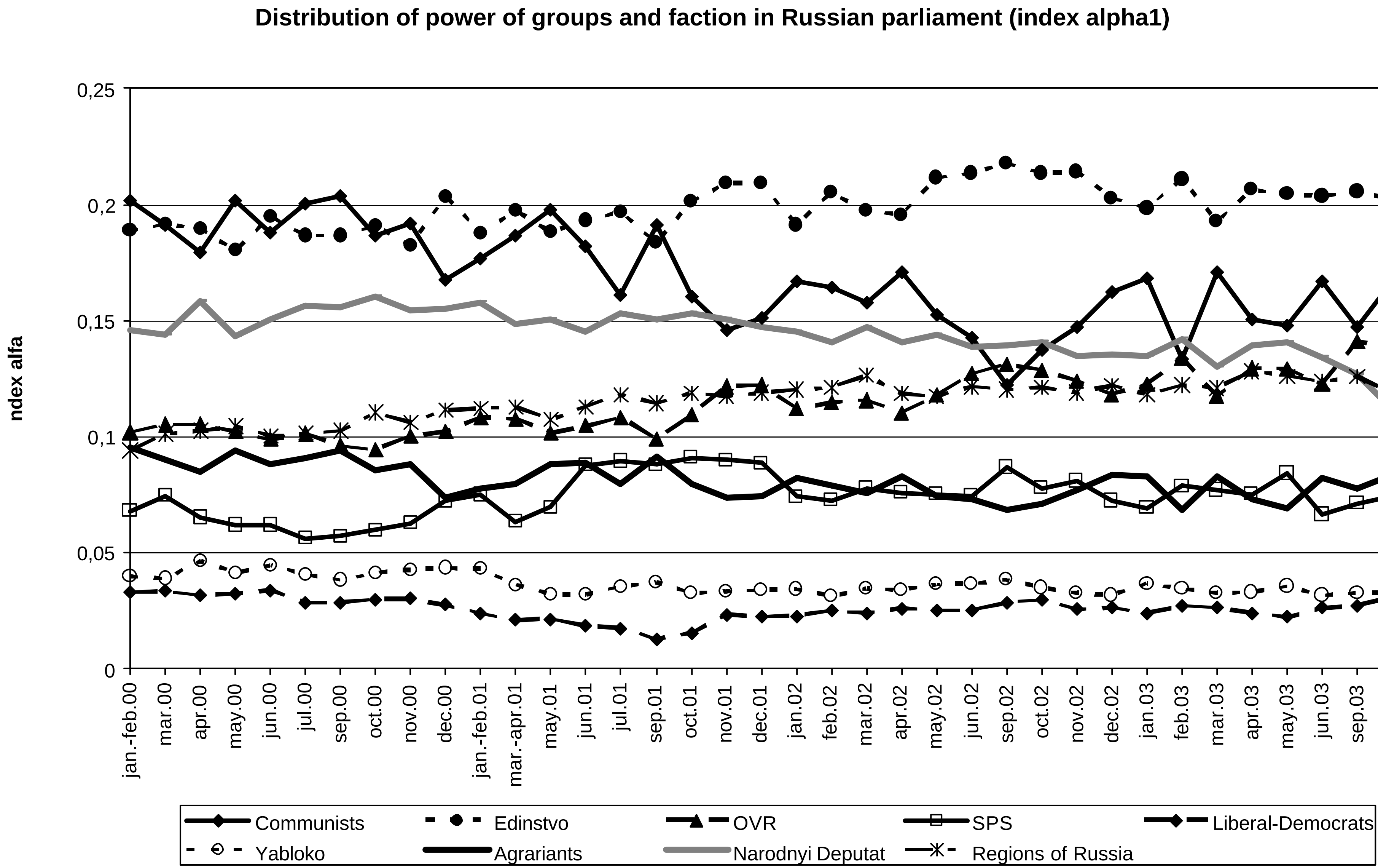

Distribution of power of groups and faction in Russian parliament (index alpha1)
ndex alfa
0,25
0,2
0,15
0,1
0,05
0
jan.-feb.00
mar.00
apr.00
may.00
jun.00
jul.00
sep.00
oct.00
nov.00
dec.00
jan.-feb.01
mar.-apr.01
may.01
jun.01
jul.01
sep.01
oct.01
nov.01
dec.01
jan.02
feb.02
mar.02
apr.02
may.02
jun.02
sep.02
oct.02
nov.02
dec.02
jan.03
feb.03
mar.03
apr.03
may.03
jun.03
sep.03
oct.03
nov.03
Communists
Edinstvo
OVR
SPS
Liberal-Democrats
Yabloko
Agrariants
Narodnyi Deputat
Regions of Russia

Figure 6.

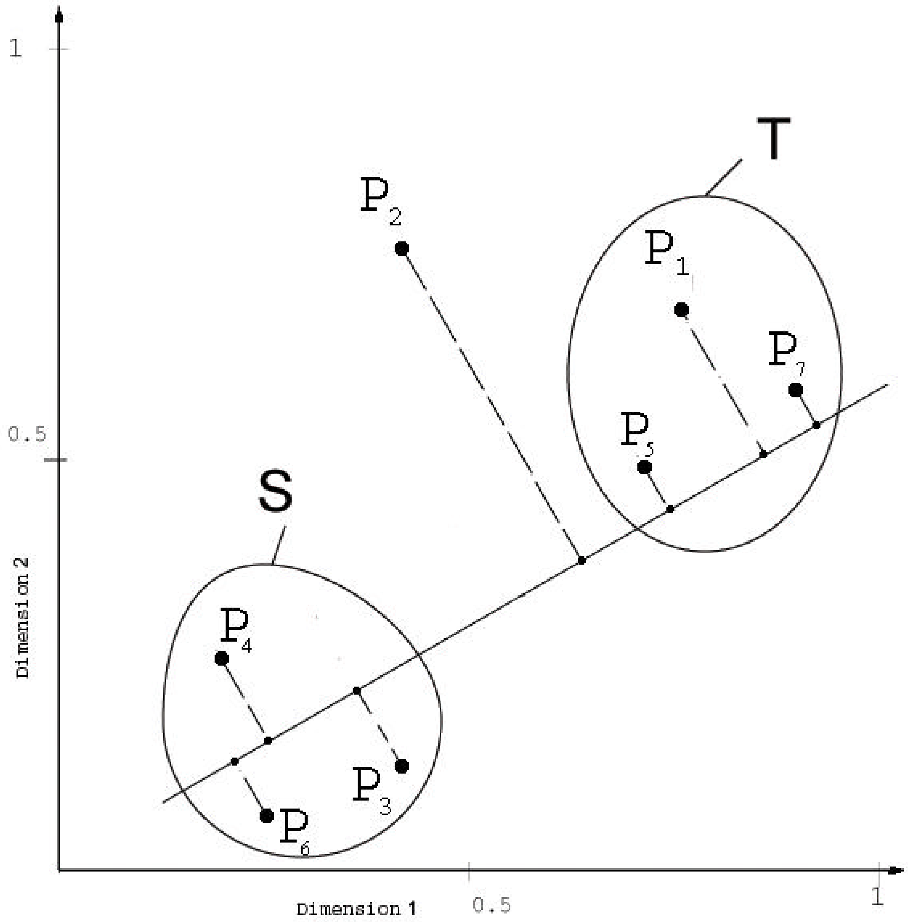


Figure 7.

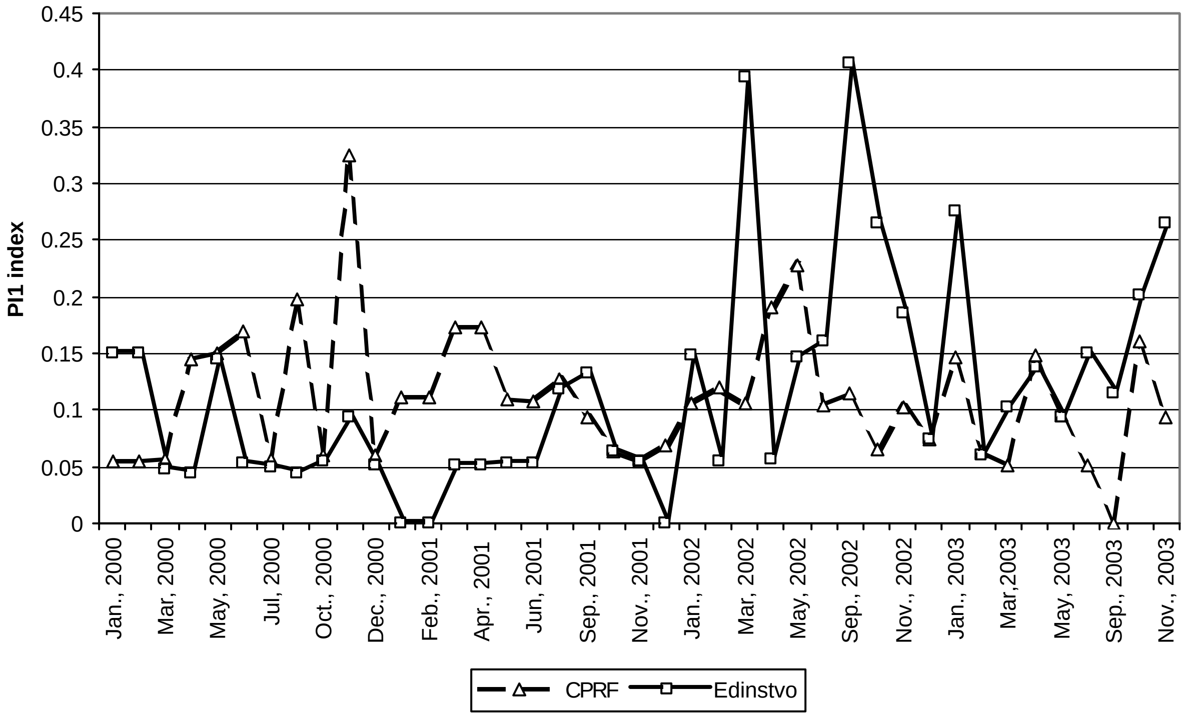


Figure 8. Extended power index values PI_1 for the III State Duma (Edinstvo, CPRF)

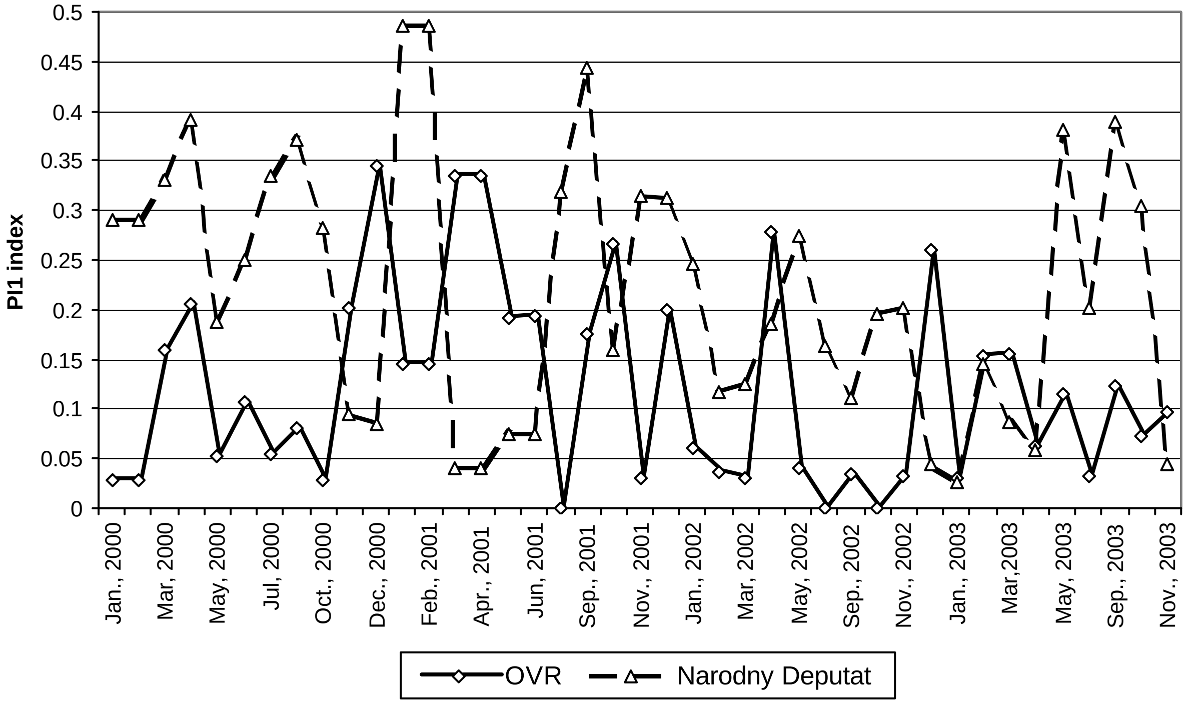


Figure 9. Extended power index values PI_1 for the III State Duma (Narodny Deputat, OVR)

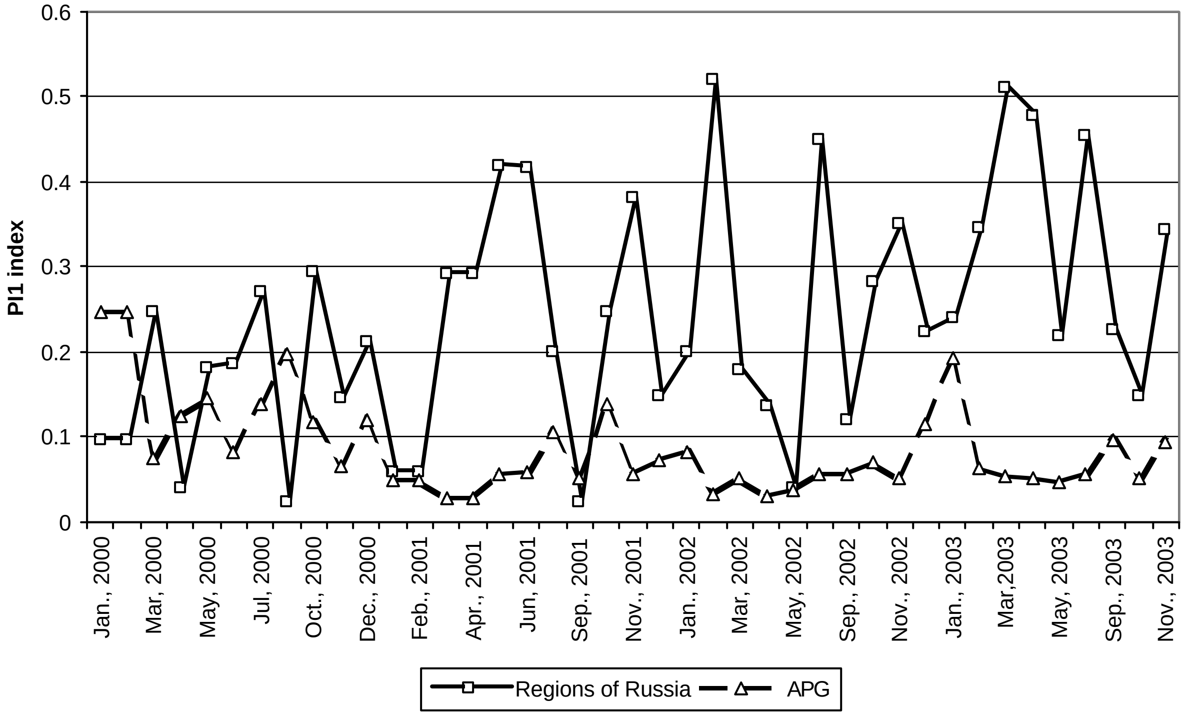


Figure 10. Extended power index values PI_1 for the III State Duma (Regions of Russia, APG)

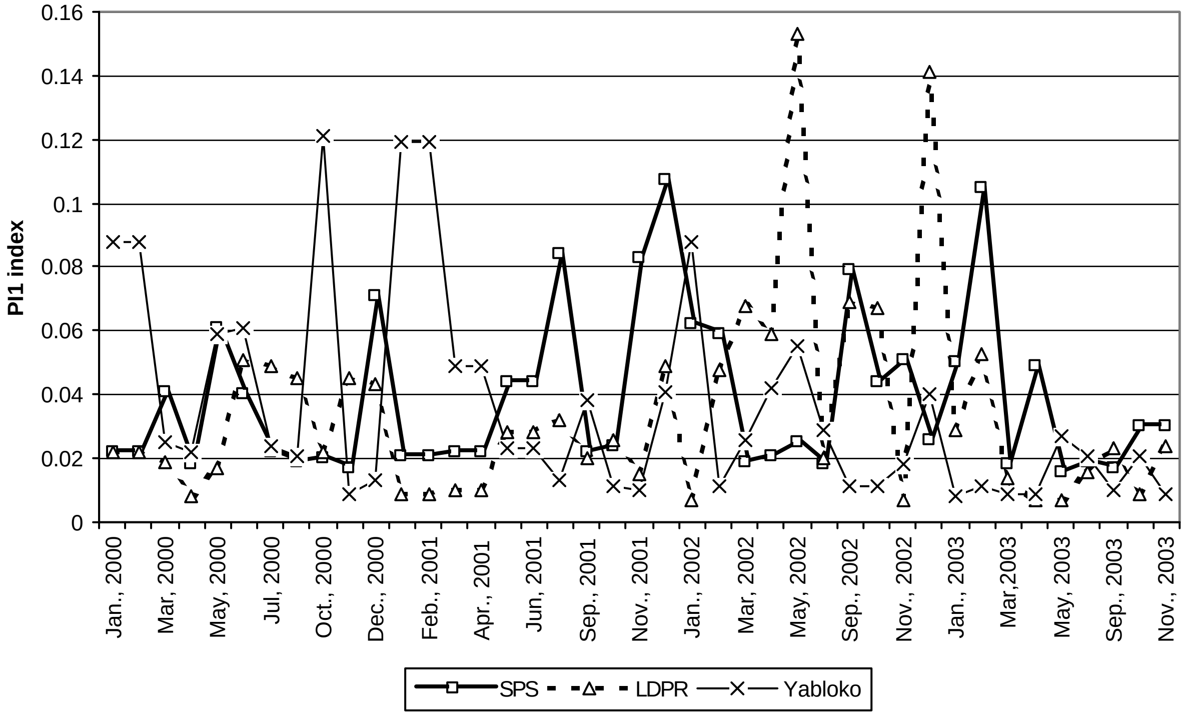


Figure 11. Extended power index values PI_1 for the III State Duma (SPS, LDPR, Yabloko)

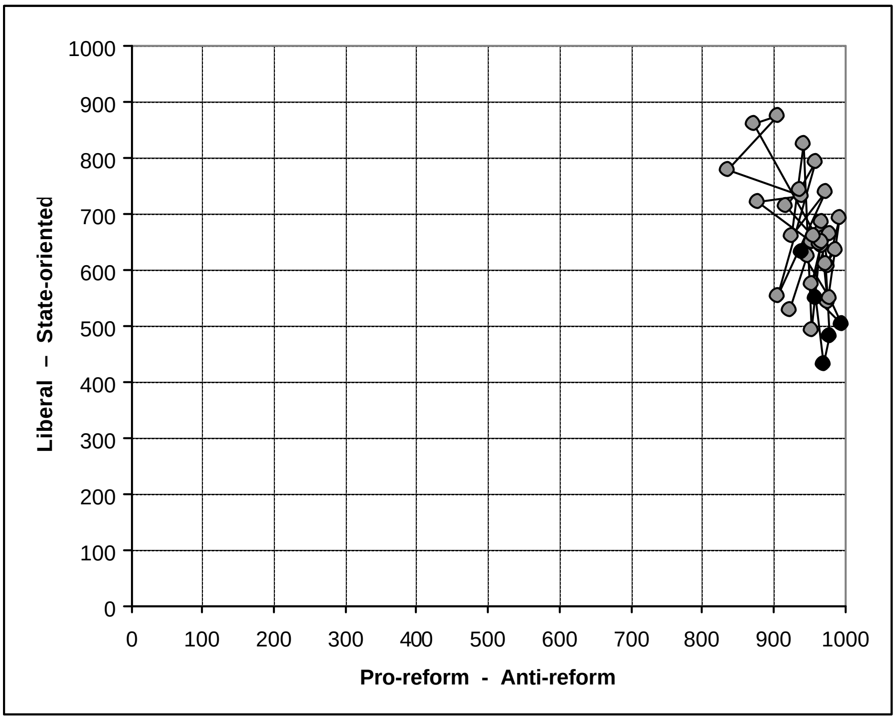


Figure 12. Dynamics of political positions (CPRF)

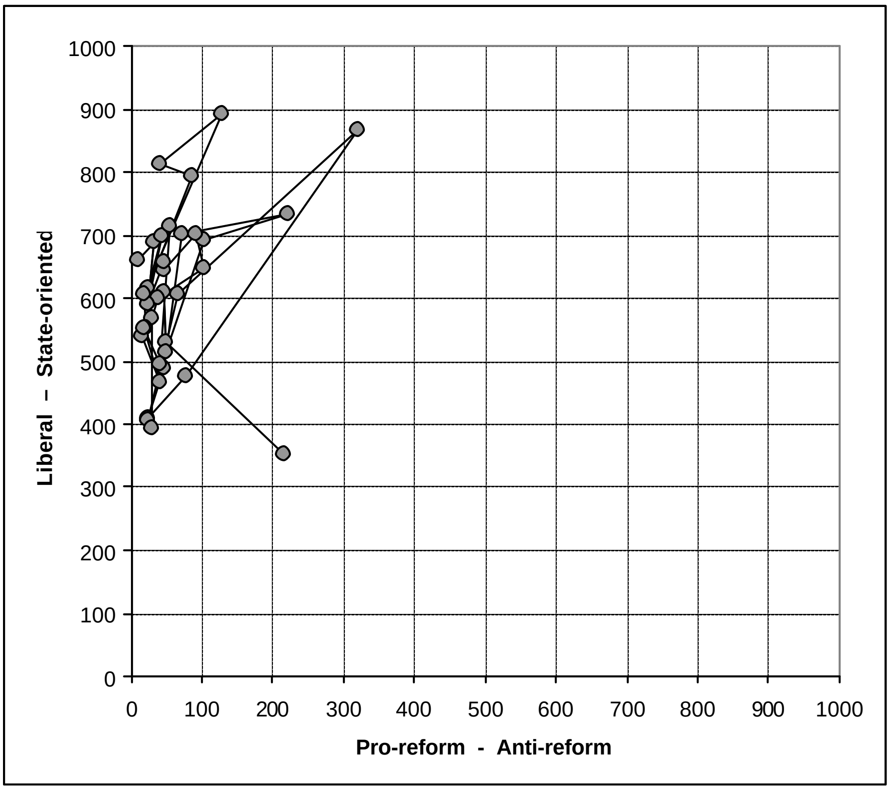


Figure 13. Dynamics of political positions (Edinstvo)

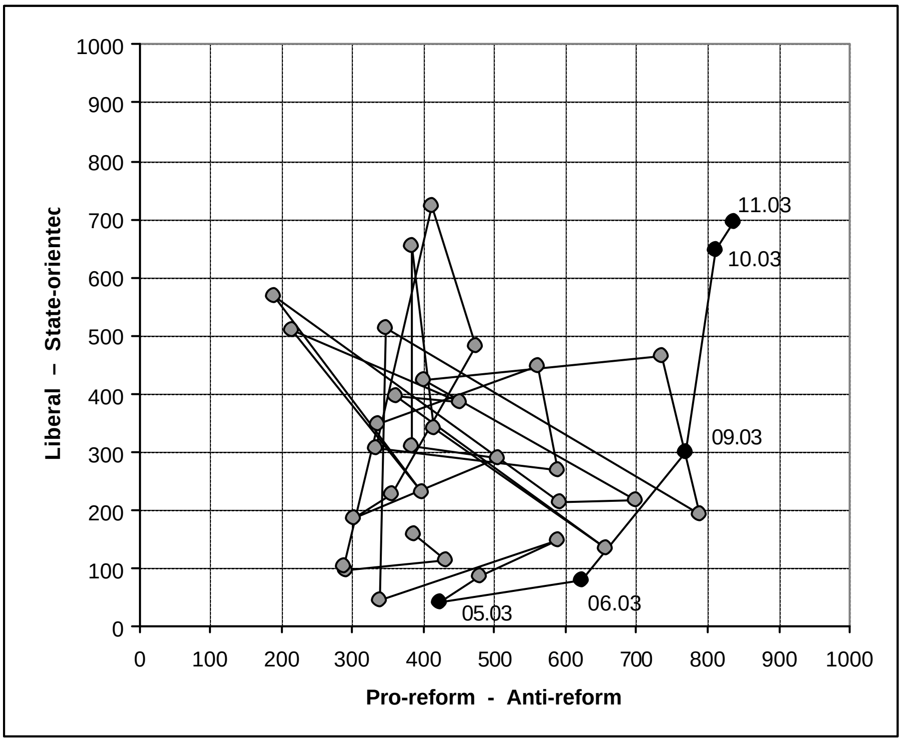


Figure 14. Dynamics of political positions (Yabloko)

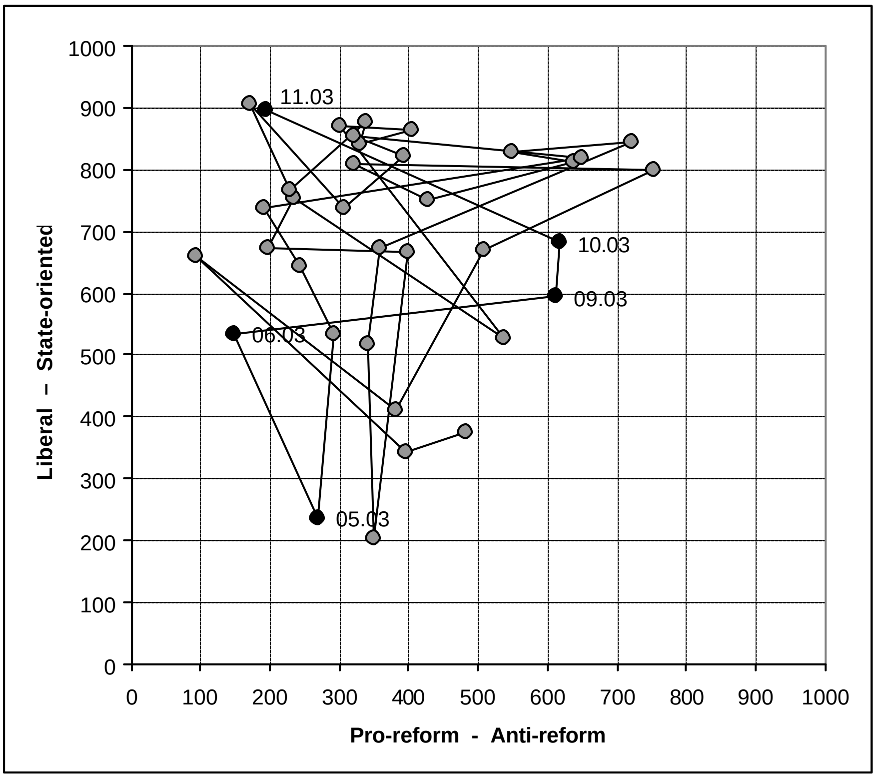


Figure 15. Dynamics of political positions (Narodny Deputat)