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An Agent-based Model of Prehistoric Settlement Patterns and Political Consolidation in the Lake Titicaca Basin of Peru and Bolivia

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Abstract

Insights into prehistoric region-wide political consolidation were suggested by simulation results from an agent-based model of pre-state societies. The case study was the late prehistoric period circa 2500 BC to AD 1000 in the Lake Titicaca basin of Peru and Bolivia. Over a series of simulation runs the model produced a range of alternative political pre-histories. A substantial proportion of those runs were classified as matching the scenario archeologists believe actually happened during this time period. Classification was based on multidimensional quantitative measures of empirical criteria for the emergence of simulated macro-level patterns corresponding to observed patterns in the archaeological record.

The model's structure consisted of a grid of cells, each scaled to 1.5 km × 1.5 km, representing the geography, hydrology, and agricultural potential of the 50,000 sq. km basin. A collection of multiple agents—Settlements, Peoples, Polities, and Chiefs (political leaders) – interacted with the environmental grid and with each other. The agents' behavior was modeled as micro-level condition-action rules based on the hypothesized causal factors of: agriculture, migration, competition, and trade. Internal structure and dynamics as well as the simulation results showed strong indicators of the model's structural realism. Uncertainties associated with the model were also assessed.

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Keywords: agent-based, Lake Titicaca, social complexity, polity formation, settlement pattern, socio-ecological simulation, trade routes, political consolidation, inter-settlement migration, pre-state societies, Repast, simulation model

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Introduction

The Lake Titicaca basin of the South Central Andes was part of the Inca Empire in the 15th century (Figure 1). In prior centuries the basin was home to the Tiwanaku state (ca. AD 550-1000) and a number of complex chiefdoms prior to that period. What factors may have accounted for the ascendancy and decline of these complex polities and how did these factors interact with each other?



Figure 1: Titicaca Basin in Context of the Inca Empire c. 1530.

One of us (CS) has proposed in previous publications that the primary factors in development of complex societies in the Titicaca Basin were both ecological and social. The ecological factors were land use potential for agriculture and pastoralism (Stanish 1994) and the effect of rainfall variation between different parts of the basin (Stanish 2003). Social factors include political competition and conflict (Arkush and Stanish 2005), inter-settlement migration (Stanish 2003) and long-distance trade (Plourde and Stanish 2006). All of these factors have been identified by numerous scholars, and the

impact of each is the subject of ongoing debates within anthropology and especially archaeology. Interested readers should refer to the sources cited here.

We have attempted to gain insight into the dynamic interplay of these factors by using spatial agent-based modeling (ABM) and simulation. Joshua Epstein summed up his comparison of this technology to other complex systems analysis tools (state space equations, regression analysis, game theory, ...) as follows: "... agent-based modeling is clearly a powerful tool in the analysis of *spatially distributed systems of heterogeneous autonomous actors with bounded information and computing capacity.*" (Epstein 2006, p 38) As should become apparent, the phrase in italics is a very apt description of our problem space, hence the motivation for employing ABM.

Epstein also argues that a necessary condition for an ABM to "explain" an observed phenomenon is an empirical demonstration that the model can recreate what actually happened in the real world (2006, pp 9-14). In this sense the model reported here was designed to explain the role of agriculture, migration, competition, and trade leading to the political consolidation observed in the prehistoric Titicaca basin. These hypothesized explanatory factors were included in simple local rules¹ within the model. During simulations each independently operating agent followed these rules as it interacted with other similarly autonomous agents. Our objective was to model these rules for micro-level interactions in such a way that the collective effect would be the emergence of the following macro-level patterns indicated in the archaeological record:

- Repeating cycles of polity fission followed by consolidation into restructured polities – sometimes called chiefly recycling (Marcus 1992, Read 2002)
- Concentration of population in primary and regional centers (Stanish 2003)
- Primary centers located at north and south ends of the lake (Stanish 2003)
- Basin-wide population: 0.1% average growth rate reaching level of 500,000 at Tiwanaku's height (Stanish 2003)

Each of these patterns did indeed emerge during our simulations. In addition an overarching pattern unexpectedly emerged that simultaneously included all the above patterns. It appeared to be a scenario that temporally and spatially matched the broad-brush political prehistory of the Titicaca Basin. Each of these patterns, expected and unexpected, will be presented and discussed in subsequent sections, as will the rules for agent behavior.

The remainder of this section attempts to place the current model in the context of other relevant simulations published in the literature and at the same time introduce the sections that follow. In most reported instances of spatial ABM's, agents operate on a small idealized grid of resources (e.g. 25 x 25 cells) that represents no specific geographic location or time interval. Two well known examples are Thomas Schelling's classic simulation to explain segregated residential neighborhoods (Schelling 1978, pp147-55) and the Sugarscape models by Joshua Epstein and Robert Axtell (1996) that explore a range of social and economic issues. The simplicity of this modeling style has a number of practical advantages: models can be built quickly, fast program execution allows comprehensive simulation experiments, detection of emergent behavior is

¹ Simple local rules prescribe how one agent interacts with each neighboring agent without control by or knowledge of anything outside that immediate neighborhood, where neighbors are defined at more than one scale, i.e., neighboring settlements and neighboring polities.

unambiguous, model dynamics are easily understood, and the level of effort is feasible for an individual investigator or a small team.

Modeling studies without geographic or time correlates, as above, typically do not include validation based on comparing simulation results to specific empirical data. One interesting exception to this was a model of aggression in which cross-cultural ethnographic data were used to validate the simulation results (Burtsev and Korotayev 2004). This is also one of the few models of complex pre-state societies found in the literature.

There have been a few published ABM simulations of pre-state agrarian cultures that are geographically and time specific. Two such projects modeled the site selection behavior of individual farming families spanning 700 years from AD 600 to 1300 in two different areas of the American Southwest, in: the Mesa Verde region of southwestern Colorado (Kohler et al. 1999, 2005, 2007), and the Long House Valley of Arizona (Dean, et al. 1999, Axtell, et al. 2002). Sited in another part of the world, a very fine grained model simulated the impact of ecological stresses on the planting, harvesting, herding, and exchange behavior of individuals in a bronze age (c. 3000-1000 BC) Mesopotamian village over a 20 year period (Wilkinson, 2007). Relevant lessons learned from these models are included in the sections that follow.

An important benefit of simulating a specific time period is the potential to validate the model by comparing model outcomes to empirical data or a chronology of events indicated in the archaeological record during the corresponding time span. This validation approach was a key feature in both of the American Southwest simulations, above, and was also employed in the current model. However a departure from earlier simulations was our use of quantitative empirical validation based on multi-dimensional measures. Looking at a single feature, e.g. population level, would not have captured the many facets of long-term political trajectories. Subsequent sections discuss 12 numerical match criteria each aimed at a specific spatial-temporal dimension of the emergent macro-level patterns. The number and extent to which these criteria were met by numerical measures, collected during thousands of simulation runs, provided quantified estimates of how accurately the model had recreated reality as depicted by the archaeological record.

Another major advantage, geo/time-specific modeling enforces reality-based feasibility constraints on space, time, and energy during simulations. The current model is both geographically and time specific. As with the other such models cited above, we also modeled the ecology and agriculture, although in less detail, in order to create a physically realizable substrate on which other factors in cultural change could operate, namely competition and political consolidation.

Our approach to modeling political dynamics was inspired by Lars-Erik Cederman's agent-based Emergent Polarity model of early nation-state geopolitics (1997, 2002), which we adapted for pre-state polities. This model simulated the consolidation of small polities into large ones which may then fission back into small independent entities and subsequently consolidate again, reminiscent of the recycling pattern observed in pre-state chiefdoms.

We also followed Cederman in the manner of interpreting simulation results. He classified the spatial end state of each simulation run as one of several alternative political configurations, based on the number of sovereign states remaining: unipolar,

bipolar, multipolar, or nonpolar. In the same way, each simulation run of the current model was classified as one of seven alternative Titicaca political prehistories, one of which corresponded to what the record indicates to us actually happened. We differ, however, by adding the temporal dimension to the classification scheme to distinguish not only the end state but the trajectory through time to reach that configuration. (See Results section following)

As important as the benefits of geo/time specific modeling are, we also have been very mindful of the virtues of model simplicity listed earlier. Every opportunity was taken to avoid model complexity wherever simpler alternatives were apparent. For example, our model of polities' structural changes following conflicts is a greatly simplified version of Cederman's approach, which in turn was far simpler than earlier models by other political scientists (1997, p 83). This is one of a number of instances listed in Appendix B Table B-1 where we favored simplicity over more complex reality.

Volker Grimm (2005) estimated that ecology has produced as many ABM's as all other disciplines combined. From a continuing review of this large body of published models, the concept of pattern-oriented modeling has evolved as a set of guidelines for developing ABM's. The main thrust of this approach is that modeling and simulation should focus on multiple patterns observed in nature at different scales and hierarchical levels, such as the five patterns introduced above. Grimm concluded that the more a model exhibited these interlocking constraints, the higher the likelihood that the model's structure corresponded to that of the real-world system being modeled. This and other pattern-oriented characteristics of the current model are considered in the Model Dynamics and Structural Realism section.

Grimm also observed that establishing the appropriate resolution and level of aggregation is a distinguishing characteristic of "extremely useful models" (1999, p 142). In the current model, portraying political consolidation required a different scale of modeling than individual people and their households as in existing geo/time-specific models, above. The lowest level of aggregation in our model was entire settlements, which in turn were the building blocks for complex polities. As a corollary to the higher level granularity, our temporal-spatial simulation space was quite large, with a time span of 3600 years and covering a geographic area of the entire Titicaca Basin, some 50,000 square kilometers.

Prehistory of the Titicaca Basin

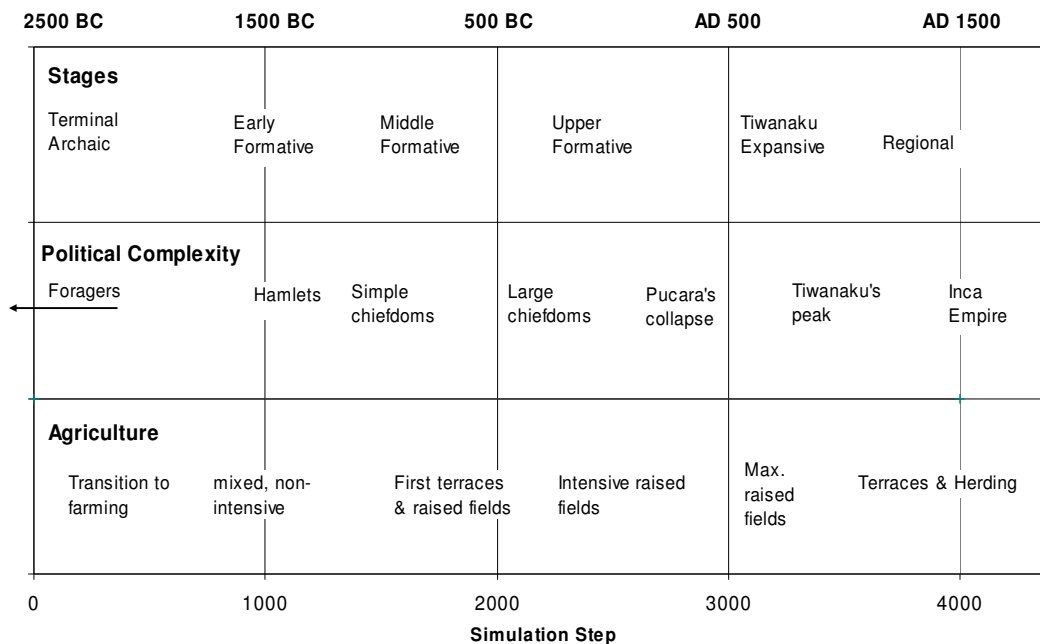
There is a very rich archaeological record of the prehistoric Titicaca basin that we have drawn upon to develop the current model. The record has provided two complementary sources of information: 1) macro-level temporal and spatial patterns used to assess the accuracy of the simulation results, and 2) micro-level empirical data that were incorporated into the behavior rules of the agents. The differences in scale and hierarchical level between micro data and macro patterns mean that the modeling and validation sources are quite independent, even though both have common roots. We will focus on the macro-level in this section, and then highlight the micro-level data in the subsequent Model Structure section.

The 50,000 square kilometers of Lake Titicaca and the surrounding hydrological basin are nestled between the eastern and western cordillera of the south Central Andes mountains. The lake is notable for its high elevation, large size, and marked salinity. At

3800 m above sea level, it is the world's highest navigable lake. The lake itself has an area of approximately 8500 square kilometers. For modeling purposes we assume that lake water was unsuitable for plant agriculture due to its salinity.²

One of the three archaic states that developed in South America arose in the Titicaca basin. That state, known as Tiwanaku, emerged in the 6th century AD as a regional power, a few centuries after its counterpart on the north coast—Moche—and contemporary with its counterpart, Wari, north of the basin. While all three states developed in the "Andean region", there is no evidence of interaction between Moche and Tiwanaku, and no evidence of Tiwanaku or Wari in either of their respective core areas. In other words, Tiwanaku developed largely as an autochthonous phenomenon.

During Tiwanaku times and earlier, the Titicaca basin provides a long time frame of millennia in which we can assess models of chiefdom and state development. It was centuries later that the basin became part of the Inca Empire for about 100 years until the Incas were conquered by the Spaniards in the mid 16th century AD. The chronologies in Figure 2 summarize the prehistory of the region extending back approximately four millennia before the Inca period. The date in years and the corresponding simulation time steps are shown at the top and bottom of the figure respectively. In between are three parallel chronologies: the archaeological stages, or periods, referenced throughout this paper, the changes in political complexity, and agricultural methods employed.



Sources: (Bauer & Stanish 2001, Cipolla 2005, Klink 2005, Stanish 1994, 2003, 2005)

File: Timeline

Figure 2: Chronologies of political complexity and agriculture in the Titicaca basin 2500 BC – AD 1500 (the key to agricultural methods is continued below)

² The salinity of the lake varies across space — less saline in the north and more saline in the south — and through time, as indicated by the work of paleolimnologists. There are some rare instances where lake water today is used for some irrigation. However, the total effect of lake resources is a relatively small fraction of the Titicaca Basin economy today and in the past.

Prehistoric Agricultural Methods

During the time period modeled (2500 BC – AD 1000) Titicaca basin peoples employed a number of agricultural techniques to take advantage of the Titicaca basin's virtually unique environment.

Herds of camelids, llamas and alpacas, were important as portable wealth, as pack animals, for wool, to fertilize fields, as well as a source of meat (along with fish and some deer). Herds could be pastured in the puna zone above 4000 m where crops cannot grow. There were also low pastures on the flat pampas. Today, farmers irrigate the pampas in some areas to provide better pasturage for their grazing animals (Stanish 2003).

Plant agriculture was complex and intensive, based upon a huge variety of tubers and some grains. These plants were cultivated in a number of areas of the basin using a variety of techniques. Once the Titicaca basin people began to cultivate land, they employed the full suite of techniques to most effectively exploit the landscape (Stanish 2006).

Terraces constructed on hillsides were the most common use of the land for agriculture. These terraces served to catch rain runoff as well as prevent erosion. Any overflow cascaded to the next lower terrace downhill. Very modest raised canals or aqueducts were also built to direct additional water to the terraces from other sources such as springs (Treacy 1989)

Cocha were artificial depressions in the low pampa areas dug in the ground to reach the groundwater that collects in front of the lake and to hold water during the dry season. Crops were planted in small furrows that farmers dug out from the cocha. (Stanish 1994, 2006)

Raised fields were a series of earthen platforms built up in swampy or low lying areas along the lake shore and river banks. The platforms increased soil depth and provided the appropriate drainage for otherwise waterlogged soil. The areas between the platforms where soil was removed became canals that filled with water. The canals collected and produced organic sediment used as fertilizer and were a source of water for splash irrigation. Some have suggested that the fields stored heat from solar radiation to protect against overnight frosts. (Kolata 1986, Erickson and Candler 1989, Erickson 1993)

Our study period begins at the end of Terminal Archaic stage circa 2500-2000 BC. Archaeological sites from this period have been found primarily along rivers. There was a slow population growth paralleled by an increasing reliance on horticulture and intensive collecting and fishing. There was also a gradual increase in sedentary lifestyles with semi-permanent villages established by 2500 BC. By ca 1800-1500, settlements were effectively sedentary although the use of wild resources continued for millennia afterwards.³ By the Early Formative (circa 2000-1500 BC), the population was concentrated near the lakeshore in small, largely permanent hamlets. There is no evidence of social or political ranking during this period. Site sizes were restricted to a narrow range, probably no larger than one hectare and usually much smaller. Subsistence was characterized by a mixture of rain-fed farming, domestic animals, hunting, gathering, and fishing.

Socio-political ranking and economic heterogeneity developed during the Middle Formative Period (circa 1300-500). Regional centers were established, occupied by aggregations of people around architecturally complex settlements. It is likely that an emergent elite established itself over time in the centers as a permanent class or coalition. Throughout the Middle Formative, there was an increasing elaboration in economic intensification and the concomitant development of political ranking. Chiefdom political organization, in the sense formulated by Joyce Marcus (1992), Carneiro (1970), Spencer (1993) and others, emerged as the dominant political type on the landscape at this time. That is, there is evidence for regional polities in which a central place had some kind of influence or control over smaller settlements in its immediate area. Major settlement was

³ Research on the Island of the Sun (Stanish 2002) suggests that populations were effectively sedentary by the end of the third millennium BC.

no longer restricted to the lake edge or along the rivers, and the first signs of terraced and raised field agriculture appeared in the Middle Formative (Erickson and Chandler 1989, Stanish 1994, Stanish et al 2002, Bauer and Stanish 2001).

The Upper Formative circa 500 BC – AD 400 saw the rise of complex competing chiefdoms and large political centers. The largest and most influential political centers were Pucara and Taraco in the north and Kala Uyuni (Bandy 2006), Khonkho Wankane (Janusek 2007) and Tiwanaku in the south of the lake. Figure 3, left, shows the hypothesized polities in the basin at the peak of Pucara’s influence circa AD 100-200. Intensification of raised field agriculture during this stage was evident. All these developments suggest that elites and their factions organized and possibly controlled labor to create an economic surplus for their unique kinds of Titicaca political economies.

Access to exotic goods and commodities outside the basin, e.g., obsidian and copper, also became important to leaders for maintaining the integrity of their factions and keeping political power. Matthew Bandy (2004b) has argued for the centrality of long-distance exchange in the region via caravans, a position also discussed much earlier by David Browman (1984), Luis Lumbreras (1974), and Núñez and Dillehay (1979) among others.

It was also in the Upper Formative period that we see unequivocal evidence for formalized inter-polity conflict including trophy head iconography, decapitation scenes on pottery and stone sculpture, and evidence of the burning of structures on settlements. Prior to this, there is no direct evidence of formal conflict but we assume that there was low level raiding between “ethnic” groups, as seen in the ethnographic record for societies of this time throughout the world.⁴

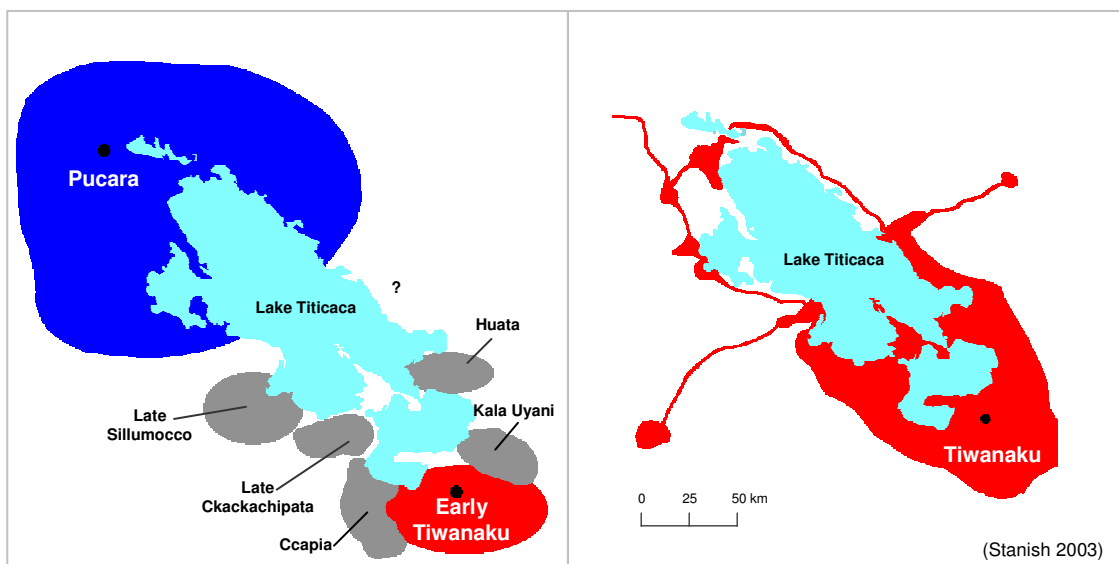


Figure 3: Large Titicaca Polities at their Peaks: Pucara (left) in Upper Formative c. AD 100-200 and (right) Tiwanaku c. AD 800-900.

⁴ The earliest evidence of a fortified site comes from a pukara in the Putina region. Collected by Lisa Cipolla (personal communication 2003), the site was dated to the Upper Formative.

Pucara collapsed as a regional polity near the end of the Upper Formative, around AD 200-300, and continued to contract thereafter. The reasons for this decline are unknown although drought, political competition, and altered trading patterns are likely contributing factors. Meanwhile in the south Tiwanaku was emerging as a powerful polity and eventually became an expansive archaic state by AD 600-700. Tiwanaku grew to be one of the largest cities in the ancient Americas with a population in the entire Tiwanaku valley estimated to be as large as 50,000. This large urban population was supported by vast areas of raised fields, camelid herds, trade, and specialized craft production. Figure 3, right, indicates what is believed to have been the extent of Tiwanaku's influence and control at its height around AD 800-900. Tiwanaku's end around the 11th through 12th century AD defines the end of our study period.

Overview

The computer displays shown in Figure 4 provide an introduction to the model structure and the simulation results which follow in subsequent sections. A computer program that implemented the underlying model and collected measures during simulations also generated these displays. This software was written in the Java programming language making use of a class library for agent-based modeling called Repast that was originally developed at the University of Chicago and now is maintained by the Argonne National Laboratory (North et al. 2006). Complete source code of the Titicaca model software is available from the authors upon request.

The displays in Figure 4 are snapshots of the model's state at three key points in a simulation run. The run shown consisted of 3600 consecutive steps, each corresponding to one year, beginning in 2500 BC through AD 1100. The displays depict three model elements: 1) the ecology, 2) Settlements, and 3) multi-Settlement Polities (model agents are capitalized throughout this paper). The ecology was modeled as a 200 x 200 grid of Patches each representing a 1.5 x 1.5 km portion of the basin. In Figure 4 (a – c) the color of each Patch represents its elevation (darker colors are higher), one of several Patch properties. Others include geographic zone, presence of river, spring or lake, normal rainfall, and potential agricultural methods. See Appendix A for details.

Settlements are indicated by a square outline around Patches. Figure 4(a) shows initial conditions of the model with Settlements along major rivers and at the edge of the lake. This is believed to approximate the settlement pattern c. 2500 BC based on field surveys from several areas in the basin (Klink 2005, Cipolla 2005)⁵. Through succeeding simulation steps Settlements colonize adjacent Patches, spawn new Settlements, may be abandoned, plant and harvest crops, and gain and lose population from inter-Settlement migration, net births (including deaths from natural causes), and deaths due to food shortages and conflicts.

⁵ All surveys reported on here relied on full-coverage regional survey methodologies used in contemporary Andean settlement archaeology. The spacing of individuals in survey crews is usually reported to be less than 25 meters, with others as low as 10.

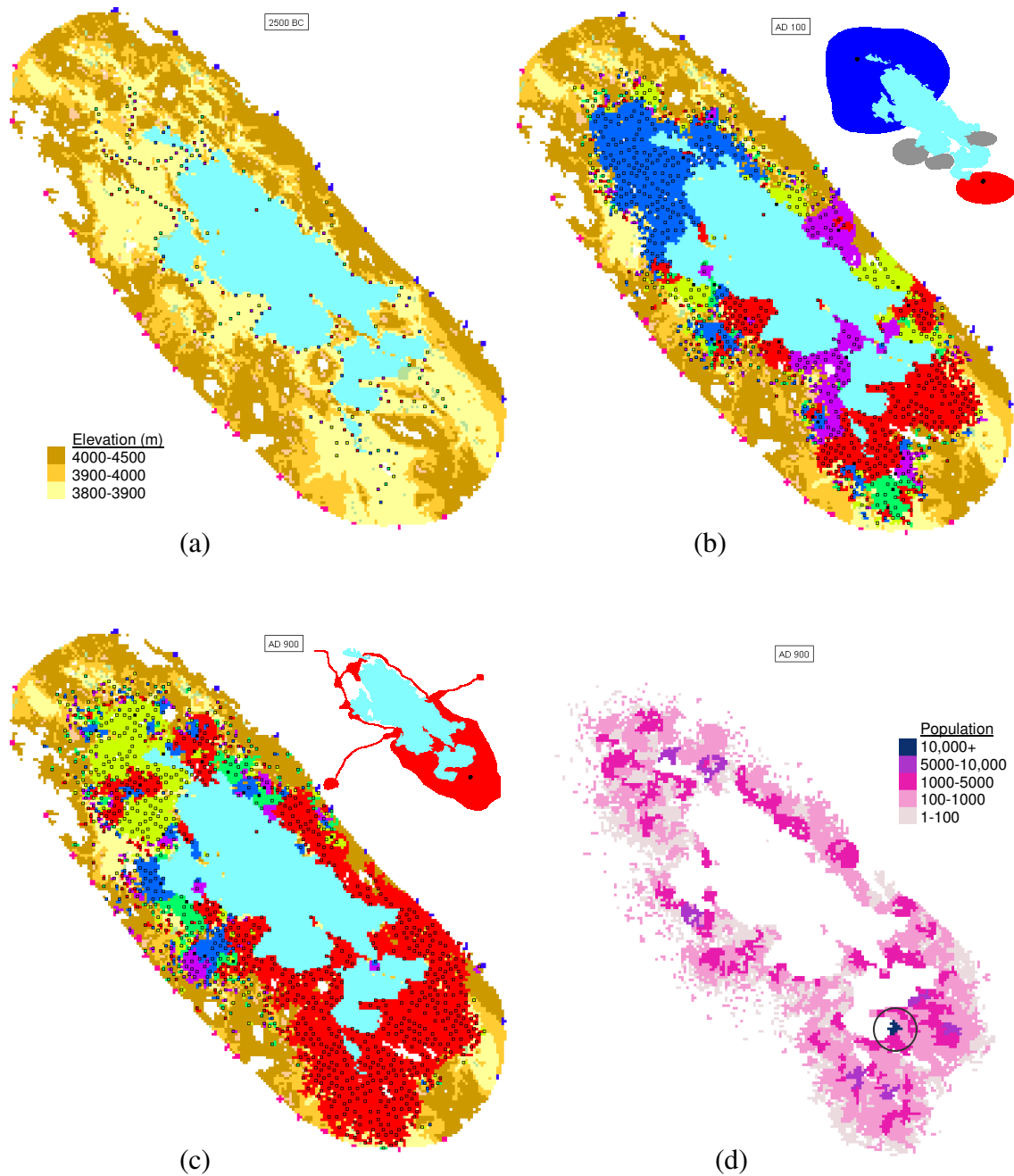


Figure 4 : Simulation Displays: (a) 2500 BC (step 1) initial settlement pattern; (b) AD 100 (step 2600) Polities at Pucara's peak; (c) AD 900 (step 3400) Polities at Tiwanaku's height; (d) AD 900, darker colors indicate dense population of regional centers

Large Polities are shown as continuous areas of color in Figure 4 (b) and (c) overlaying the Patches to indicate Settlements under leadership of the same Chief. As in Figure 3, a Polity's center Settlement is denoted as a solid black Patch. (The color of a Polity has no significance other than to show the borders with adjacent Polities.) Initially each Polity consists of a single independent Settlement. A multi-Settlement Polity forms and grows when its Settlements have no room to expand, at which point the Chief will

attempt to gain control over additional Settlements and crop fields from a neighboring Chief. Polities also may fission if the Chief cannot maintain control over internal factions. Further, a Chief's power in this continuing internal and external competition will be increased if he has access to commodities for trade from outside the basin via passes through the mountains to the east and west. These passes are shown in Figure 4 (a) as red or blue Patches on the periphery of the basin.

The run pictured here clearly shows the emergence of the long-term, basin-wide pattern of polity consolidation and fission alluded to in the Introduction. In this scenario a large Polity developed at the north end of the lake, Figure 4(b), but fissioned by step 2700 corresponding to AD 300, which is about the time that Pucara is believed to have collapsed. The north remained unconsolidated thereafter. An even larger Polity then developed in the south, Figure 4(c), remaining stable through step 3400 (AD 900, Tiwanaku's peak). We refer to this pattern throughout the paper as: South After, North Before Collapse (SANBC, pronounced San-B-C).

Population of the Settlement that controls each Patch at step 3400 is displayed in Figure 4(d). (Patches covered include a Settlement's adjacent fields in addition to its residential areas). The darker colors, as well as the dominant south Polity, circled, mark the high population regional centers and primary center that emerged throughout the basin.

Model Structure

The macro-level chronology described above was not explicitly modeled. Rather it emerged as a consequence of micro-level interactions between autonomous agents. Following is a top level description of the model structure. Additional features are described as needed in the next section on Results. The Appendices contain full specifications of the ecological properties and agents' behavioral rules.

The tables in the current section compile more specific structural properties. Table 1 summarizes basic assumptions about agents' behavior. Table 2 defines the model parameters including calculations dependent on them.

Model Agents and Rules

Rules were defined that reflected the hypothesized causal factors of: agriculture, migration, competition, and trade. Consistent with the guidelines for pattern-oriented modeling (Grimm 2005), the rules initially were created with the four patterns in mind that were expected to emerge: recycling Polities, population concentration, locations of dominant population centers, and basin-wide population levels and growth rate (the four patterns and causal factors presented in the Introduction). After the SANBC pattern was discovered, all five patterns guided subsequent rules refinements.

The model consists of one type of ecology object, Patch, and four types of agents: Settlement, People, Chief, and Polity. Each type of agent has a set of condition::action rules that control its behavior. Conditions refer only to an agent's local knowledge and produce similarly local actions. The conditions are dependent on: 1) the properties of the Patch on which the agent is located or a limited neighborhood of surrounding Patches, and 2) its own state and that of neighboring agents. "Local" and "neighboring" are defined on different scales for Polities and Chiefs however. In the case of neighboring Polities (having bordering Patches), the respective Chiefs know each other's strength and

can be in conflict even though their home Settlements may be separated by many intervening Patches and other Settlements. Similarly a Chief has knowledge of all Settlements in his own Polity each of which may be some distance from the Chief's home at the Polity's center Settlement.

The actions and properties of Patches and the four agent types are summarized below with the details in Appendices A and B respectively:

- 1) Each year a Settlement decides: how large a crop to plant, if it needs to colonize additional Patches for planting, or spawn a new Settlement, or abandon its site altogether, and whether to resist the Chief's leadership if part of a complex Polity.
- 2) A Settlement is the lowest level of aggregation; families or individuals are not modeled. However some portion of a Settlement's People can decide to migrate to another Settlement. Each People also determines its own net birth rate.
- 3) A Chief dictates his share of the harvest, can move his Center to another Settlement, tries to obtain remote commodities needed for trade, can attempt to take over the Settlements of another Chief, and will assert his leadership when it is resisted by a Settlement in his Polity.
- 4) A Polity consists of all the Settlements led by one Chief. Adjacent Polities can be repeatedly consolidated to form larger Polities. A Polity also can fission into independent Settlements.
- 5) Patches have properties and relationships that can constrain the agent's actions listed above. For example, the population that a Settlement can support is impacted by crop yield, which is a function of rainfall, which in turn is dependent on the location within the basin of each Patch. Properties of each Patch include: xy coordinates, elevation, presence of river or spring, geographic zone, and types of agricultural land use supported.

Basis for Model Rules

What is the basis for the Titicaca micro-level condition::action rules? The preferred source for agent behavioral rules was empirical micro-level data from archaeological surveys and excavations of the Titicaca basin dated to the time period being modeled. For example one of the conditions required for a Settlement to split, i.e., create a new Settlement, is that a vacant site exists no closer than 3 km to any existing Settlement. This was based on a survey of the Taraco Peninsula at the south end of the lake where the typical distance between settlements was found to be 3 to 5 km. (Bandy 2001, 2004a).

Rules concerning agricultural productivity and methods make use of appropriate modern-day data specifically from the Titicaca basin, if they are also valid in the prehistoric context, such as camelid meat yields (kcal/ha) as detailed in Appendix A. The per-hectare crop yields (kcal/ha) and labor requirements for different agricultural methods (person-yrs/ha), also given in Appendix A, are based on the recreation of prehistoric practices as part of small-scale demonstration projects in the basin during modern times. (Erickson and Candler 1989, Graffam 1992, Treacy 1989)

Table 1 lists the major operational assumptions on which agent behavior rules were based. Where data specific to the Titicaca basin were not available, we made assumptions that were "ethnographically plausible" (Axtell 2002) for pre-state agrarian societies in general during comparable time periods.

Table 1: Operational Modeling Assumptions

Settlements

People lived in permanent nucleated settlements that were located near a fresh water source.
 Residential area was proportional to population size.
 Area adjacent to a residential area was colonized for planting or pasturing and when fully used was expanded if land was available.
 A settlement split off a new Settlement when its population reached some level if land was available.
 A settlement was abandoned when its population fell below some level.

Population

People migrated from economic need to economic opportunity over the shortest possible distance without regard to polity boundaries.
 People were killed in conflicts with people from other settlements.
 There was no significant migration into or out of the basin during the study period.
 There were no significant fluctuations in population due to epidemic disease during the study period.
 Population density was higher in areas with a higher ecological carrying capacity.
 In the absence of food shortages, migration, and conflicts, a settlement's population increased in proportion to its then current level.

Agriculture

Fields were fallowed periodically to maintain crop yield.
 There was some level of labor specialization, i.e., other than food production.
 An independent settlement planted or pastured only enough area to support its then current population.
 Settlements that were part of a multi-settlement polity optimized land use and maximized area planted or pastured.

Political followers

People submitted to political leadership but also could resist it.
 People provided some level of labor and/or agricultural produce to their leader but also could withhold it.

Political leaders

Each polity began as an independent settlement with a political leader.
 A leader's strength was his overall potential to influence others, be it his ability to: persuade, befriend, reward, organize, marry, protect, intimidate, attack, defend, etc., or any combination thereof.
 A leader attempted to maximize his strength, which was:
 directly related to the size of the population of his own polity,
 enhanced by access to remote trade goods,
 diminished as the distance increased between himself and the target of influence.

Political dynamics

Adjacent settlements came into conflict when no empty land remained for expansion to accommodate population growth.
 Overt conflicts were suppressed between settlements in the same polity.
 Conflict between settlements could be resolved by consolidation into a multi-settlement polity.
 When polities consolidated, the stronger of the original competing leaders usually became the leader of the new polity; the opposing leader was permanently removed from leadership.
 The consolidation process continued between multi-settlement polities.
 Consolidation occurred through many processes: overt conquest, allying with weaker partners, switching allegiances, encroaching on buffer zones, etc.
 A polity's leadership regime continued from generation to generation until it lost a consolidation conflict.
 There was competition between factions within a multi-settlement polity.
 A faction could resist its polity's leader, which if successful caused the entire polity to fission into independent settlements.

Many of the operational assumptions in Table 1 flowed from more abstract generalizations, e.g., agriculture intensified in chiefdoms to generate a surplus that supported craft and other specialists for the benefit of the Chief and his factions (Earle 1997). Based on the ethnographic record from different parts of the world, this sweeping statement was operationalized by secondary assumptions: in chiefdoms farmers optimized the cultivatable area each year. Conversely outside of chiefdoms only areas required to support the current population were planted. Additional subsidiary assumptions are included with the detailed rules in Appendix B for instances in which the corresponding assumptions from Table 1 are not obvious.

These assumptions reflect how we abstracted reality for modeling purposes. Table B-1 in Appendix B lists further simplifications made in the course of rendering assumptions into detailed agent rules. The assumptions also reflect choices we made on a number of issues that have rich histories of debate among archaeologists. For example, we accept optimization behavior on the part of the modeled populations, recognizing that cultural norms and local historical trajectories often deviate from such a generality. Full enumeration and discussion of the opposing arguments for each such supposition are beyond the scope of the current paper.

Model Parameters

The strength of certain relationships could not be determined from any of the sources described above; in these cases, rules were implemented with parameters, which allowed us to observe the impact of different strengths during simulations. The eleven parameters listed in Table 2 define the parameter space.

Our primary interest during the simulations reported here was finding parameter values that minimized the error between model output and patterns indicated by the record. Said another way, we wished to maximize the detection of the patterns. The Model Dynamics section below touches on a secondary interest in these parameters: a simulation experiment in which parameter values were varied to gain understanding of internal model dynamics.

The values shown in Table 2 were established using optimal parameter estimation (also called calibration or tuning). This technique has been used widely in physical as well as social sciences, from climatology (Intsiful 2004) to economics (Winkler 2001), for determining model parameter values. In all such applications an objective function was defined which measured the difference, or similarity, between model output and a reference set of empirical data produced by the real-world system being modeled. The parameter space was then searched to find a set of values that optimized (minimized or maximized) the objective function (Press 1992, p 446).

The objective function used for the current model was evaluated as the frequency of runs within a batch of runs that matched the archaeological record. A run was classified as matching if multiple numerical measures collected during the run met a set of quantitative criteria, which are defined in the Results section below. Sets of values throughout the parameter space were tested until the maximum match frequency was achieved (which could be one of many local maxima). Based on the results of 2400 value-testing runs (execution time of about 320 hours at 8 min/run) the optimal parameter values listed in Table 2 were estimated. The Acceptable column in the table shows the

range of values surrounding the optimal for which match frequency remained relatively constant, given that all other parameters were set to their optimal values.

Table 2: Model Parameters and Sensitivity Analysis

Parameter	Definition	Optimal Value	Acceptable Values
Birth rate	Annual rate of births less natural deaths as a percent of each Settlement's population assuming no food shortage; population decreases at twice this rate during each year of a food shortfall in a Settlement (transitions smoothed by a 5 yr. running average).	0.18%	0.16% -0.18%
Defense Multiplier	Multiplier of a Chief's strength when defending his home Settlement	2.1	2.1, 2.3
Force-at-Distance (Fad) Decay Rate	Percent compounded reduction in Chief's strength for every km between his Center and the point where strength is applied. Strength at distance D km = Strength at Center/(1+Fad rate) ^D	10%	3% - 13%
Kill ratio	Determines the lethality of competition between Chiefs. Average number of Chief's people killed by each combatant of the competitive Chief in a Conflict	0.1	0.08 - 0.1
Migrate Rate	Annual rate People migrate to a larger nearby Settlement for opportunity	0.018%	0.018%-0.030%
Probability Resist	Probability during each step that a satellite Settlement will resist the Chief and attempt to secede when it cannot expand by splitting or colonizing.	0.002	0.002
Rain Constant	Determines relative impact on crop yield of north-south rain gradient; actual Yield = normal Yield*(1 + Rain Constant*NS rain index)	1.4	1.4, 1.6
Risk Threshold	Highest probability of losing that a Chief will accept when considering whether to start a conflict	0.5	0.3 - 0.8
Stop Resist Strength	Portion of Chief's strength (at his Center) employed to reestablish his leadership over a secessionist Settlement. (Remaining strength committed to maintaining loyalty of other satellite Settlements)	18%	15% - 25%
Trade Constant	Determines impact of trade on Fad Decay rate and hence on Chief's strength at a distance. Range is 0 to 1 where 0 results in no trade impact and 1 gives the maximum impact. r _{fad} = r _{fad0} *(1 - trade access * Trade Constant)	0.7	0.5 - 0.9
Work Days per Year	Maximum number of days available each year for a farmer to farm or a herder to head; includes all irrigation-related labor: assumes 5 work hours a day	270	270 - 365

Testing the parameter space involved running a series of sweeps, sets of multiple run batches, at a number of points across a range of *realistic* values for each parameter. After each sweep the value ranges were narrowed until further adjustment no longer increased the frequency of matching runs. In two cases independent data sources were

available as guides to selecting *realistic* parameter ranges. Values considered for the Rain Constant had to produce differences in the modeled crop yields that were within or close to the reported variance of modern potato yields in the Titicaca basin (Orlove 2000).⁶ In the same way, we judged the values chosen for Defense Multiplier to be appropriate for pre-state polities in comparison to its value for early nation-states (Cederman 1997).

We relied on subjective judgment to set *realistic* ranges for the remaining parameters. This was possible because we were able to define parameters having a real-world meaning. As shown in the definitions given in Table 2, each parameter is surrounded by the context of model variables with familiar and meaningful dimensions of: kilometers, hectares, persons, calories, and years. These parameters are not arbitrary coefficients, as in some models with parameters detached from reality that can take on a wide range of values from minus to plus infinity.

Simulation Results

The simulation displays in Figure 4 graphically illustrate the overall, macro-level, spatial and temporal scenario generated by the model and how it compared with the archaeological record during the study period. How often simulation runs matched the record was determined for 540 runs. For each run the model parameters were set to the optimal values (from Table 2) and stochastic relationships were initialized to a unique random seed. Of the 540 runs, 34% produced a scenario that matched the record by virtue of matching all five patterns (from the Introduction, plus SANBC).

Runs were counted as matching all the patterns if at least 11 of 12 criteria were met that were established to evaluate a run and classify it as one of several alternative prehistories. Runs meeting 11 criteria were included in the match count because, upon visual inspection, many of these runs appeared to have a strong pattern match even though one measure was marginally outside the limit of its criterion. A few runs that met just 10 criteria also appeared to have been good matches, but these were not classified as such.

Table 3 shows the percent of the 540 runs that met 11 or 12 criteria along with the frequencies of several alternative scenarios based on which criteria were not met.

Table 3: Frequency of jointly meeting criteria for alternative scenarios (540 runs)

Scenario	Frequency
Met at least 11 of 12 criteria	34%
(15% met all 12 criteria)	
Population too large or small	1%
Basin not consolidated	32%
Not at N and S ends of lake	8%
North always larger than South	17%
South always larger than North	2%
Unclassified	6%
Total	100%

The 12 match criteria are listed in Table 4 along with the corresponding emergent patterns (from Introduction). Each criterion is associated with a quantitative measure that

⁶ Tubers were the primary staple plant crop in the Titicaca region.

was collected during simulation runs. Each criterion also has a constant upper or lower limit or range of values, as well as a fixed time step or step range during which the measure is compared with the limits. This comparison determines if the criterion was met. Values for these measure limits and step ranges were chosen to achieve a balance between preventing false positives and accommodating acceptable variability in the patterns. Appendix C gives detailed specifications of all the criteria.

We opted for this relatively simple approach to pattern recognition even though some runs were misclassified. From past experience attempting to increase the accuracy with more sophisticated processing would have been an enterprise of rapidly diminishing returns. Rather we assumed that over many runs, errors in classification counts due to misses would tend to be canceled out by false hits.

Table 4 also shows the frequency with which each criterion was met individually in the same set of 540 runs. The high frequencies shown indicate that, in addition to jointly matching all the patterns including SANBC, the model consistently matched each of the four originally expected patterns by virtue of meeting the individual criteria.

Table 4: Frequency of independently meeting each criterion (540 runs).

ID	Criteria	Pattern(a)	Met(b)
1	Final Population Close to Expected	Basin pop. level /growth	99%
2	Early Population Close to Expected	Basin pop. level /growth	100%
3	N Polity Larger than S Before	SANBC (c)	92%
4	S Polity Twice size of N After	SANBC	48%
5	N Polity Stable Before (d)	Recycling, SANBC	94%
6	N Polity Unstable After	Recycling, SANBC	65%
7	S Polity Stable After	Recycling, SANBC	76%
8	Center at N End of Lake	Centers' location	98%
9	Center at S End of Lake	Centers' location	83%
10	Basin Consolidated After	SANBC	62%
11	Early S Polity Not Concentrated	Pop. concentration	83%
12	Late S Polity Very Concentrated	Pop. concentration	82%

(a) Emergent pattern (see Introduction) associated with each criterion

(b) Same batch of 540 runs as the joint frequencies in Table 3 (data file: posa3)

(c) SANBC stands for South After, North Before Collapse (see Overview)

(d) Stable/Unstable refers to the degree of Polity recycling

The ID numbers in the Table 4 are referenced within Figure 5 below, which illustrates the significance of each criterion related to the SANBC pattern and the intertwined pattern of recycling. The other criteria listed are similarly referenced throughout the remainder of this section in topics discussing each of the three other emergent patterns: basin-wide population levels and growth, population concentration centers, and locations of primary centers.

South After, North Before Collapse (SANBC) Pattern

The bubble chart in Figure 5 is a convenient way to visualize the temporal as well as spatial dimension of a simulation run. Simulation steps are plotted on the horizontal axis of the chart and the vertical axis represents the distance in grid space from the origin of the model grid to the center of a Polity. The area of each bubble is proportional to the population of the largest Polity in the north and south, sampled every 100 years. Each key

feature of this run that matched the archaeological record is referenced by the ID number of the relevant criterion listed in Table 4.

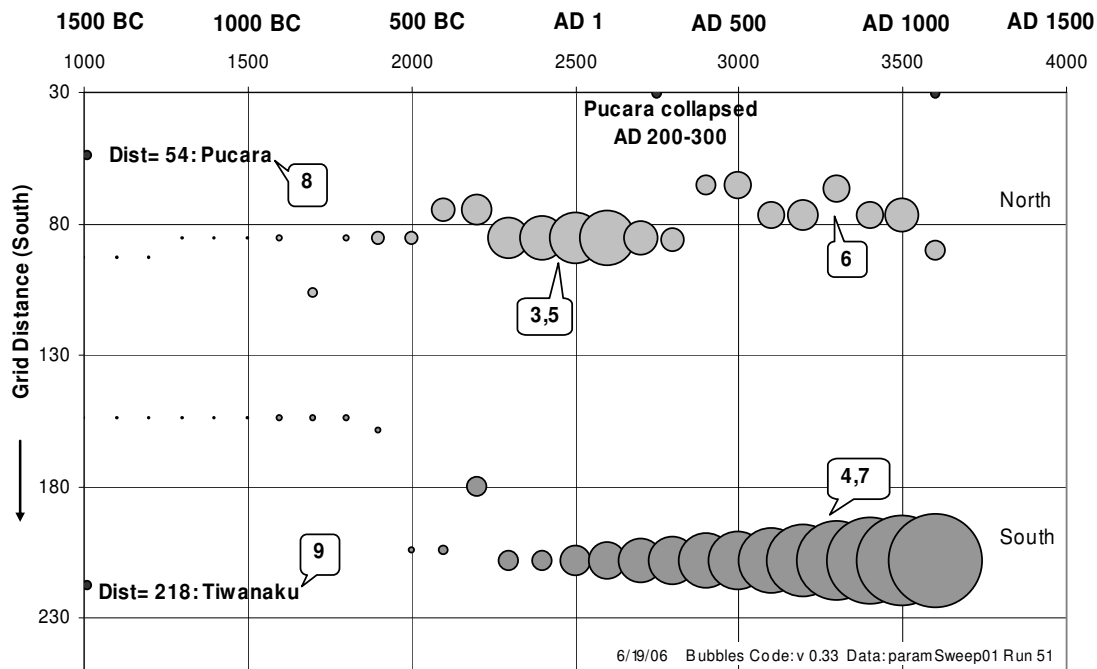


Figure 5: Overall temporal-spatial pattern of a simulation run meeting all criteria for matching the archaeological record; each criterion is referenced by its ID number

In runs that match the record well, as in the one shown here (same run from the Introduction), the North Polity emerges at the north end of the Lake, its center not far from the Pucara site (criterion 8) and stabilizes, i.e., is not recycled, between steps 2100 and 2700 (criterion 5), and by that time is more populous than the largest South Polity (criterion 3). The North Polity fissions shortly after step 2700 and thereafter continues to be unstable (criterion 6) and is less than half the size of the largest South Polity (criterion 4). Meanwhile the South Polity emerges at the south end of the lake, with its center near the Tiwanaku site, by step 2800 (criterion 9) and remains stable thereafter (criterion 7). By step 3400 a significant part of the basin's population has consolidated into one or two large polities (criterion 10, not referenced in Figure 5).

The bubble chart in the Figure 5 shows a run that clearly matched the SANBC pattern. There were also runs that just as clearly did not match. Between these two extremes was a continuum of degrees to which runs matched the pattern. The 12 criteria define a boundary on that continuum between match and no match. We have included as supplementary material the bubble charts and 12 criteria match indicators for each of 100 runs in a typical simulation batch. These have been ordered by the number of criteria met and secondarily by a weighted score indicating how strongly criteria were met.

How likely is it that the pattern from a run classified as a match could have occurred by chance alone? The answer depends on the random process to which the pattern's frequency of occurrence is compared. For the model of Mesa Verde in the American Southwest (Kohl 2000), a random process was defined to compare one pattern,

farm site locations, using a single measure, the distance between empirical data of actual prehistoric site locations and the closest randomly assigned location. These random distance differentials were then compared to the distances between each simulated site and the closest actual site. For the reported runs the simulated sites were markedly closer to the actual sites than were the same number of random points.

Performing a similar comparison for the current model required defining a random process corresponding to 12 heterogeneous criteria measures of five spatial-temporal patterns at different scales and hierarchical levels. Because of these rather daunting requirements, most of the candidate processes considered were fraught with varying degrees of dueling interpretations, arbitrary assumptions, complex logic, and opaque calculations. While not fully immune to all of these maladies, the random process selected for the following comparison seemed fair (fair in the probabilistic sense) and was intuitive.

Suppose a coin is tossed for each of the 12 criteria to determine if it is met or not; the probability of meeting one criterion is 0.5. Since each toss is an independent event, the probability of jointly meeting all 12 criteria is $0.5^{12} = 1/4096$ or about 0.025%. Compare this probability of meeting all 12 criteria by chance to 15% from Table 3, which is the probability of the model meeting all 12 criteria. (The strict match definition, which requires meeting all 12 criteria, is used here to allow a simple probability calculation.) Thus, based on this comparison, the current model was 600 times more likely to have produced a simulation run matching the archaeological record than by chance alone (15% / 0.025%).

Basin-wide Population Levels and Growth

Population is a central model variable that impacts and is impacted by many model elements. The importance of explicitly including a realistic population pattern as a model expectation was made clear by the Long House Valley model, mentioned in the Introduction (Dean et al 1999). The simulated population levels for an early version of that model were reported to be 500% larger than indicated by the corresponding empirical data. (However a later version of the model produced remarkably accurate simulated population levels (Axtell 2002).)

The population of the Titicaca basin in prehistory is still hotly debated. Our best estimate is that about 500,000 people inhabited the entire basin at Tiwanaku's peak in AD 900 (Tschopik 1947; Stanish 2003).

We also accepted an estimate that the total basin population grew on average 0.1% each year, which has been reported as a typical average rate for non-industrial agricultural village populations worldwide (Bandy 2001). Titicaca settlement size data from two survey areas, plotted in Figure 6, spanning 1500 BC to AD 1500 were consistent with a 0.1% average rate.

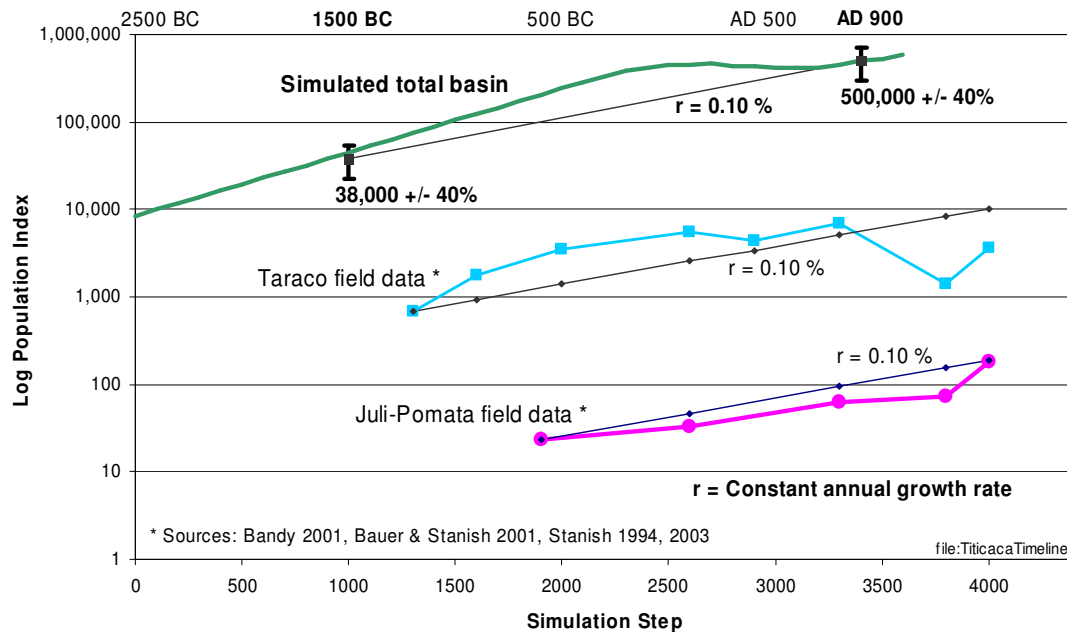


Figure 6: Basin population levels and growth rate; simulation results compared to field data

In order to reach a population level of 500,000 by AD 900 growing at a constant annual compounded rate of 0.1%, the population in 1500 BC must have been near 38,000. So a simulation run was considered to match the population growth indicated by the archaeological record if the total basin population at 1500 BC and AD 900 were within plus or minus 40% of 38,000 and 500,000 respectively. These two population ranges and associated dates constitute match Criteria 1 and 2. The plot at the top of Figure 6 shows these error ranges and the basin population (the sum of all individual Settlement populations) from a typical simulation run. Nearly all runs matched these two criteria for population levels and the implicit average growth rate of 0.1%.

The expected mean rate of 0.1% should not be confused with the parameter Birth Rate optimal value of 0.18% (Table 2). This parameter is one of many factors that determine the time varying simulated populations of each Settlement at each time step.

Centers of Population Concentration

Figure 7 shows the shift in the population distribution within the Tiwanaku valley from relatively uniform and autonomous villages in the Formative period (on the left) to a center with a high population concentration, surrounded by much smaller satellite settlements, when the Tiwanaku polity was at its height (on the right). The charts at the top of the figure show field data for the area of each settlement (commonly used as a population index) plotted against its rank order by size (McAndrews et al. 1997). A logarithmic scale was used for both axes so the size-rank data could be easily compared with a Zipf distribution, which appears as a straight line in a log-log plot, hence is also referred to as a log-normal distribution (also called a power law distribution). Because of its rather remarkable universality, this linear relationship has become a standard of comparison for rank size distributions of quantities as disparate as: English language

word frequencies, the population of cities, the size of business firms, and web site popularity (Axtell 2001).

Criteria 11 and 12 require the Settlement rank-size distribution emerging from the model to be spatially and temporally similar to Figure 7, top. The bottom of Figure 7 plots Settlement size data from a simulation run that clearly meets these criteria. We defined a simple measure to quantify the degree of similarity of rank-order distributions between simulation results and the published field data (zPop in Appendix C). It measures the net percent of Settlements surrounding the dominant south Polity that have populations above or below the expected log-normal distribution. For example if all Settlement populations were above the Zipf line (as in Figure 7 bottom left) zPop = 100% and when all were below the line (bottom right) zPop = -100%. This measure was collected each run during step intervals corresponding to the early and late date ranges and covering the same spatial range as the field data plotted in Figure 7, top.

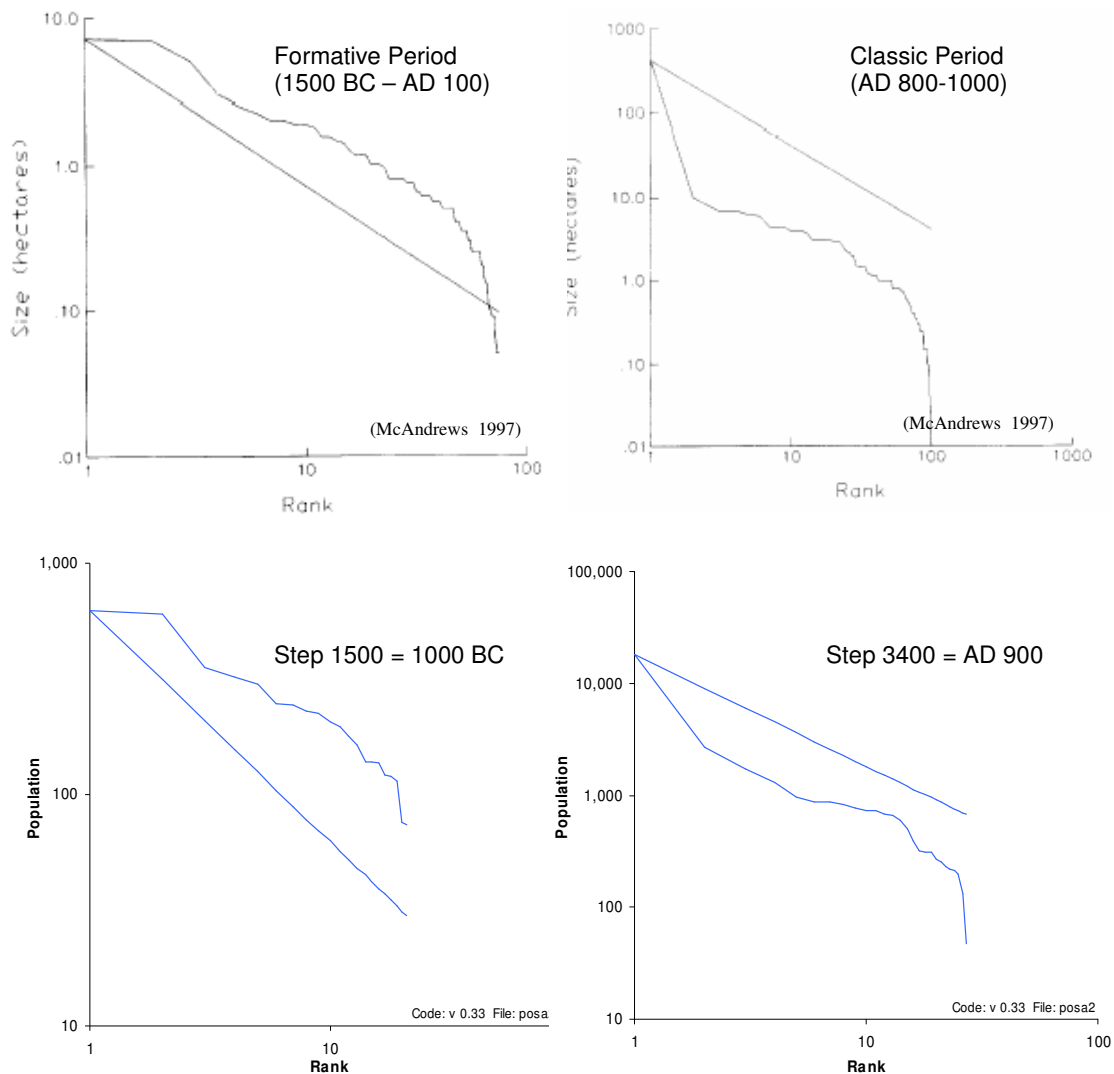


Figure 7: Comparing field data and simulated rank-size distributions: Top - Tiwanaku Valley field data (McAndrews et al. 1997); Bottom - dominant South Polity from sample simulation run

Based on these measures, the population distributions of the simulated Settlements in the dominant south Polity appear quite consistent with field data collected in the Tiwanaku valley dated to corresponding time periods (82% and 83% respectively met Criteria 11 and 12 in Table 4). More generally, settlement rank-size distributions shown in Figure 7 appear very similar to those for a number of other pre-state and early-state societies in different parts of the world at comparable stages of integration (Johnson 1980).

Locations of Primary Centers

The centers of the dominant north and south Polities consistently emerged during simulations at the north and south ends of the lake (criteria 8, 9 met 98%, 83% respectively). These results were typical of all runs over a wide range of parameter values. The xy plot in Figure 8 shows the location of centers for the dominant Polities in the north (small black squares) and south (small black triangles).

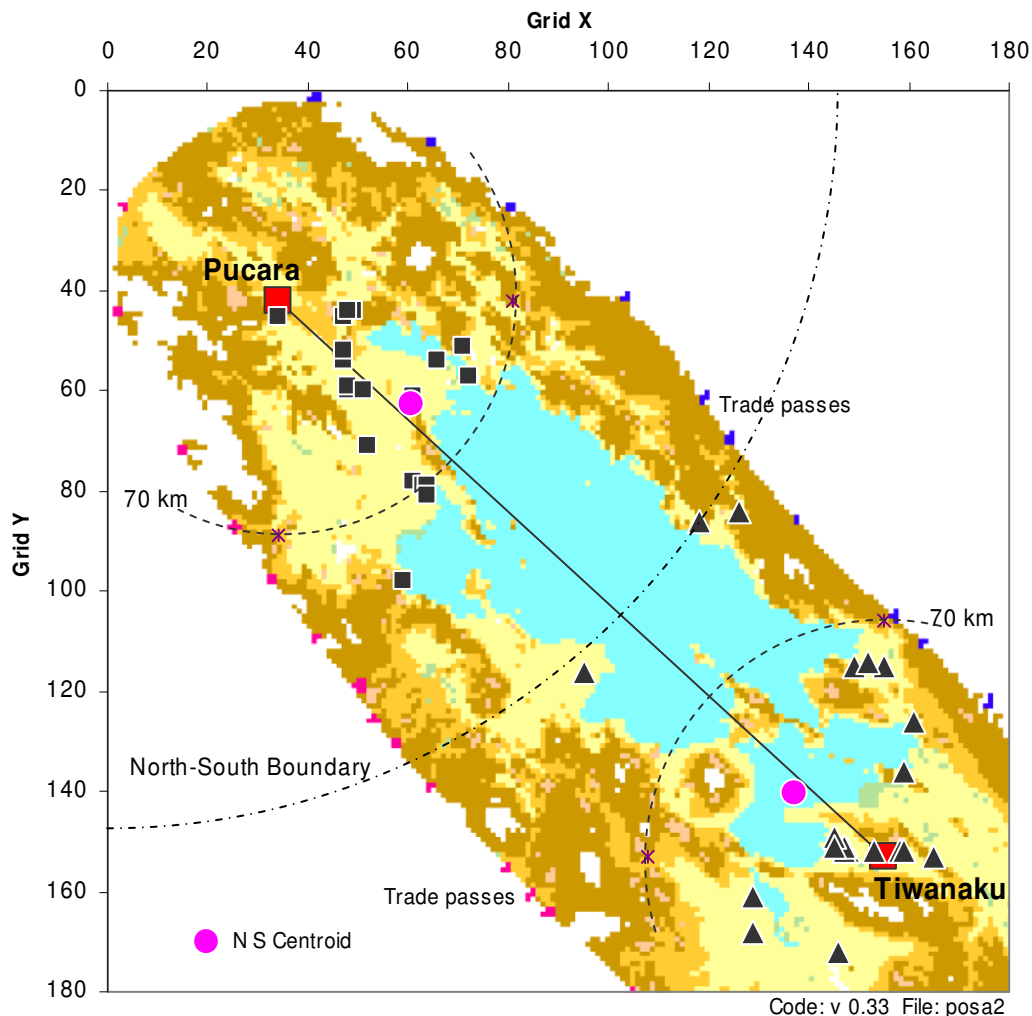


Figure 8: Locations of dominant Polity centers in the north (small squares) and south (triangles), from 40 simulation runs, during periods corresponding to the years when Pucara (AD 100-300) and Tiwanaku (AD 900) were at their peaks – compared to their actual site locations (large red squares). Solid pink circles are north and south centroids.

The north and south Polity locations are shown at different time steps corresponding to the dates when Pucara and Tiwanaku were thought to have been at their respective peak size and influence. Simulated Polity centers were considered to be at the north or south end of the lake respectively, if they were located within 70 km of the presumed sites of Pucara and Tiwanaku (large red squares) during periods when each was at its height. This plot illustrates how consistently Polity center locations were within the limits of criteria 8 and 9.

Model Dynamics and Structural Realism

The previous section presented the frequency of occurrence of simulation runs that jointly met at least 11 of these 12 criteria for matching all five temporal-spatial patterns. Results were also given for independently matching criteria for the original four patterns as measured by: population level and growth rate, Settlement rank size distribution, and dominant polity locations. In other words we tried to answer the question: how well did the simulation results match the empirical archeological record?

This section considers three questions regarding the model's internal structure and its behavior:

- 1) Model dynamics – how do the interactions of model agents explain the causal links between the micro-level rules and the macro-level patterns?
- 2) Structural realism – how well does the internal model structure correlate to that of the real-world system being modeled?
- 3) Equifinality – could another model have produced the same results?

Model Dynamics

The following analyses identify the central dynamics of the model. The first topic is an examination of the dynamics that result in the location of dominant Polities. Next, the key dynamics responsible for the recycling and SANBC patterns are delineated.

Location of Dominant Polities – Observations of the model display (Figure 4) during simulations suggested that one important factor influencing the location of simulated dominant Polity centers in the basin was geography. For purposes of analysis another way of saying the same thing is: the probability of a dominant center occurring at any given location is a function of the basin's geometry. To understand this relationship remember that in the model the continued existence of a Polity and its dominance are directly related to the strength of that Polity's Chief whose home is at the location in question.

Consider the locations of four chiefdoms in an idealized rendering of the basin as two concentric ellipses shown in Figure 9. The ellipses were drawn to scale of the actual basin perimeter and lakeshore. On the left, initially assume there is no lake and the four chiefdoms are equal in strength of 1000 arbitrary units. Also assume a uniform population distribution. The Chiefs' strength diminishes with distance equally in all directions, so the Polities have the shape of a circle with the Chief at its center. Since strength is determined primarily by population, each Chief's strength is proportional to the area of his Polity. Thus there is an equal probability of a Polity center occurring at each of the four points shown in the drawing on the left.

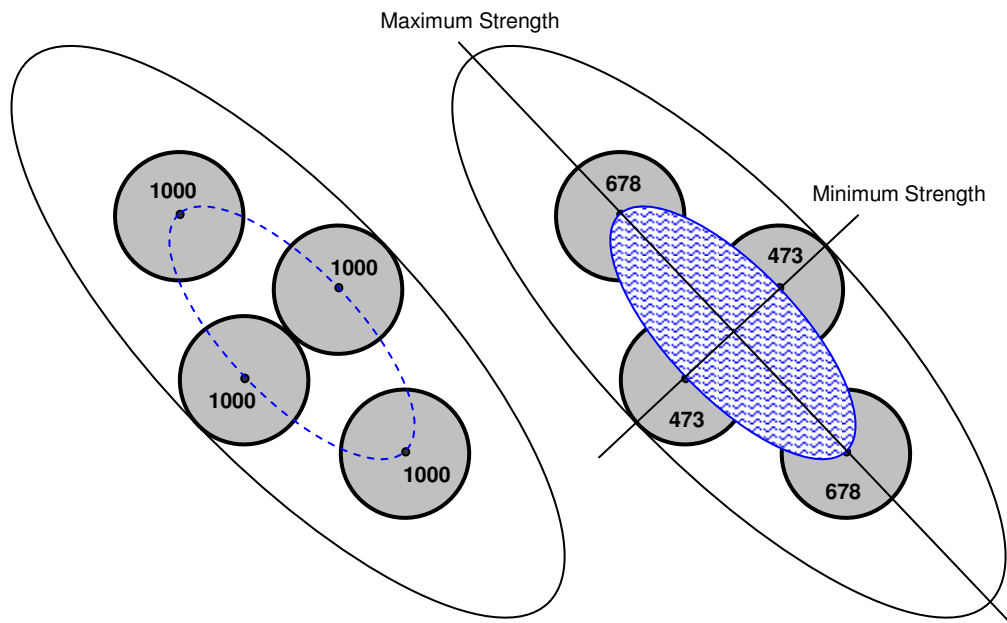


Figure 9: Idealized basin shows the impact of geography on location of dominant Polities

The right of Figure 9 shows what happens with the lake in place: the areas of influence for the north and south Chiefs, and hence their strengths, are much larger than those to the east and west. If we similarly determine the strength for each non-lake point in this abstract basin, the center locations of maximum strength surround its major axis running north-south and those of minimum strength converge on the east-west minor axis. Thus considering only the geometry of the basin the most likely locations of dominant Polities are at the north and south ends of the lake and are least likely to occur on the east and west sides.

The simulation results shown in Figure 8 are consistent with the above analysis of the basin's geography. The unweighted centroids (average x and y shown as pink filled circles) of the simulated north and south centers are nearly coincident with the hypothesized major axis of the lake. It is intriguing that the actual locations of both Pucara and Tiwanaku (large red squares) are on the lake's major axis as well.

It is also interesting to note that the analysis above is an example of a modeling strategy recognized as particularly useful in ecology (Grimm 1999, p 140) where the results from a bottom-up simulation model in Figure 8 and those of a top-down mathematical model, Figure 9, mutually reinforce each other.

The simulation results also suggested that access to long distance trade was another important factor favoring the emergence of dominant polities at the north and south ends of the lake. In our model, access to trade increases the strength of a Polity's Chief. Any Chief has access to any trade pass within a given distance range as long as at least one path to the pass is not blocked by the lake or by other Polities. Blocked by the lake, east and west Polities have access to fewer passes on average than do Polities to the north and south. Hence the strength potential of east and west Chiefs is limited by lack of

trade access, compared to that of Chiefs north and south. (See Chief, Trade Access in Appendix B for modeling details.)

Table 5: Influence of Trade on the Location of Dominant Polities

Trade Constant Parameter	% Runs with Dominant Center in East or West region ^(a)	Total Runs
0.0	32 %	60
0.3	27 %	60
0.5	22 %	60
0.7	12 %	60
0.9	12 %	60

Chi square (4 df) = 11.7 $p \leq 0.025$

(a) Dominant south center is between N-S boundary and 70 km limit from Tiwanaku when Criteria 9 not met. Refer to Figure 8.

Table 5 clearly indicates the simulated effect of blocked trade access on east and west Polities, as distinct from the impact of the basin's geometry discussed above. The Trade Constant parameter determines the relative impact that trade has on a Chief's strength (refer to the Trade Constant parameter in Table 2). Table 5 lists values of the Trade Constant for batches of 60 runs and the percent of runs from each batch in which the dominant south center develops in the lower east-west region, the region in Figure 8 above the 70 km limit for criteria 9 and below the N-S boundary. When the impact of trade on strength was completely turned off by setting Trade Constant to 0, the dominant south center emerged in the east-west region 32% of the time. As the importance of trade on the Chiefs' strength was amplified by increasing the Trade Constant, the percentage of east-west centers steadily dropped to 12%.

Emergence of the recycling and SANBC patterns – To conclude the analysis of model dynamics, consider Figure 10 that shows conceptually the probability a Polity will fission (y axis) as a function of the number of Settlements it contains.

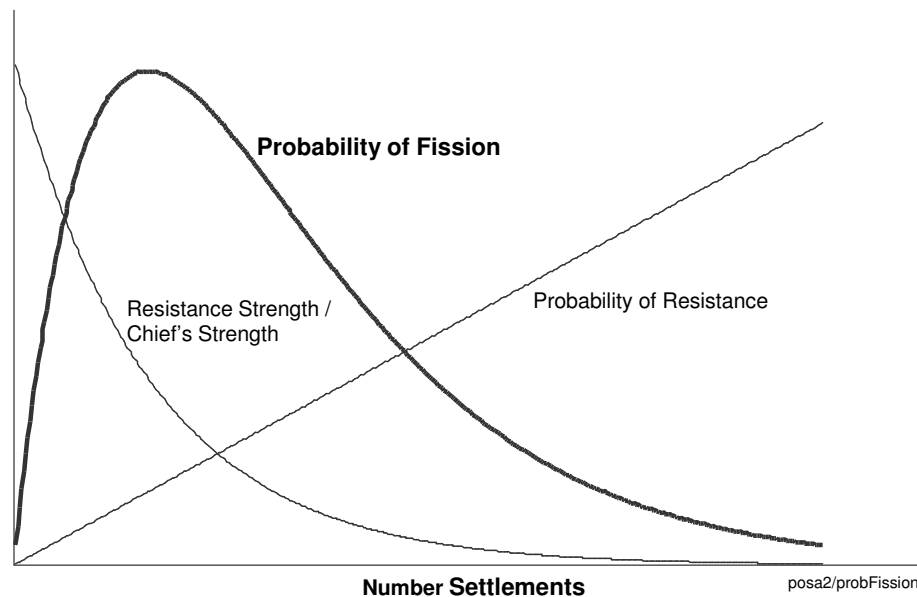


Figure 10: Opposing factors determine the probability that a Polity will fission

In our model the probability that one Settlement will resist the Chief during each step is a fixed constant. So as a Polity incorporates more Settlements the likelihood increases that at least one of its Settlements will resist. At the same time a resisting Settlement's relative strength is decreasing, compared to that of the Chief's. That's because the Chief's strength grows as the number of Settlements in his Polity increases. As the relative strength of the resisting Settlement decreases, the probability that the Settlement can prevent the Chief from reasserting his leadership also decreases. These two opposing forces come into balance at some point causing the Polity's fission probability to peak and then decrease as more Settlements are added. For a Settlement belonging to a Polity that continues to grow past this critical mass, such as the south Polity in Figure 5 bubble chart, seceding becomes less and less likely. A Polity that fails to surpass this critical size most likely will continue to be recycled as did the north Polity in Figure 5. We will have more to say about the fission probability function in a future publication.

Based on the fission probability curve, a plausible chain of causality during the course of a simulation run can be constructed leading to the emergence of recycling and ultimately the SANBC pattern:

- 1) The north end of the basin begins consolidating sooner than the south because of higher population density and attendant competition. This is due to greater rainfall in the north reflected in the model as tighter initial Settlement spacing and continuing higher crop yields for north Settlements as compared to the south.
- 2) The size of the consolidating north Polity continues to grow until it approaches the peak of the fission probability curve, at which time it is likely to fission. Continued recycling then follows because of self-reinforcing feedback: recycling is attended by continued conflicts causing population leveling among competitors so that one Polity cannot dominate, which would have to occur in order to break the cycle.
- 3) Meanwhile the south begins consolidation at a relatively slower pace and hence less conflict attrition and population leveling than in the north. The delay in becoming highly competitive allows population of south Polities to reach a level where consolidation proceeds rapidly once the process starts. This carries the dominant south polity quickly past the peak of the fission probability curve so that recycling becomes less and less likely. Population concentration driven by migration, both intra and inter Polity, also plays a role in decreasing the fission probability of the dominant Polity by shifting strength from satellites to center Settlements over time.

Structural realism

The antecedents to this discussion appeared beginning in the Model Structure section with a list of the basic modeling assumptions, followed by a complete specification of the agent rules in Appendix B, and finally an analysis of the internal model dynamics immediately above. The current model has a number of characteristics recognized in pattern-oriented modeling as indicators of high structural realism (Grimm 2005):

- Started building the model while keeping in mind multiple patterns observed in nature: the four expected patterns listed in the Introduction
- Modeled at different hierarchical levels and scales: multi-agents of Peoples inhabiting Settlements that are constituents of Polities
- Looked for patterns in simulations at different hierarchical levels and scales: the SANBC pattern superimposed over the four expected patterns
- Calibrated model parameters to reproduce multiple patterns simultaneously – estimated parameter values by maximizing frequency of meeting multiple pattern match criteria

Grimm uses one of the American Southwest models cited in the Introduction to illustrate another important characteristic. The Long House Valley simulations exhibited two spatial patterns not hypothesized that “...can be considered *independent secondary predictions*, strong indicators of the model’s structural realism.” (2005, p 990). The current model made two such predictions:

Independent Prediction 1: SANBC Pattern – During early full-basin simulations we were quite surprised that in a number of runs, the timing for consolidation and fission of the dominant Polities coincided with the presumed dates for the rise and fall of Pucara in the north followed in the south by Tiwanaku’s ascent. This was a surprise in the sense that, to this point, the model had been conceived and designed without this overarching complex scenario in mind. To the best of our knowledge no prior publications, presentations, or discussions had suggested that the timing of these events in the prehistoric Titicaca basin were part of a meaningful pattern. We had no expectation that the originally hypothesized four patterns would appear embedded, temporally and spatially, in an even higher-level fifth emergent scenario, the SANBC pattern.

After discovering this pattern, we defined the 12 match criteria, including six associated with SANBC, to quantify the similarity of model simulations to the archaeological record. The sequence of these events is important, because a pattern used as the basis for parameter calibration cannot be considered an independent predication (Grimm 2005). However that is not the case here. The previously unrecognized SANBC pattern was first observed in simulations *prior to parameter optimization*. Only subsequently were the associated criteria defined and used as part of the objective function for estimating parameters. Any doubt that prediction preceded calibration should be allayed by considering the impossibility of the alternative, i.e., that we defined the SANBC criteria *before* the pattern’s existence was known to us.

Independent Prediction 2: Conflict deaths per capita – From cross-cultural ethnographic data compiled for 23 pre-state societies (after max and min outliers were removed), the annual war deaths as percent of population ranged from 0.02% to 1.0% with a median of 0.45% (Keeley 1996, p 195). The maximum comparable rate of simulated conflict deaths was 0.21%, which falls within the above range and reasonably close to its median value. The simulated rate was determined for 100 runs from a typical batch. This rate was determined by first calculating the average percentage of conflict deaths for the basin-wide population during each successive interval of 100 time steps. The rate of 0.21% was the maximum over all the intervals. Note that the war death rate, or any other conflict measure, was not part of the parameter estimation objective function. So the independence of this prediction is unequivocal.

Interestingly the maximum war death rate occurred at step 2700, which corresponds to the presumed time of Pucara's collapse, AD 200-300 (that time step also defines the boundary between the Before and After periods in the match criteria). We will have more to say about the temporal pattern of conflict deaths in a future publication.

Equifinality – Equifinality has been a topic of interest in the modeling literature. This is particularly true for hydrology, which is a mature discipline with a long modeling tradition (Bevin 2001). Because of critical applications such as flood management and warning, hydrologists are understandably concerned about equifinality, even where structural differences between alternative models appear to be variations on some well established themes (Butts et al 2004). However in the current model, we are concerned with equifinality of the theme itself.

So the equifinality question posed earlier should be restated: could a model with a *different theme* have produced the same results? Based on our experience considering many candidates for a comparative random process (see above, Results SANBC) we attempted to imagine other models that might simultaneously produce the five patterns discussed throughout this paper. We were able to envision alternative models that could jointly meet a few of our 12 match criteria. However the only models we could conceptualize with the potential to jointly satisfy all twelve were *variations on the same theme as the current model*.

Some level of uncertainty due to equifinality certainly exists for the current model. However we believe the risk, while still present, has been mitigated to a reasonable extent, not only through the use of multiple match criteria, but by the convergence of many corroborating factors, which are summarized in the next section.

Discussion

A highly connected web of plausibility is how we would characterize the current model and its simulation results presented in the previous sections. We find compelling the multiple demonstrations of consistency across and within different scales and hierarchical levels of the model's structure and patterns that emerged in the simulation results:

- realistic constraints on space, time, and energy provided by a geographically and time specific model of the Titicaca basin ecology
- how frequently multi-dimensional, quantitative, empirical criteria were simultaneously met for matching multiple patterns in the archeological record
- explanations of the causal links between micro-level rules and macro-level patterns in terms of the model's internal structure and dynamics
- independent secondary predictions indicative of the model's structural realism

There are also uncertainties associated with the model and simulation results that were identified and assessed in the previous sections:

- possible operation of unidentified dynamics hidden by model complexity
- risk of overestimating match frequencies due to subjective assignment of criteria limit values and time step ranges
- possibility that simulated patterns emerged by chance alone

- possible existence of other models, with different themes, capable of producing the same results

Based on the model's web of plausibility – appropriately chastened by the attendant uncertainties – we venture some insights into real-world processes that could have shaped region-wide settlement patterns and political consolidation of the Titicaca basin. Several specific insights are discussed below.

The early northern segment of the South After, North Before Collapse (SANBC) pattern suggests that the timing of Pucara's decline could have resulted from the accumulation of internal political stresses caused by competition between intra-polity factions. Pucara may have been more fortunate than the typical chiefdom at similar stages of integration and avoided being recycled for an extended period of time. This would have allowed its sphere of influence to grow quite large. However the population of this dominant polity may have failed to reach a critical size quickly enough so that the probability of fission would have decreased with additional growth (see Figure 10).

As an alternative explanation, there has been speculation that Pucara's collapse may have been brought on by a drought that occurred c. AD 1 - 200. (Binford et al. 1997, Stanish 2003). However the current model does not require the presence or absence of dry periods to explain the demise of a Polity at some distinct point in time. Drought *may* have been one of several possible factors that affected the internal dynamics of this polity, but does not directly explain the end of Pucara.

The model also helps to understand why dominant polities were located at the north and south ends of the lake. The bottom-up simulation and complementary top-down abstract analysis suggests that the basin's geometry and access to long distance trade routes can explain this phenomenon (see above Model Dynamics and Structural Realism). It provides an explanation for why no evidence has been found west or east of the lake of a dominant center on the scale of Pucara, much less Tiwanaku, even though these areas were very rich agriculturally.

The simulations further indicate that a state-level polity was more likely to have arisen in the south than in the north, but this occurrence was far from inevitable. Rather, this outcome is one of several possible alternative prehistories for the Titicaca basin, as listed in Table 3. The simulations suggest there was an inflection point at the time of Pucara's collapse in AD 200-300, give or take 200 years. After that point a number of scenarios were possible, but the north Polity was more likely than not to have fissioned and remained unstable thereafter, i.e., recycled (65% frequency of meeting criterion 6 in Table 4). At the same time the south Polity was very likely to have remained stable after this point; in fact it was more than twice as likely as the north to have been stable during this period (76% compared to 35%, the respective frequencies of meeting criterion 7 vs. not meeting criterion 6 in Table 4).

The development and subsequent collapse of pre-state chiefly societies prior to Tiwanaku expansion was very robust in our model. The basic outline of Titicaca basin Formative Period prehistory — the formation of prominent chiefly societies at the north and south ends of the lake and the lack of nascent prime political centers in the east and west — seems almost inevitable given the initial conditions and material realities within which these people lived. However in the period that followed during which Tiwanaku became a state, the model predicts a number of possible final outcomes. Although the

most frequent outcome was a dominant center in the south, the second most likely scenario, a close second at that, was the failure of one or two persistent high population polities to emerge anywhere in the basin (32% Not Consolidated in Table 3). This result is in accord with the supposition that compared to chiefly societies, state formation is much more historically contingent due to factors that we do not or even cannot know.

The model's efficacy has potentially important theoretical and epistemological implications for understanding the nature of political and economic evolution of human societies in general. The efficacy of the model refers to its apparent ability to match the settlement patterns and illuminate the political dynamics of one of the great areas of autochthonous state formation in world prehistory. We were able to simulate three millennia of basin-wide political and economic shifts with recourse to a few ecological factors and attributing to human agents some simple local rules of behavior. The model implies that — beginning with some empirically-derived initial conditions and modeling micro-level agent strategies for agriculture, migration, competition, and trade — simulated patterns can emerge that recreate the broad, macro-level, regional patterns in human prehistory. Such a model provides a multidimensional network of plausible hypotheses that could be tested in the field and with further simulation experiments.

APPENDICES

Appendix A: Properties of Patches for Ecology and Agriculture

Each Patch in the model was assigned a pair of potential agricultural land uses shown in Table A-1 and a corresponding annual nutritional yield and labor cost from Table A-2. In the current model we assumed that all Patches in the same geographic zone had the same pair of uses. The use employed during any one year depends on the political complexity of the Polity currently controlling that Patch. Intensity is low in simple Polities and high for complex ones.

Table A-1: Agricultural Land Use by Geography, Hydrology, and Labor Intensity

Geographic Zone	River Spring or	Low Intensity Land Use (id)	High Intensity Land Use (id)
lake edge	Yes	unraised-cocha (3)	raised-canals (5)
	No	unraised-cocha (3)	unraised-cocha (3)
swampy	Yes	none (0)	raised-canals (5)
	No	none (0)	raised-canals (5)
low pampas (3800 m)	Yes	unraised-cocha (3)	raised-canals (5)
	No	pasture-low (2)	pasture-low (2)
high pampas (3900 m)	Yes	pasture-low (2)	pasture-low (2)
	No	pasture-low (2)	pasture-low (2)
hillside (3900 m and 4000 m)	Yes	terrace-rain (6)	terrace-aqueducts (7)
	No	terrace-rain (6)	terrace-rain (6)
puna (4000 m)	Yes	pasture-high (1)	pasture-high (1)
	No	pasture-high (1)	pasture-high (1)

Elevation and hydrology were derived from digitized 100 meter contour maps of present day southern Peru and northern Bolivia. Lake shorelines, rivers, swamps, and elevation contour lines were manually traced to create a composite thematic map image of the Titicaca basin. This image was read by the model software to create a grid of Patches with the specified dimensions (1.5 x 1.5 km for all data included in this paper). The software imputed geographic zones from elevation and adjacency to the lake. The swampy zone was captured directly from the map image. Agricultural land uses were assigned to each Patch depending on geographic zone and presence/absence of water suitable for irrigation. The land use designations in Table A-1 were based on the archaeological record (Stanish 1994, 2003).

Table A-2 lists estimates for each land use of nutritional yield (from crops and herd animals) and labor requirements in the Titicaca Basin. These data were compiled from published sources and converted to equivalent units of kcal/ha and person-days/ha respectively. The numbers for plant agriculture were based on the recreation of prehistoric farming practices as part of small-scale demonstration projects conducted in modern times.

Table A-2 Properties of Food Production Areas by Land Use

Land Use id	Area type	Irrigation	Moisture source	Fallow time (yrs) (n)	Yield with normal moisture (kcal/ha)	Production cost (person-days/ha)
0	Not usable	none	na	na	0	na
1	High Pasture	none	Rainfall	na	160 (a)	6 (f)
2	Low Pasture	none	Rainfall	2	160 (h)	6 (f)
3	Un-raised Fields	cocha ditches	Ground water and rainfall	7	1000 (b)	80 (m)
4	Un-raised Fields	aqueducts	Springs, river and rainfall	7	4000 (b)	160 (m)
5	Raised Fields	canals	Groundwater and rainfall	4	10,000 (c)	270 (j)
6	Terraces	none	Rainfall and runoff	7	3900 (e)	284 (k)
7	Irrigated Terraces	aqueducts	Springs, rainfall, and runoff	7	7800 (d)	426 (l)

- (a) Graffam 1992, Calculated from reported 35 kg of meat per llama, 1420 cal/kg, 10% of herds slaughtered annually and estimated basin-wide population of 1,000,000 herd animals during Late Intermediate period.
- (b) Erickson and Candler 1989, p 240: modern Puno potato yield: 1000-4000 kg/ha (1 to 4 metric tons reported; 1 kcal per 1 kg of potatoes).
- (c) Erickson and Candler 1989, p 240: 10 metric-tons/ha = 10,000 kg/ha = 10,000 kcal/ha (1 kcal/kg potato); based on planting plus canal areas.
- (d) Treacy 1989, (p 222, Table 3) potato yield from modern reconstructed irrigated terraces in Peru (17,206 kg/ha * 0.9kcal/kg * 50% arable = 7743 round to 7800).
- (e) Assume rain fed terrace has half the yield per ha of irrigated terrace.
- (f) Graffam 1992, One herder can tend 60 animals; animals per ha calculated same as for Yield in (a).
- (g) Graffam 1992, expense/capital ratio for raised fields = 40,800/43,700 (10 yr amortize) or about 1.0.
- (h) Erickson and Candler 1989, p 243: modern day native vegetation of pampas supports one sheep per ha; assume comparable to yield for llamas in high pasture as in note (a).
- (i) Graffam 1992, p. 889, 2 herders for 60 animals; area per animal imputed same as for yield.
- (j) Erickson and Candler 1989 p.239-240; 270 p-days/ha (at 5 hrs/day) annually including rebuilding raised fields every 10 years.
- (k) Treacy 1989, p 221: reported 1416 worker-days (at 5 hrs/day) to construct 1 ha of terraces, not including aqueducts; assume amortized uniformly over 10 years = 142 p-days/ha; assume expense/capital ratio of 1.0 (same as for raised fields (g)), so total annual cost $2 * 142 = 284$ p-days/ha.
- (l) To include aqueduct cost, increase by 100% the amortized capital cost of rain fed terraces: $142 * 2 = 284$ p-days/ha ; assume expense costs same as for rain fed terraces, so total annual cost = $284 + 142 = 426$ p-days/ha.
- (m) Estimated based on degree of similarity to labor requirements for terraces.
- (n) Bandy (2005) used calculations similar to those performed here that included fallow times (although different values) to compare labor requirements and crop yields of raised field and dry land agriculture.

Appendix B: Agent Rules and Properties

Rule dependencies

There are five types of dependencies, or linkage, between the various agent rules that are represented in Figure B-1: Population, Agriculture, Ecology, Competition, and Trade

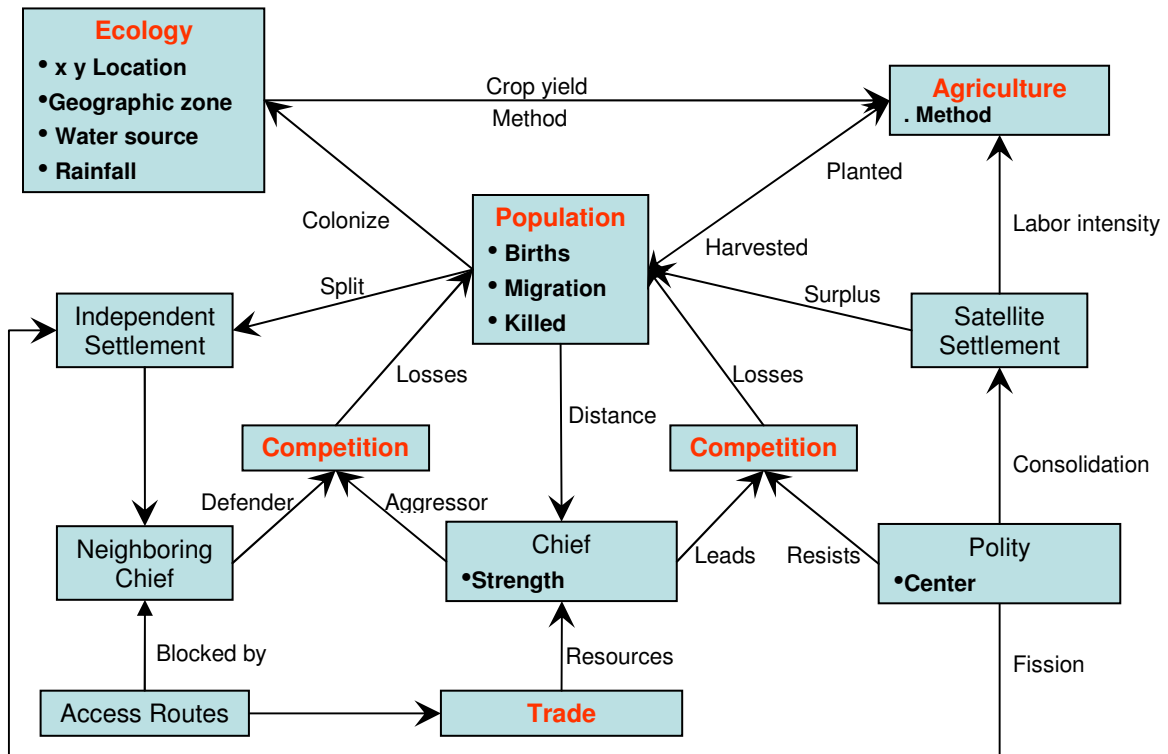


Figure B-1: Model Rule Dependencies

At the center is Population which impacts and is impacted by the other model elements. Population levels of each Settlement are dependent on the area and yield of crops planted which is determined by the agricultural land use, the Ecology constraints on crop yield at that location, and the labor intensity applied. Population also drives the extent to which a Settlement colonizes adjacent Patches for crops and spawns new Settlements.

In the model Polities fission or are consolidated as a result of Competition. A chief's strength is key to determining the political outcome of Competition with neighboring Chiefs and with internal factions of his own Polity. A Chief's strength is dependent primarily on the Population in his Polity, the distance from his competitor, and on his access to commodities needed for Trade goods. One Chief can block another Chief's access.

Stochastic Elements

A number of the dependencies between rules outlined above are stochastic in nature. It would not be realistic or practical to model all relationships deterministically:

1. Springs are randomly assigned to Patches at the same time that its deterministic properties are initialized. Assignment is based on the estimated probability of occurrence within each elevation zone. That's because there is no practical way to determine the actual location of each fresh water spring throughout the basin.
2. Initial Settlement population is set as a uniformly distributed random number within upper and lower bounds.
3. When a new Settlement is spawned its site is chosen randomly from among all suitable Patches within a set range of the parent Settlement. The same approach is used when a Settlement chooses a Patch to colonize for planting additional crops.
4. The outcome of a Chief's attempt to consolidate the Settlements of a neighboring Chief with his own is stochastically determined based on his strength compared to that of his competitor. Say this comparison predicts a 70% chance of success. The actual outcome on the current step will be randomly selected from a population composed of 70% wins and 30% losses.
5. Each Settlement in a complex Polity will attempt to secede with a fixed probability of occurrence during a single step. The outcome of a secession attempt will be determined in the same manner as an attempted consolidation, i.e. at random with the probability of success based on the strength ratio of the secessionist Settlement to that of the Polity's Chief. See Polity below for details about the rules for consolidation and fission.

Execution Order

During each simulation step, rules are applied in the following order of sub-steps.

1. Chiefs of complex Polities consider moving their Center, based on Polity structure and population at the end of the previous step.
2. Settlements plant crops or establish herds based on population and available land at the end of the previous step.
3. Settlements harvest food.
4. Chiefs take their share of the harvest.
5. Settlements consider (in the order listed): colonizing more Patches, spawning a new Settlement, abandoning the Settlement, or seceding from a complex Polity. One or none of these actions is applied.
6. Chiefs consider starting a Conflict.
7. Determine outcomes of Conflicts started in sub-step 6 above.
8. Determine Peoples' population changes from net births, migration, and Conflict losses.
9. Apply Peoples' population changes from sub-step 8 above.
10. Change structure (consolidate or fission) of Polities involved in Conflicts during this step according to outcomes from sub-step 7 above.

It is important to note two things about this sequence: 1) the rules are applied for all the agents of the class indicated before going to the next sub-step, and 2) applying actions

that can change the population or Polity structure are separate from determining what action to apply. The net effect is that actions are applied simultaneously to all agents during each step. This avoids situations such as a Conflict outcome based on the state of one Chief's Polity from the current step and the other's state from the previous step.

The Execution Order above can be thought of as the God's-eye-view during each time step of the simulation (although its linear structure is primarily an artifact of sequential computing rather than an essential feature of the model). In contrast, the sets of detailed rules that follow offer four very different local views as experienced simultaneously by each type of agent: Settlement, People, Chief, and Polity.

Settlement

Initial Conditions

All Settlements are initialized at the start of each run as simple Polities with populations randomly distributed between 20 and 60 people. Settlements are randomly assigned over a set of acceptable early sites, Patches through which a river runs and are not swampy. Lakeshore Patches with a fresh water spring also are acceptable. Sites must be a minimum distance apart. For locations receiving more than 880 mm/yr of rain this minimum spacing is 3 km, and for those receiving less rain the closest spacing is 1.5 times that, or 4.5 km. The assumption is that Settlements with more rain required less surrounding area to support the same number of people. The 880 mm/yr threshold corresponds to the north-south boundary defined for data collection, which was chosen as the north-south midpoint of the lake. About the same numbers of Settlements are initially sited above as below this boundary.

Expand/Abandon

A Settlement splits when population > 150 if a suitable site is available at least 3 km from nearest existing Settlement. This minimum spacing is 7 km for Settlements in the Puna geographic zone. Population splits evenly between original and new Settlement. Original Settlement retains control of current Patches. New Settlement colonizes new Patches as needed.

A Settlement colonizes a new Patch when its current Patches are 90% planted and if suitable land is available. Suitable means adjacent to current Patches and no more than 3 km from the Settlement (5 km if part of complex Polity, 7 km in Puna geographic zone.)

A Settlement is abandoned when population falls below 5 people. See People, Migration to Opportunity below.

Resist Leadership

A satellite Settlement that cannot split or colonize will attempt to secede from its Polity with a fixed probability (parameter Probability Resist, Table 2). A secessionist Settlement will establish a rebel Chief and the harvest will not be shared with the Polity's Chief. The Polity's Chief will then attempt to reassert his authority over the rebels – see Chief below.

Plant/Harvest crops

An independent Settlement plants only the area needed to feed the current population including Chief's share + 10% (given crop yield and labor productivity). So the area planted is limited by need, labor, and non-fallow land available in the current year for each Settlement.

A Settlement that is part of a complex Polity will intensify agriculture by maximizing area planted, colonizing additional Patches limited only by labor and non-fallow land but not constrained by subsistence need.

The area planted each year is a fraction of land available because fields must lay fallow some number years between each year planted, i.e., non-fallow area = land available / N+1 where N is the fallow time for each type of land use as listed in Table A-2.

Crop yield (kcal/ha) is determined by the agricultural land use and rainfall – see Table A-2. Labor productivity is a function of agricultural land use; see Table A-2. In turn, land use is determined by political complexity of the Settlement, geographic zone, and availability of water as given in Table A-1.

Maximum area planted by Settlement (ha) =
 Population * % producers * Work days per year / Production costs (p-days/ha)

where:

% producers = proportion of population producing food (see People below)

Work days per year = maximum work days per person each year (model parameter, Table 2)

Production costs = labor to produce normal yield from one ha (Table A-2).

People

The total change in People's population each step is the sum of the following four components:

1) Net Births (births less natural deaths)

Population grows at a set annual rate each step when there is no food shortage that step (parameter Birth rate in Table 2). The rate becomes negative and doubles if there is a shortage, which occurs when harvest per capita falls below a set annual level (2200 cal/day * 365 days/yr). Transitions between a positive and negative rate are smoothed by a 5 year running average.

2) Migration to Opportunity

People migrate at a fixed rate (Migrate Rate parameter, Table 2) to the largest nearby Settlement that offers opportunity. A Settlement is nearby if it's no more than 2 km beyond the minimum inter-Settlement spacing (see Settlements Expand/Abandon above). Opportunity exists at the largest nearby Settlement if it is a Center or its population is larger than this People's. People can migrate across Polity borders.

When population falls below 1% of the largest nearby Settlement's population, out-migration rate accelerates to 50%. (This causes the Settlement to be abandoned after a few steps – see Settlement Expand/Abandon above)

3) Migration from Need

When birth rate goes negative, due to a poor harvest or loss of Chief's share from satellite Settlements, People will migrate to a Settlement within 15 km that has the most food per person, provided this amount is greater than that of People's current Settlement. People stay put if there is no such Settlement. The rate of migration from need becomes 100 times the rate for opportunity.

4) Conflict losses

Population will be decreased when a Settlement's Chief is involved in a competitive Conflict, either external or internal – see Chief below. Chief's losses will be proportional to the strength of the opposing Chief. (Cederman (1997) refers to this relationship as the rule of war). However a Chief's losses will be limited to half his current strength. The loss proportion applied depends on: 1) the Kill Ratio parameter (KRP); 2) whether the Chief in question was the aggressor or defender; and 3) whether he was the winner or loser of the Conflict. Thus:

	Winner of Conflict:	
	<u>Aggressor</u>	<u>Defender</u>
Losses:		
<u>Aggressor</u>	0.25*KRP	0.5*KRP
<u>Defender</u>	1.0*KRP	0.25*KRP

Food Producers

The proportion of the population that farms and/or herds depends on the political complexity of the Settlement: 70% for independent and satellite Settlements, and 25% in center Settlements. These numbers are based on two assumptions: 1) in all Settlements 30% of People's population is not productive (too young, too old, etc.), thus is excluded from the labor force; 2) there is a high degree of labor specialization in centers which further lowers the percentage of food producers.

Chief

Share in harvest

Chief receives a fixed share of the harvest from each Settlement in his Polity currently not resisting his leadership (see Settlement Resists Leadership above), 10% for simple Polities and 25% for complex ones.

Strength

A Chief's strength is the sum of the populations from each of his Settlements, adjusted

for the distance from his Polity's Center. The strength a Chief can draw from each of his Settlements decreases by a fixed rate (Fad Decay Rate parameter, Table 2) compounded for each km between his Center and that of the Settlement, i.e.,

Strength at Center = sum all Settlements (Strength of Settlement i at distance d_i from Center $/(1+\text{Fad rate})^{d_i}$).

The same rate of decrease applies to determine the force a Chief can apply against a competitor at a distance, either internal or external, i.e.,

Strength at distance d km = Strength at Center $/(1+\text{Fad rate})^d$.

The rate at which effective force decreases with distance can be lowered by the Chief's access to trade, see Chief's Trade Access below.

Trade Access

Figure B-2 shows the impact of trade access on Chief's strength-at-distance. The more access a Chief has to trade commodities the less his strength will be diminished with distance. The highest level of access, 1.0, occurs when a Chief can reach mountain passes to the east and to the west, one or more of each type. Access to only east or west will have proportionally less value, 0.65 and 0.35 respectively, and is 0.0 for no access.

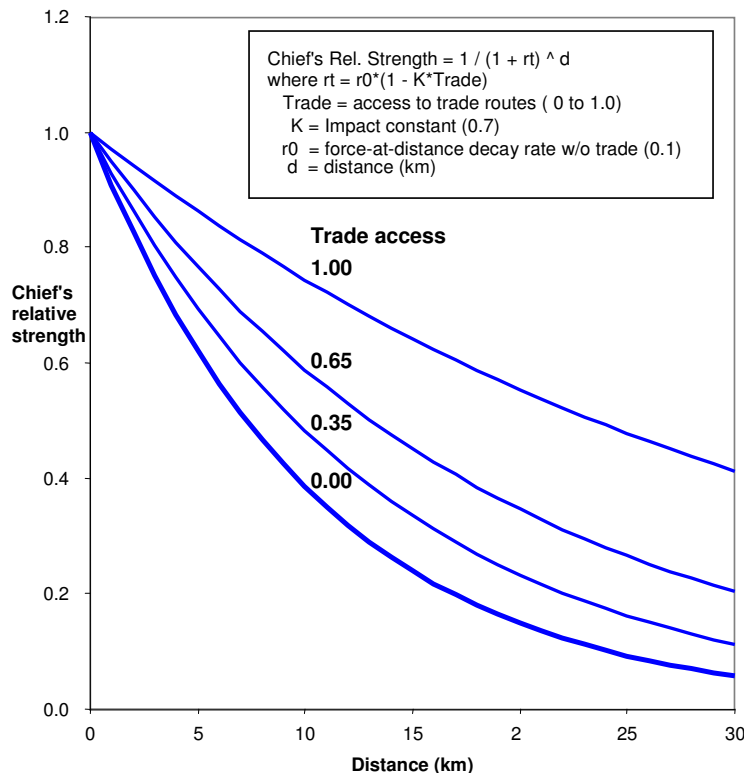


Figure B-2: Impact of trade access on Chief's strength-at-distance

To have access, the Chief's Center must meet the following conditions for at least one pass: 1) be within a fixed range of the pass (80 km in the current model); and 2) there must be an unobstructed path from his Center to the pass in question. A path can be obstructed by the lake or by another Chief's Polity, which can change from step-to-step.

External Competition

When the proportion of his Polity's population that can still expand falls below 10% the Chief will consider taking over an adjacent Polity. Expand means that a Settlement can either split to form a new Settlement or can colonize additional Patches. However a Chief in the Puna geographic zone (see Table A-1) will never consider a takeover (assumes resources too limited).

Chief will estimate the probability he can prevail over each adjacent Polity based on the ratio of his effective force to that of his neighboring Chief. The weakest one will be targeted for takeover.

If not already in conflict, the Chief will attempt a takeover of the targeted Polity if the estimated chances of losing do not exceed the Chief's risk threshold (Risk Threshold parameter, Table 2).

The takeover conflict will occur at the targeted Chief's Center, so his strength to resist will not be decreased by distance, as will the force applied by his opponent. The targeted Chief's strength also will be multiplied by a fixed factor to reflect his defensive advantage (Defense Multiplier parameter, Table 2).

A successful takeover will occur stochastically, its probability of occurrence the same as the Chief's estimate to prevail that was based the ratio of his strength to that of his opponent.

Internal Competition

The Chief will apply a fixed proportion of his strength (parameter Stop Resist Strength, Table 2) to reassert his authority over a secessionist Settlement in his Polity (the balance used to maintain order in his remaining Settlements.) The winner of this conflict will be determined stochastically the same as for external competition.

Move Center

After a Chief has taken over another Polity he will consider moving the center to any of his Settlements that would increase his total strength (adjusted for distance to all other Settlements in his Polity).

Polity

Consolidation

When a Polity's Chief prevails in an external conflict (see Chief's Internal Competition above), the opposing Chief is removed and all his Settlements become satellites of the winner. Otherwise the structure of both Polities is unchanged.

Fission

If the Polity's Chief prevails in an internal conflict (see Chief's Internal Competition above) the rebel Chief will be removed and the secessionist Settlement remains part of the original Polity. However if the rebels win the Polity will fission, meaning that all its Settlements become simple Polities each with an independent Chief.

Model limitations

To understand a model, John Holland suggested, it is important to know what has been left out (1995, p 137). Table B-1 lists design choices made for the sake of simplicity that may limit the realism of the current model. However the software was structured to allow many of the excluded features to be easily added in the future as needed for specific simulation experiments.

Table B-1: Model Limitations

Property or action	Modeling considered but not included	Simplification used in current model
Terrain elevation model	Elevation in meters for each cell in grided digital elevation model (DEM)	Four elevation zones – used to assign agricultural land use potential
Geographic data	Acquire, register, and combine multiple GIS layers (shoreline, hydrology, elevation, etc) using special-purpose software; model accesses external GIS database	Single thematic map image in gif format manually traced, using Photoshop, from scanned contour maps; input and initialization via standard Java image methods
Hydrology	Hydrologic GIS layers – edit out minor rivers and streams; see Geographic data above	Manually trace lake, major rivers, and swampy areas from contour maps; randomly locate springs as function of elevation zone
Terrain slope	Based on DEM, exclude areas where terrain too steep for Settlement sites or pasturing	Not modeled: excluded areas above 4500 meter elevation
Soil moisture	Multi-year ground water storage model – function of elevation, soil type, and stochastic annual rainfall	Not modeled – see Crop yield and Rainfall
Crop yield	Non-linear function of time-varying soil moisture	Increasing linear function of rainfall
Rainfall	Precipitation map from modern rain gauge data (NOAA 2003); add annual stochastic variation; long-term wet and dry periods based on cores from lake bottom and surrounding glaciers (Binford 1997)	Average level with no variation over time; linear north-south gradient
Lake level and shoreline	Lake shoreline retreats in shallow areas during dry periods (Bandy 2004b)	Fixed shoreline and lake level
Agricultural land use	Multiple land uses: function of wet/dry periods as well as political complexity. (Moseley 2001)	One of two uses for each elevation zone, one for simple Polities and one for complex (can be same use for both)

Property or action	Modeling considered but not included	Simplification used in current model
Size of Settlement's residential area	Annex all or part of adjacent agricultural Patches for residential use when density exceeds some limit.	Residential area contained in single 1.5 km x 1.5 km Patch , the original Settlement site
Locations of defensible high ground	Settlements on high ground have enhanced defense multiplier during conflicts (Stanish 2003); identify candidates for high ground locations with DEM software algorithm.	Input as property of Patches but not used in current model of conflicts; located by visual inspection of terrain maps – traced areas on thematic map image for input
Probability that satellite Settlement resists Chief	Probability for each Settlement is function of its food shortfall history	Probability constant over time and Settlements
Chief's strength during competition	Two step decay – low rate inside own Polity to reach staging area at boarder and high rate after crossing into opponent's Polity.	Constant decay rate of Chief's strength at a distance
Chief's selection of consolidation target and decision to start conflict.	As well as current strength each Chief has a dominance factor that mediates conflict decisions; the factor increases for each past win and decreases with losses. (Hemelrijk 2000)	Decisions based only on current strength of neighboring Chiefs' compared to his own
Polity political complexity	Multi-level hierarchies with sub-Chiefs (Stanish 2002)	Two level hierarchy: simple (1 Settlement) or complex (>1 Settlement)
Polity structural changes due to secession	Only resisting Settlement secedes plus others that would otherwise create non-contiguous enclaves (Cederman 1997)	All satellite Settlements become independent
Polity structural changes due to consolidation	One Chief takes over only targeted Settlement from neighboring Chief (Cederman 1997)	One Chief takes over all Settlements of neighboring Chief

Appendix C: Definition of Match Criteria and Measures

Table C-1: Pattern Match Criteria Definitions

	Criterion and time (a)(b)	Relational quantity based on measures	Measures (sampled every 100 steps)	Time step or range	Lower (Upper) limit
1	Final Population Close to Expected	Percent above or below the expected population of 500,000 in AD 900	pop	3400	(40%)
2	Early Population Close to Expected	Percent above or below expected population of 38,000 in 1500 BC (c)	pop	1000	(40%)
3	N Polity Larger than S Before	Number of steps largest N Polity was <u>greater than</u> largest S Polity (d)(f)	popPol N, S	2100–2700	300
4	S Polity Twice Size of N After	Number of steps largest S Polity was <u>two times or more greater than</u> largest N Polity (e)(f)	popPol N, S	2800–3400	300
5	N Polity Stable Before	Number of steps <u>with no change</u> in Chief of largest N Polity (e)(f)	chief N	2100–2700	200
6	N Polity Unstable After	Number of steps <u>with change</u> in Chief of largest N Polity (f)	chief N	2100–2700	300
7	S Polity Stable After	Number of steps with <u>no change</u> in Chief of largest S Polity (e)(f)	chief S	2800–3400	300
8	Center at N End of Lake	Average distance in km of largest N Polity's Center from ancient site of <u>Pucara</u>	dref N	2100–2700	(70 km)
9	Center at S End of Lake	Average distance in km of largest S Polity's Center from ancient site of <u>Tiwanaku</u>	dref S	2800–3400	(70 km)
10	Basin Consolidated After	Average percent of total basin population accounted for by combination of largest N and S Polities	pop popPol N, S	3200–3400	30%
11	Early S Polity Not Concentrated	Surrounding the Center of the largest S Polity, the peak percentage of Settlements during the <u>Early</u> period, whose population was <u>above</u> the level predicted by Zipf's rule	zPop S	1500–1800	60%
12	Late S Polity Very Concentrated	Surrounding the Center of the largest S Polity, the peak percentage of Settlements during the <u>Late</u> period, whose population was <u>below</u> the level predicted by Zipf's rule	zPop S	3400–3600	100%

Notes for Table C-1

- (a) Before/After periods = 700 years preceding/following expected collapse of largest N Polity in AD 200.
- (b) Early S/Late S periods = 300 to 400 years during the Formative/Classic Tiwanaku periods.
- (c) Early basin population in 1500 BC that would grow to the expected level of 500,000 in AD 900 given an average annual growth rate of 0.1%.
- (d) Polity size is based on total population of all its Settlements.
- (e) Assumed Tiwanaku would have been much more integrated during the After period than Pucara was Before. Accordingly S Polity limits After are more stringent than those on N Polity Before, both for relative size (2 times larger) and stability (300 vs. 200 steps).
- (f) Sampled value of measure taken as typical of values between samples

Each match criterion in Table C-1 consists of: 1) one or more numerical measures, listed in Table C-2, that were sampled every 100 time steps during simulations, 2) a relational quantity calculated from the measure(s), 3) an upper or lower limit, and 4) a time step or step range during which the relational quantity is tested against the limit.

Table C-2: Selected Measures Collected during Simulation Runs (sampled every 100 steps)

Symbol	Extent (a)(b)	Definition
pop	Basin	Total Basin Population
dref	N, S	Distance in km from the Center of dominant North (South) Polity to ancient site of Pucara (Tiwanaku)
popPol	N, S	Population of the dominant North (South) Polity
chief	N, S	Unique Id number for the Chief of the dominant North (South) Polity. Different id in adjacent samples indicates a change in Chiefs
zPop	N, S	Net percent of Settlement's within 15 km of dominant N(S) Polity Center whose population is higher (lower) than that predicted by Zipf's rule for the size rank of each Settlement, where net percent = $100\% * (\text{count greater than Zipf} - \text{count less than Zipf}) / \text{total count}$

- (a) Distance of North Centers from the grid origin is less than 145 grid coordinate units. For South Centers this distance is greater than or equal 145. Using this distance assigns about the same number of initial Settlements and population to North and to South.
- (b) Dominant North Polity has the largest population (sum of all its Settlement populations) of all North Polities: same is true for dominant South Polity

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